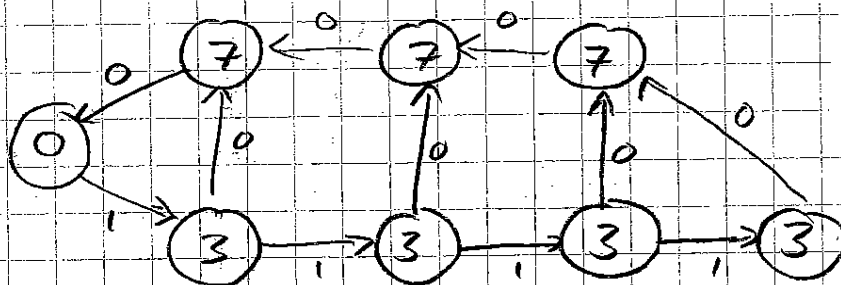


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L8.1Clock periodCircuit  $G = (V, E, d, w)$ 

- $V$  = comb elements
- $E$  = interconnections
- $d$  = propagation delay of comb elem.
- $w$  = latch (register) count on edge.

Ex.

Def. Path weight  $p = v_0 \xrightarrow{e_0} v_1 \xrightarrow{e_1} \dots \xrightarrow{e_{k-1}} v_k$  is

$$w(p) = \sum_{i=0}^{k-1} w(e_i)$$
Path delay

$$d(p) = \sum_{i=0}^k d(v_i)$$

&lt;&lt;Note fencepost.&gt;&gt;

Legality constraints

- D1.  $d(v) \geq 0 \quad \forall v \in V$   
 W1.  $w(e) \geq 0 \quad \forall e \in E$   
 W2. In any directed cycle  $p$  of  $G$ ,  $\exists$  edge  $e$  such that  $w(e) \geq 1$ .

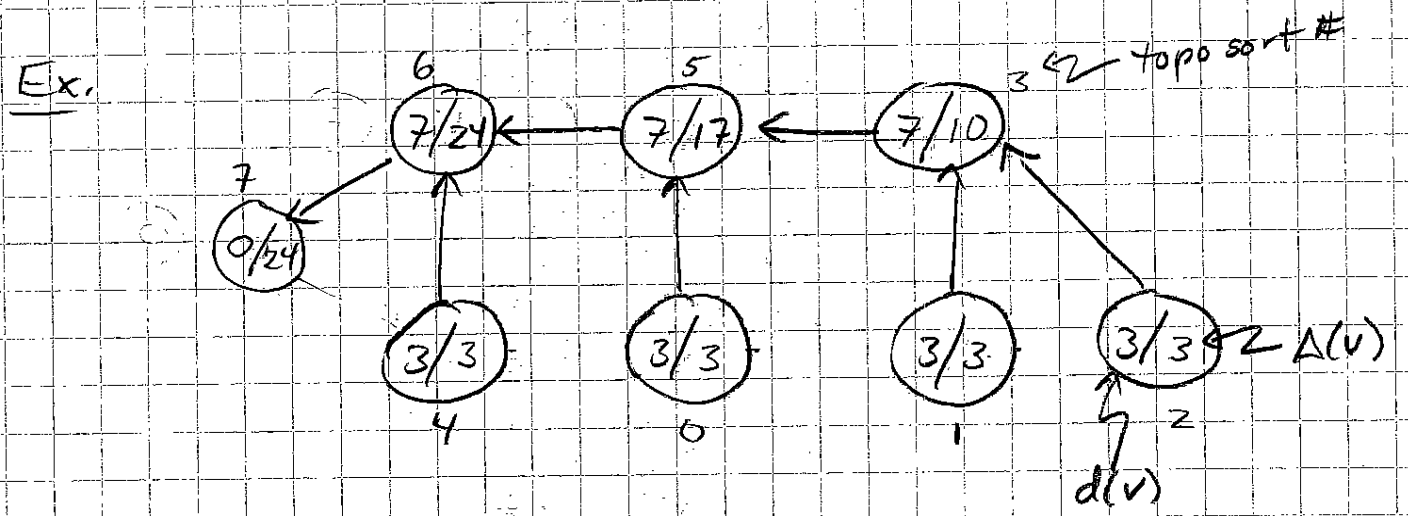
W2: Synchronous circuits - no unlocked state.

