Recall: (L18)

**Bounded 2-player Constraint Logic (ACL)**
- each edge is either white or black
- each edge can be reversed only once
- goal: each player has target edge & wins if they reverse it

- PSPACE-complete for planar constraint graphs with white AND, SPLIT, OR, CHOICE & VARIABLE vertex
- reduction from impartial game positive CNF SAT
- players take turns setting variables
- positive \(\Rightarrow\) white wants true, black wants false
- black can't win (edge irreversible)
- white wins \(\Leftrightarrow\) formula satisfied
- crossover gadget (only use of CHOICE)
- can make OR protected using free edge no constraint at degree-1 end
Amazons: [Walter Zamkauskas 1988]
- queens on chessboard
- move = queen move + queen shot
  destroy board position at queen-reaching location
- last player to move wins

- PSPACE-complete [Hearn 2005]
  - polynomial # moves: shot consumes board
  - reduction from Bounded 2CL
Konane [Hawaii - ancient Hawaiian Polynesians]
documented by Captain James Cook in 1778

- move = jump your piece over 1 or more opponent pieces in a straight line:
  \[ \bigcirc \rightarrow \bigcirc \rightarrow \bigcirc \rightarrow \bigcirc \]  
  \( \rightarrow \) remove captured opponent pieces
- last player to move wins

- PSPACE-complete [Hearn 2005]
  - polynomial # moves: move consumes \( \geq 1 \) piece
  - reduction from Bounded 2CL
  - conditional gadget for AND, SPLIT, shift:
    - can traverse input 2 \( \rightarrow \) output 2
      only after input 1 \( \rightarrow \) output 1 (else captured)
    - ignore output 1 \( \Rightarrow \) AND
    - prime input 2 \( \Rightarrow \) SPLIT
    - both \( \Rightarrow \) parity shift
Cross Purposes: [Michael Albert 2004]
- black stones = 1x1x2 towers
- white stones = fallen towers
- move = \( \bigcirc \rightarrow \times \bigcirc \) \( \text{(right)} \)
- vertical player can only move up/down
- horizontal player can only move left/right
- last player to move wins

- PSPACE-complete [Hearn 2005]
  - polynomial \# moves: move consumes black stone
  - reduction from Bounded 2CL
  - H forced to help V after variable settings
  - protected OR (\& free edge) to avoid second activation terminating leaving H w/o move

Stochastic games: [Papadimitriou - JCSS 1985]
- one player (of 2) plays randomly "nature"
- PSPACE-complete to win with probability \( > \frac{1}{2} \) (via amplification)
- SSAT: \( \exists x_1 : \exists x_2 : \exists x_3 : \exists x_4 : \cdots : \text{Pr} \{ F \} > \frac{1}{2} \)
- OPEN: real games?
Unbounded formula games: EXPTIME-complete

[Stockmeyer & Chandra - SICOMP 1979]

- start with arbitrary variable assignment
- can set variables to 0 or 1 many times (unlimited)
- all partizan: black & white variables (except t)

\[ G_1: \text{move} = \text{set all variables of your color} \]
\[ \text{lose if you satisfy your } 3 \text{DNF formula (2 of them)} \]
\[ \text{ie move must satisfy your } 3 \text{CNF formula} \]
\[ \text{equivalently: shared } 4 \text{DNF formula involving} \]
\[ \text{shared turn variable } t = \begin{cases} 0 & \text{if player 2} \\ 1 & \text{if player 1} \end{cases} \]

\[ G_2: \text{move} = \text{set one variable of your color} \]
\[ \text{(can pass by not changing it)} \]
\[ \text{win if you satisfy your } 12 \text{DNF formula (2 of them)} \]

\[ G_3: \text{move} = \text{flip one variable of your color} \]
\[ \text{(no pass)} \]
\[ \text{lose if you satisfy your } 12 \text{DNF formula (2 of them)} \]

\[ G_4: \text{move} = \text{set one variable of your color} \]
\[ \text{(can pass)} \]
\[ \text{win if you satisfy (common) } 13 \text{DNF formula} \]
\[ (G_5 = G_6 \text{ but without CNF constraint}) \]

\[ G_6: \text{move} = \text{set one variable of your color (can pass)} \]
\[ \text{player 1 wins if anyone satisfies (single) CNF formula} \]

