**6.895 Theory of Parallel Systems** Due: Wednesday, November 12th

# Problems 7–11

**Problem 7.** In this problem we develop an alternative analysis for work stealing. We use a potential-function argument rather than a delay-sequence argument. The advantage is that we need not use the augmented DAG $G'$, but instead we use more mathematics. Let $d(u)$ be the depth of $u$ in $G$. Define the *weight* of thread $u$ to be

$$w(u) = T_\infty - d(u) \ .$$

Define the *potential* of thread $u$ to be

$$\Phi(u) = 3^{w(u)} \ .$$

Define the potential of a processor $p$ to be the sum of the potentials of the nodes on $p$ (i.e., in its deque or executing). That is:

$$\Phi(p) = \sum_{\text{u on p}} \Phi(u) \ .$$

Define the potential of the system to be the sum of the potentials of the processors. That is,

$$\Phi(S) = \sum_{i=1}^{p} \Phi(i) \ .$$

Now divide the execution into *rounds* as in class.

**Problem 7.a.** Show that a *constant fraction* of the potential on a processor is at the top of the deque. What fraction?

**Problem 7.b.** Show that there is a *constant probability* in each round that the potential decreases by at least a constant fraction. Calculate the constants.

**Problem 7.c.** Assume you have a coin that comes up heads with probability $p$. Show that with error probability exponentially small in $n$, if you flip $n$ times, there will be $\Theta(np)$ heads. Calculate constant.

**Problem 7.d.** Conclude that with probability at least $1 - \epsilon$, only $(T_\infty + \log(1/\epsilon))$ rounds (where $\epsilon$ is polynomial in $P$) are necessary before all threads are complete.

Hint: You will need linearity of expectation, 0/1 random variables, and Markov's Inequality. See CLRS.

**Problem 8.** For the Cilk scheduler, show that the *expected* number of steals is $O(T_\infty \cdot P)$.

**Problem 9.** Show that the MESI protocol, as presented in Lecture 13, guarantees sequential consistency.

**Problem 10.** (Extra Credit) Show that the directory based protocol is correct. Note that this involves more carefully specifying how the protocol works.

**Problem 11.** Recall from class the SPIN-BLOCK PROBLEM: in order to acquire some resource, you can either *spin*, which has cost proportional to how long you spin, or you can *block*, which has fixed cost $c$.

In class, we saw for the 1-Random-Choice Spin Block Problem that if we block at time $c/2$ with probability $p = 2/5$, then the competitive ratio is 1.8.

Suppose that we get to make one random choice at time $t = \beta \cdot c$ with probability $p$ to block. (The rest of the protocol must be determinisitc.) What are the optimal values of $\beta$ and $p$ that minimize the competitive ratio? What is the resulting competitive ratio?