Projects

Permuting data on parallel disks

Disk access time $\approx 10^{-2}$ sec
Data transfer rate $\approx 10^{-6}$ words/sec

We want to do as few disk accesses as possible.

Convenient engineering assumption:
Disk is broken into large fixed-size blocks, e.g., of 1000 words.

\[ \frac{N}{PB} \]

Computer memory holds $M$ data records total.
Assume $M \gg PB$.

Permuting disk blocks

* Off-line (perm fixed in advance)

\[ n = \frac{N}{P} \text{ tracks} \]

\[ \frac{P \text{ disks}}{N \text{ data records}} \]

Block = $B$ data record s

$\equiv$ parallel I/O's to read all data = $N/PB$.

Theorem: Can permute with $O(N/P)$ parallel I/O's (not in place)
Conflict graph

Source disk

Dest disk

All degrees

\[ v = N/p. \]

Fact: Any d-regular bipartite multigraph can be edge-colored with d colors. (Color = step at which block is moved.)


Hall's Thm.
For \( A \in V_1 \), let \( N(A) \subseteq V_2 \) be the set of neighbors of \( A \).
Then, a perfect matching exists if \( |N(A)| \geq |A| \cup A \).

Proof. Let \( f \) be maxflow, \( f(s, T) \) for some cut \((s, T)\) by maxflow-mincut thm.
Let \( A = S \cap V_1 \). Since edges from \( V \) to \( V_2 \) have \( \infty \) capacity, \( N(A) \subseteq S \). Also, \( N(V_1, A) \subseteq T \).

\[ c(s, T) \geq |V, A| + |N(A)| \]
\[ \geq |V, A| + |A| \]
Sorting (Vitter et al.)
\[ O\left(\frac{N}{PB}, \frac{\log(N/M)}{\log(M/B)}\right) \text{ IO's} \]

Idea: Internal sort \( M \) records at a time into \( N/M \) runs.
Merge runs.

Would like to merge \( M/B \) runs at a time.

\[ \frac{N}{PB} \quad \frac{N}{PB} \quad \frac{M}{B} \begin{array}{c} \uparrow \end{array} \downarrow \begin{array}{c} \log_{M/B}(N/M) \end{array} \]

Total \( N \) \( \frac{\log(N/M)}{\log(M/B)} \) \( \frac{N}{PB} \) \( \frac{\log(N/M)}{\log(M/B)} \) leaves

Problem: Can only read 1 block/run
- All of one run may be smaller than others.

Solution:
- Merge \( \sqrt{M/B} \) runs at a time (Depth of rec. doubled)
- Keep track of which blocks to read next in table
  - "Sloppy" merge. Clean up with \( O(N/PB) \) IO's.