Problem Set 3

Due: In class on Wednesday, February 25. Starred problems are optional.

Problem 3-1. Show how to divide an N-bit number by the constant 3 in $O(\lg N)$ time using $O(N)$ hardware.

Problem 3-2. A spanning tree of a graph is the tree formed by a subset of the edges of the graph such that all vertices in the graph are contained in the tree. A minimum spanning tree of an edge-weighted graph is a spanning tree of minimum weight, where the weight of the tree is defined to be the sum of the weights of the edges in the tree. Argue that a minimum spanning tree for a graph with N vertices can be computed in $O(N)$ time on an $N \times N$ mesh. You may assume that the graph is given as an $N \times N$ adjacency matrix of edge weights, and that all edge weights are distinct.

Problem 3-3. * Consider the division circuit based on the Chinese Remainder Theorem which was presented in class. The circuit computes the first N bits of $1/x$, where x is an N-bit number, in $O(\lg N)$ time. As presented, how much hardware does this circuit require asymptotically? Try to reduce the asymptotic hardware requirements without giving up the $O(\lg N)$ time. You may either restrict yourself to combinational implementations or use any fixed-connection network.