Problem Set 6

Due: In class on Wednesday, March 31. Starred problems are optional.

Problem 6-1. A comparison network with n inputs and r comparators can be described as a list $(i_1, j_1), (i_2, j_2), \ldots, (i_r, j_r)$, where $1 \le i_q, j_q \le n$ for $q = 1, 2, \ldots, r$. The list represents a topological sort of the comparators, where each ordered pair (i_q, j_q) stands for a comparison between the elements on lines i_q and j_q, with the minimum being output on line i_q and the maximum on line j_q.

(a) Give an efficient algorithm for determining the depth of a comparison network so described. (*Hint:* You can't just count the maximum number of comparators incident on a line.)

Define a comparison network as ***standard*** if $i_q < j_q$ for $q = 1, 2, \ldots, r$.

(b) Give an algorithm to convert a comparison network with n inputs and r comparators into an equivalent standard comparison network with n inputs and r comparators.

(c) Prove or give a counterexample: For any standard sorting network, if a comparator (i, j), where $i < j$, is added anywhere in the network, the network continues to sort.

Problem 6-2. A comparison network is a ***transposition network*** if each comparator connects adjacent lines. Intuitively, a transposition network represents the action over time of a linear systolic array making oblivious comparison exchanges between adjacent array elements.

(a) Show that if a transposition network with n inputs actually sorts, then it has $\Omega(n^2)$ comparators.

(b) Prove that a transposition network with n inputs is a sorting network if and only if it sorts the sequence $\langle n, n-1, \ldots, 1 \rangle$.

Problem 6-3. Give an algorithm for sorting N^3 elements on an $N \times N \times N$ mesh in $O(N)$ time.