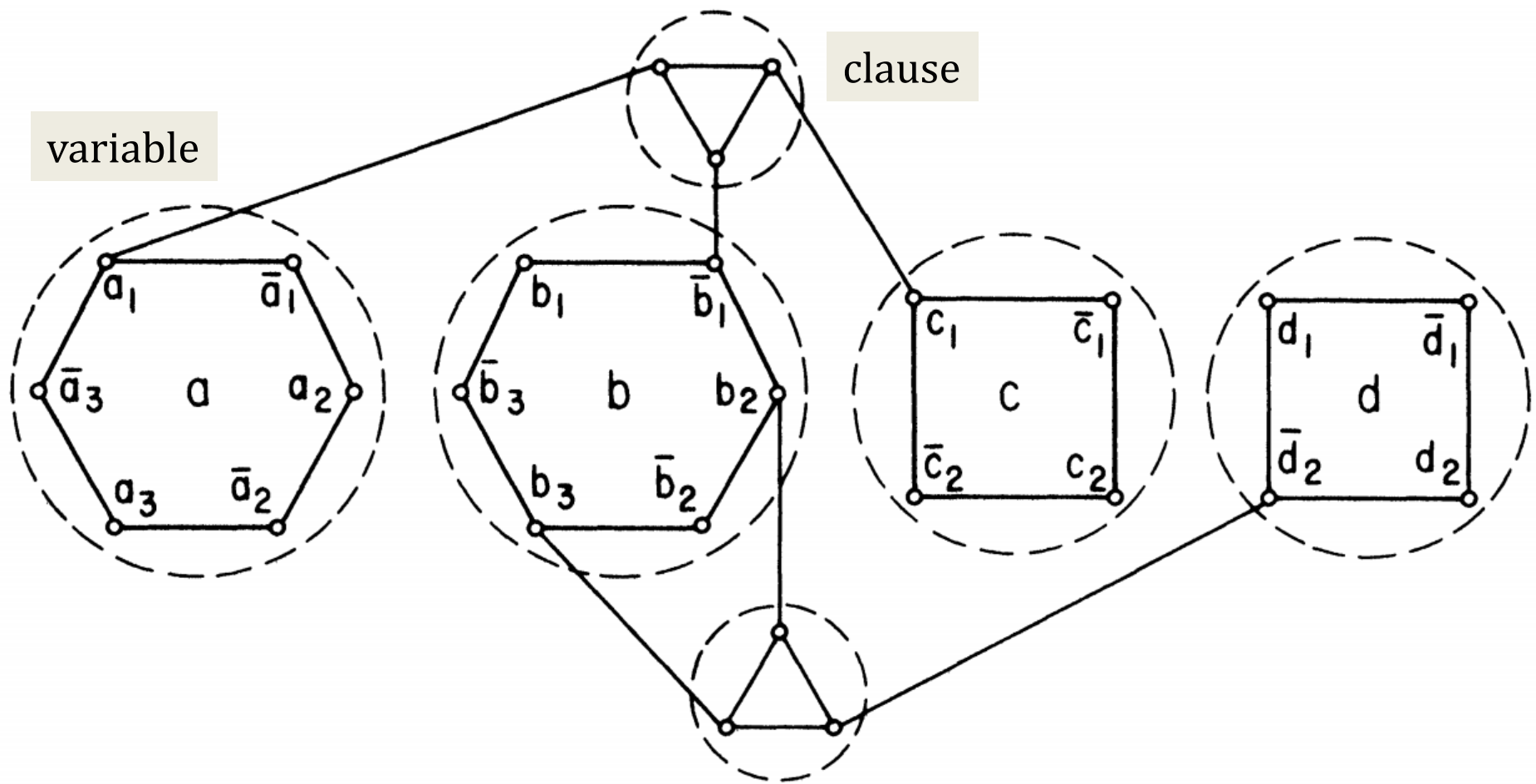


Planar Vertex Cover

[Lichtenstein 1982]