Def. The clock period of  $G$  is

$$\Phi(G) = \max_P \{d(p) : w(p) = 0\}. \quad (\text{Ex. } \Phi(G) = 24)$$

# Computing clock period

- Let  $G_0$  be subgraph of  $G$  with only 0-wt edges.  $G_0$  is acyclic (bv WZ).
- Topologically sort vertices of  $G_0$ .
- Scan through vertices in top sort order.
  - If no incoming edge to  $v$ , set  $\Delta(v) = d(v)$ .
  - Otherwise,  $\Delta(v) = d(v) + \max_{u \rightarrow v \text{ in } G_0} \{\Delta(u)\}$ .
- $\Phi(G) = \max_{v \in V} \Delta(v)$ . <<Longest path in acyclic graph>>

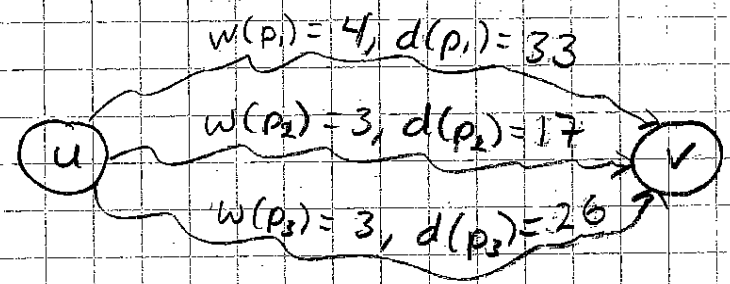


Running time:  $\Theta(V+E) = \Theta(E)$  if  $G$  connected.

## Retiming to minimize clock period

Recall: For path  $u \xrightarrow{p} v$ ,  
 $w_r(p) = w(p) - r(u) + r(v)$ .

What paths might realize  $\Phi(G_r)$ ?



Retiming reweights each path from  $u$  to  $v$  by same amount.

- Ignore  $p_1$ , since  $w_r(p_1) \geq 1 \forall$  legal retimings.
- Ignore  $p_2$ , since  $w_r(p_2) = 0 \Rightarrow w(p_2) = 0$  and  $d(p_2) > d(p_3)$ .

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Define  $W(u, v) = \min\{w(p) : u \xrightarrow{p} v\}$ .

$u \xrightarrow{p} v$  is a critical path if  $w(p) = W(u, v)$ .

Define  $D(u, v) = \max\{d(p) : u \xrightarrow{p} v \text{ and } w(p) = W(u, v)\}$ .

Lemma. For any  $c > 0$ ,  $\Phi(G) \leq c$  iff whenever  $D(u, v) > c$ , we have  $W(u, v) \geq 1$ .

Proof. ( $\Rightarrow$ ) Sup.  $\Phi(G) \leq c$ , and let  $u, v \in V$  &  $D(u, v) > c$ .  
Thus,  $\exists u \xrightarrow{p} v$  &  $d(p) > c$  and  $w(p) = W(u, v)$ .  
Must have  $W(u, v) \geq 1$ , or else  $p$  would be a  $0$ -wt path with  $d(p) > c$ . Contradiction.

( $\Leftarrow$ ) Sup.  $\forall u, v \in V$ ,  $D(u, v) > c \Rightarrow W(u, v) \geq 1$ .  
Let  $u \xrightarrow{p} v$  be  $0$ -wt path. Then,  $W(u, v) = w(p) = 0$ ,  
which implies  $d(p) \leq D(u, v) \leq c$ .  $\square$

### Computing $W$ and $D$

1. Create graph with edge weights of  $u \xrightarrow{e} v$  of  $\langle w(e), -d(u) \rangle$
2. All-pairs sh. paths (Floyd-Washall  $O(V^3)$ )  
Johnson + Fib heaps  $O(VE + V^2 \lg V)$
3. For weight  $\langle x, y \rangle$  betw.  $u$  and  $v$ ,  
 $W(u, v) = x$   
 $D(u, v) = d(v) - y$