Problem Set 1

Due: In class on Wednesday, February 11.
Starred problems are optional.

Problem 1-1. Design an efficient algorithm to sort N numbers in $\Theta(N)$ time using $\Theta(\lg N)$ processors, where each processor can hold an unlimited amount of data. You may use any fixed-connection network you wish. (*Hint:* Refresh your memory on serial sorting algorithms.)

Problem 1-2. Argue persuasively that any P-processor fixed-connection network with at most d neighbors per processor has diameter $\Omega(\log_d P)$.

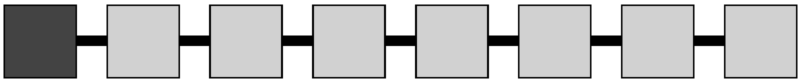

Figure 1

Problem 1-3. * A firing squad has been formed as a one-dimensional array of n soldiers, as shown in Figure 1. Each soldier is a finite automaton whose next state is dependent only on its current state and the current states of its two neighbors. There is no global communication, but the state transitions of the soldiers are synchronized by a global clock. The goal is for all the soldiers to shoot their guns at the same time.

All the soldiers begin in the same QUIESCENT state, and all soldiers are identical, except for first and last soldiers. The last soldier knows he or she is the last soldier. The first soldier is the general, who, at the start of the computation, enters the FIRE-WHEN-READY state. The goal is for all soldiers to enter the SHOOT state *on exactly the same clock tick*.

Give a design for the soldier automaton so that all soldiers fire at the same time. Your algorithm should run in $\Theta(N)$ time.