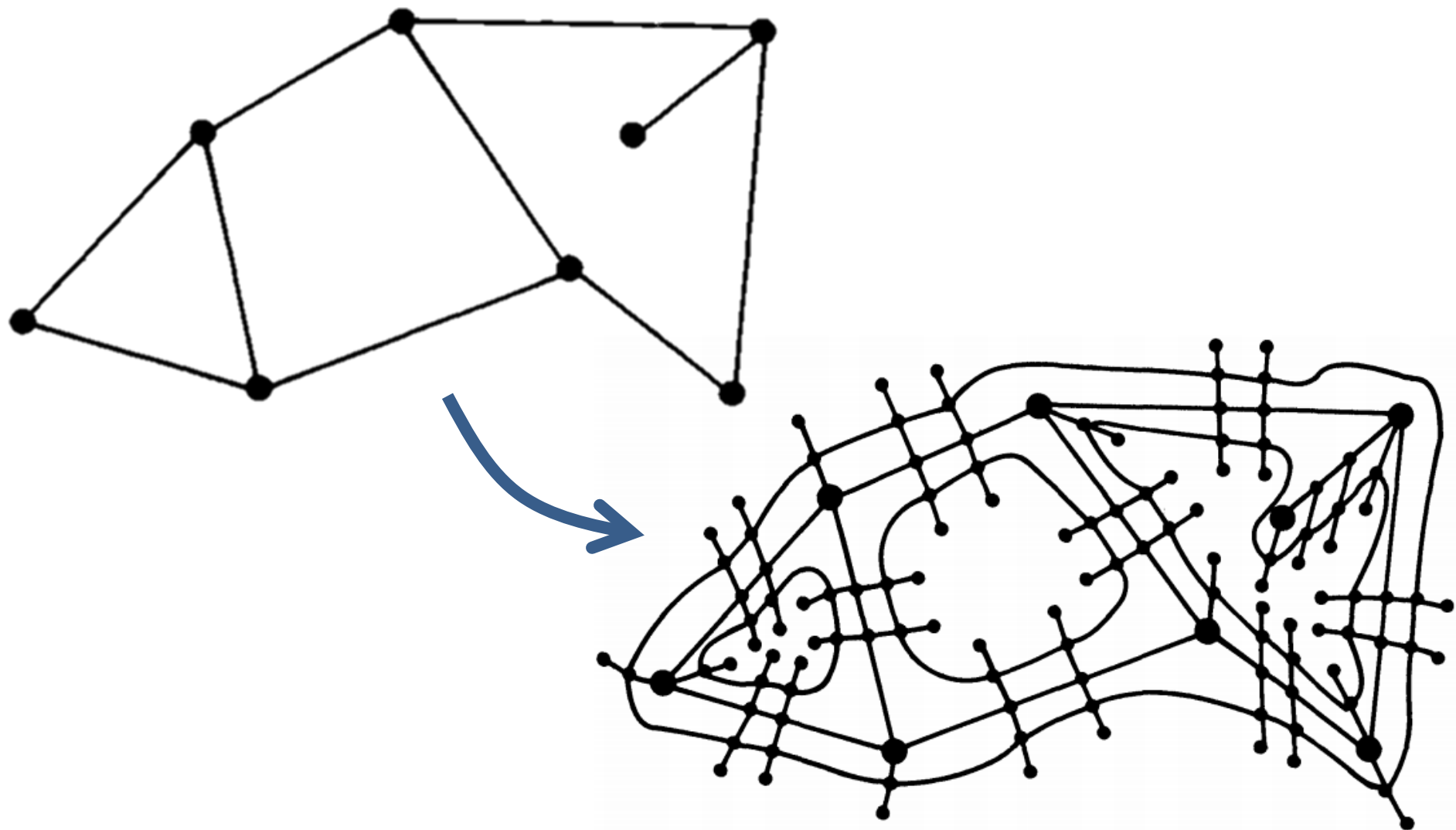


$$\text{Example : } B = (a + \bar{b} + c)(b + b + \bar{d})$$



Planar Connected Vertex Cover