Peek: stack of plates with holes; 1 fixed plate;

\[ (G_4) \]
black & white plates have 2 states, in & out
- move = manipulate one plate (can pass)
- win if hole all the way through
Membership in $\text{EXPTIME} = \text{APSPACE}$ [Chandra & Stockmeyer, Kozen - FOCS 1976]

- build set of "mate in $k$" states for $k = 0, 1, \ldots, c^n$
  \#moves \leq \#states

Unbounded graph games: $\text{EXPTIME}$-complete [Stockmeyer & Chandra - SICOMP 1979]

**HAM:**
- given simple undirected graph
- each edge black or white & in or out
- move = toggle in/out of an edge of your color
- player 1 wins if in edges form a Hamiltonian cycle (after any move)
- reduction from $G_6$

**Block:**
- given 3 graphs on the same vertex set
- each player has tokens of their color on some of the vertices (\leq 1 token per vertex)
- move = move 1 token of your color along a path in one of the 3 graphs such that target & intermediate vertices have no tokens
- player i wins if they get a token to a vertex $e_i$
- reduction from $G_3$
  - variable & clause gadget
Real games that are $\text{EXP\!TIME}$-complete: $\Rightarrow \notin P$

- Checkers [Robson - SIComp 1981]
  - reduction from $G_3$ where about to lose after every turn
  - initially players adjust kings between T/F
  - then player mounts an attack: move A or B forcing opponent to follow path, fork as desired
  - if all attack vars. set & no defense vars. set i.e. DNF clause satisfied then get $x$ free moves
  - with $x$ free moves can trigger outer spiral $\Rightarrow$ huge material advantage
  - then can form picket lines $>\text{size(\text{interior})}$ $\Rightarrow$ Win [Fraenkel, Garey, Johnson, Schaefer, Yesha - FOCS 1978]

- Chess [Fraenkel & Lichtenstein - JCTA 1981]
  - reduction from $G_3$

- Go with Japanese ko rule [Robson - IFIP 1983]
Unbounded 2CL:
- each edge is either white or black
- goal: each player has target edge & wins if they reverse it
- EXPTIME-complete even for planar graphs
  - reduction from G6
  - players flip variables
  - if formula satisfied: white (Player 1) will lock all variables & run formula
  - lock = reverse true or false edge
  - black must respond A (then B, C, D) to prevent white from fast win via F
    ⇒ black immobilized during locks
- black's slow win is 1 move longer than formula satisfaction ⇒ white can't flip its variables after any locking (no time)
- white slower win prevents black from flipping A early, e.g. instead of flipping a variable
- formula uses path equalizer so all satisfying assignments take same time
- NCL crossover
**No-repeat rule:** [Robson - MFCS 1984]

- lose if ever repeat a past game configuration

\[ \Rightarrow G_1, G_2, G_3 \text{ become EXPSPACE-complete} \]

- OPEN: is Go with superko (no-repeat) EXPSPACE-complete? (as in USA & China)

**Conditional no-repeat rule:** [Robson - MFCS 1984]

- two special variables \( x \) \& \( y \)
- lose if ever repeat a past game configuration
- \& at most 1 of \( x \) \& \( y \) have changed since

\[ \Rightarrow G_1 \text{ becomes } 2\text{EXPTIME-complete} \]

**Private-information games:** [Reif - JCSS 1984]

- you can see some but not all of opponent's state

\[ \Rightarrow G_1 \text{ DNF, } G_2 \text{ DNF become } 2\text{EXPTIME-complete} \]

- version of Peek with half of winning holes visible to each player

**Blind games:** [Reif - JCSS 1984]

- player 1's entire state is hidden from player 2

\[ \Rightarrow G_2 \text{ DNF becomes EXPSPACE-complete} \]

- version of Peek above

**OPEN:** Constraint Logic in all these settings