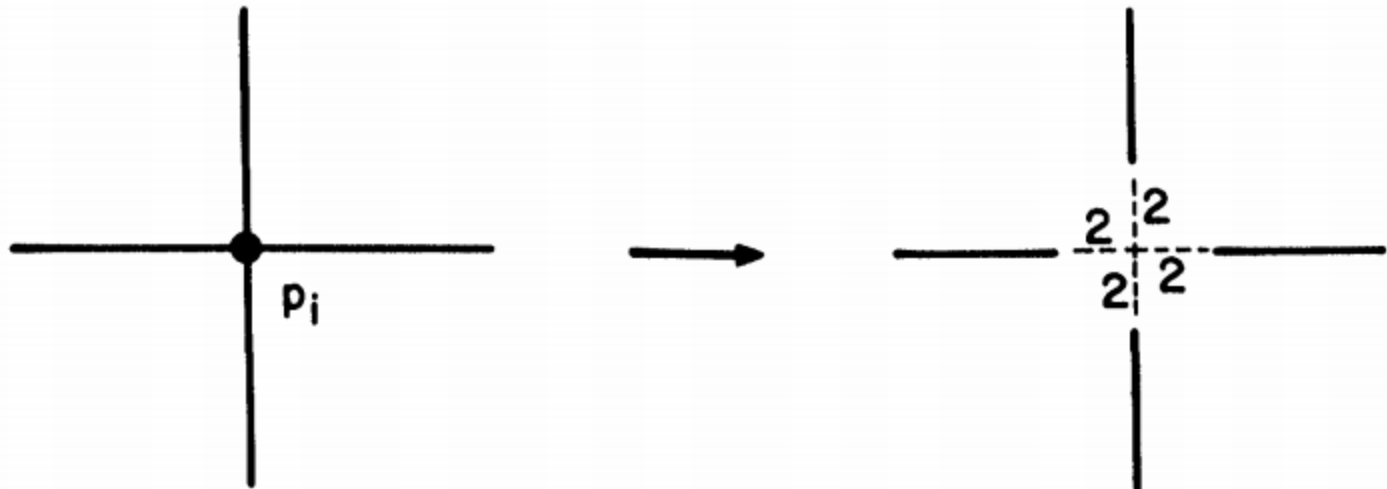
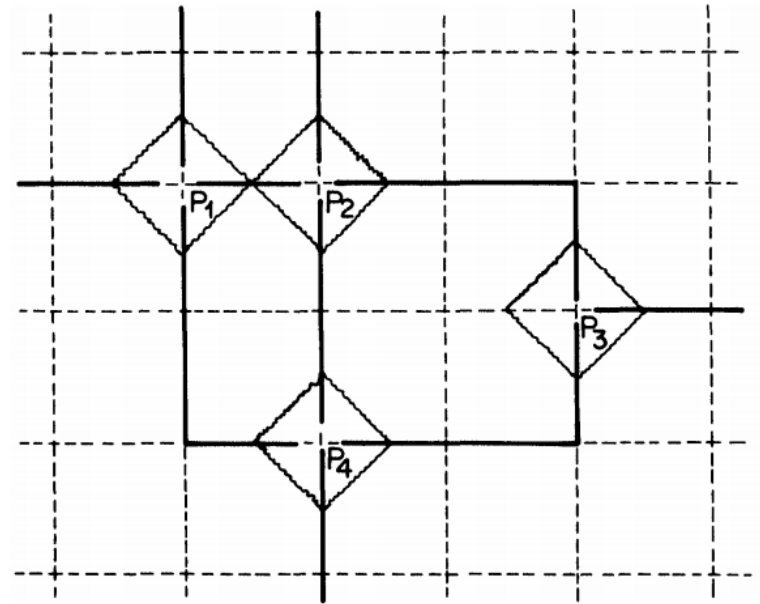
[Garey & Johnson 1977]





Rectilinear Steiner Tree

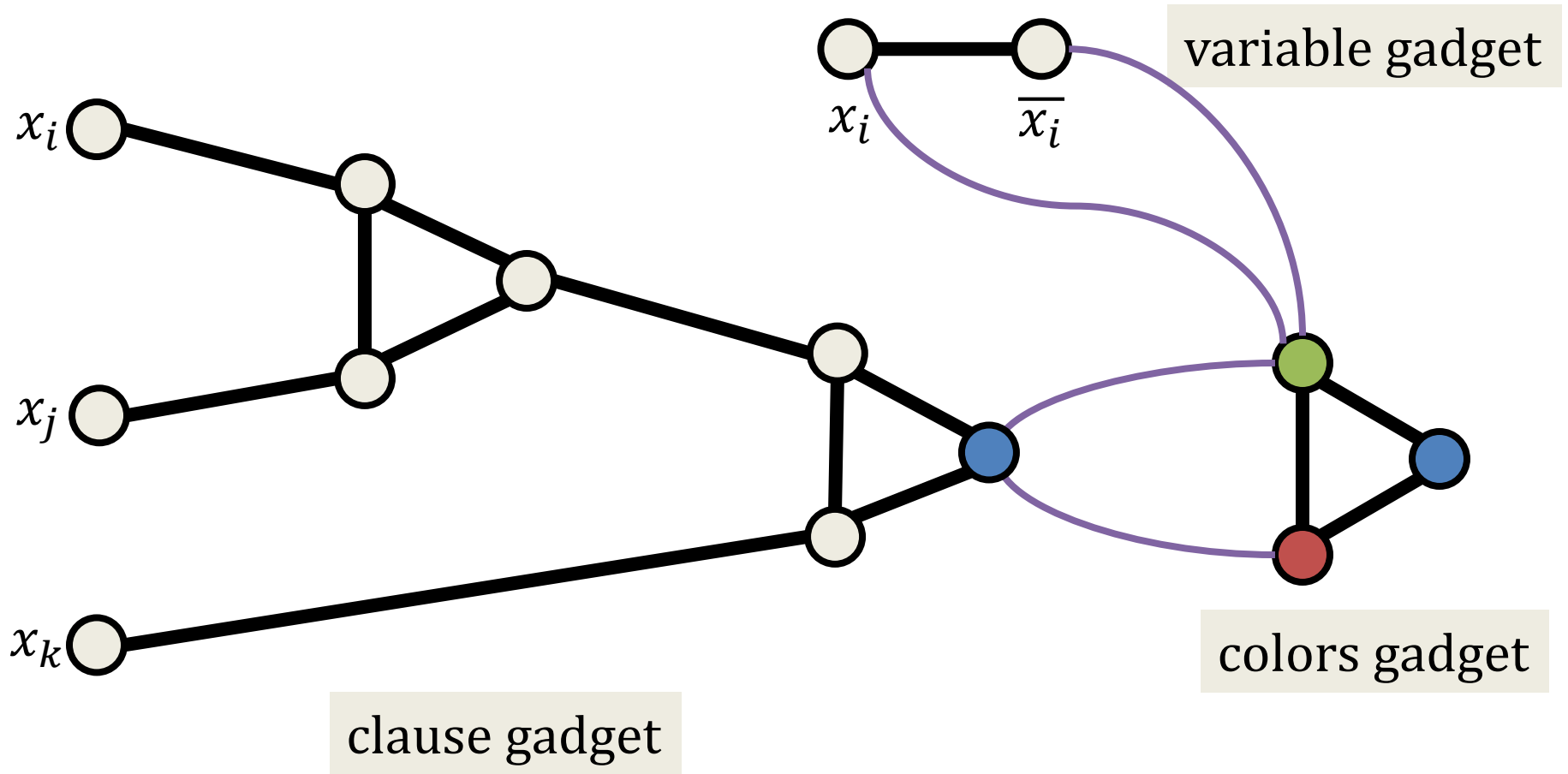
[Garey & Johnson 1977]





Vertex 3-Coloring

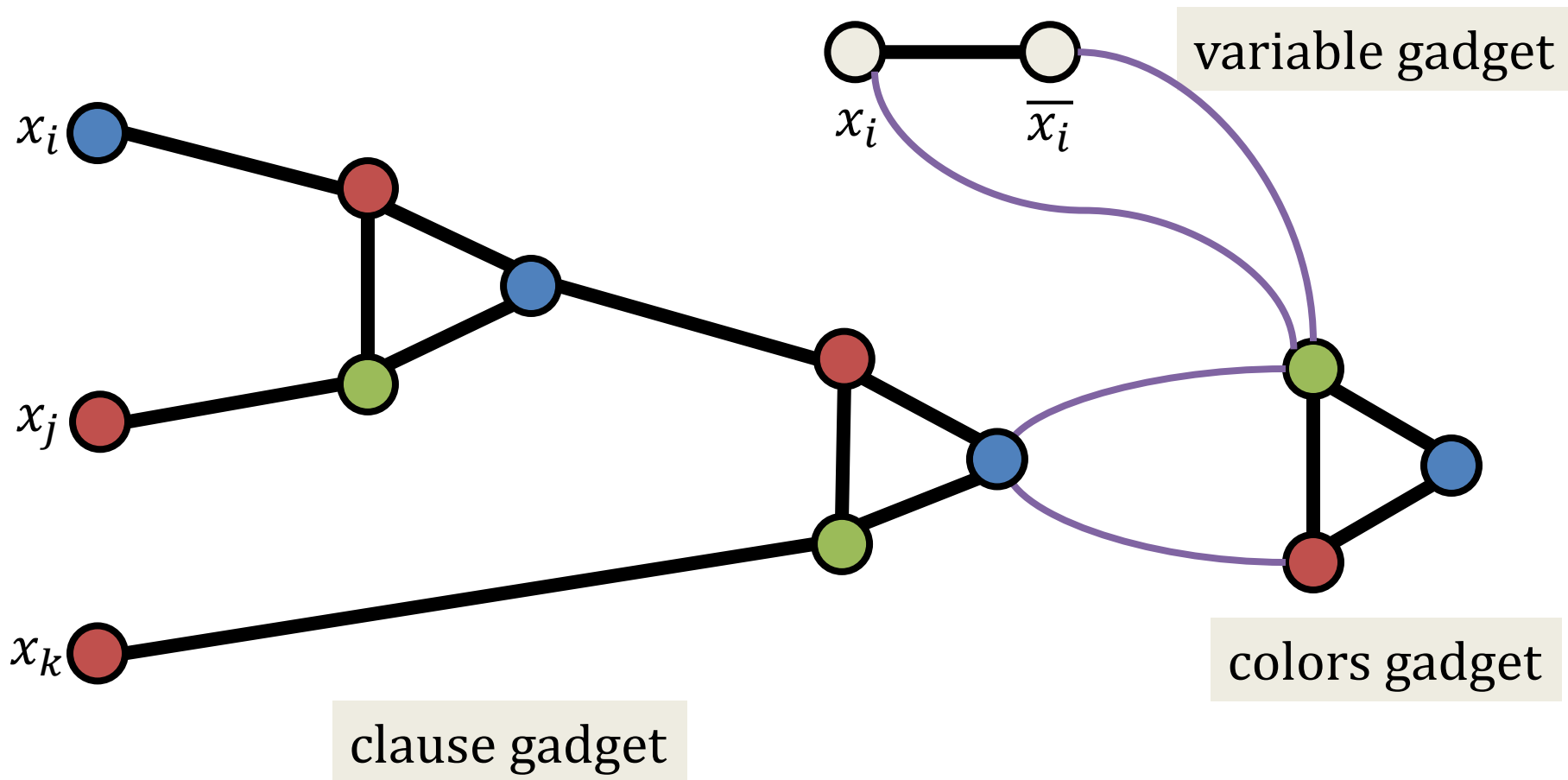
[Garey, Johnson, Stockmeyer 1976]





Vertex 3-Coloring

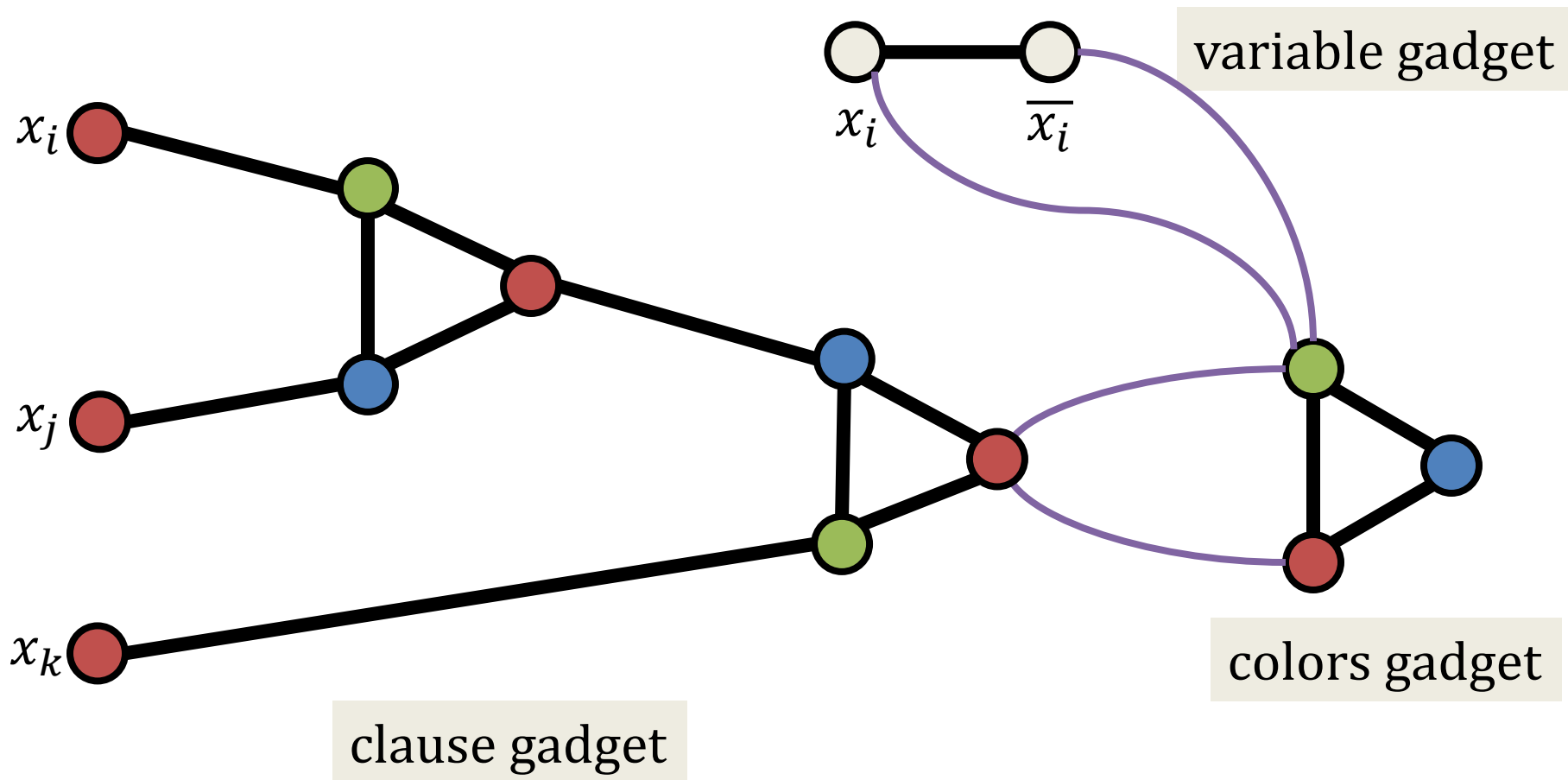
[Garey, Johnson, Stockmeyer 1976]





Vertex 3-Coloring

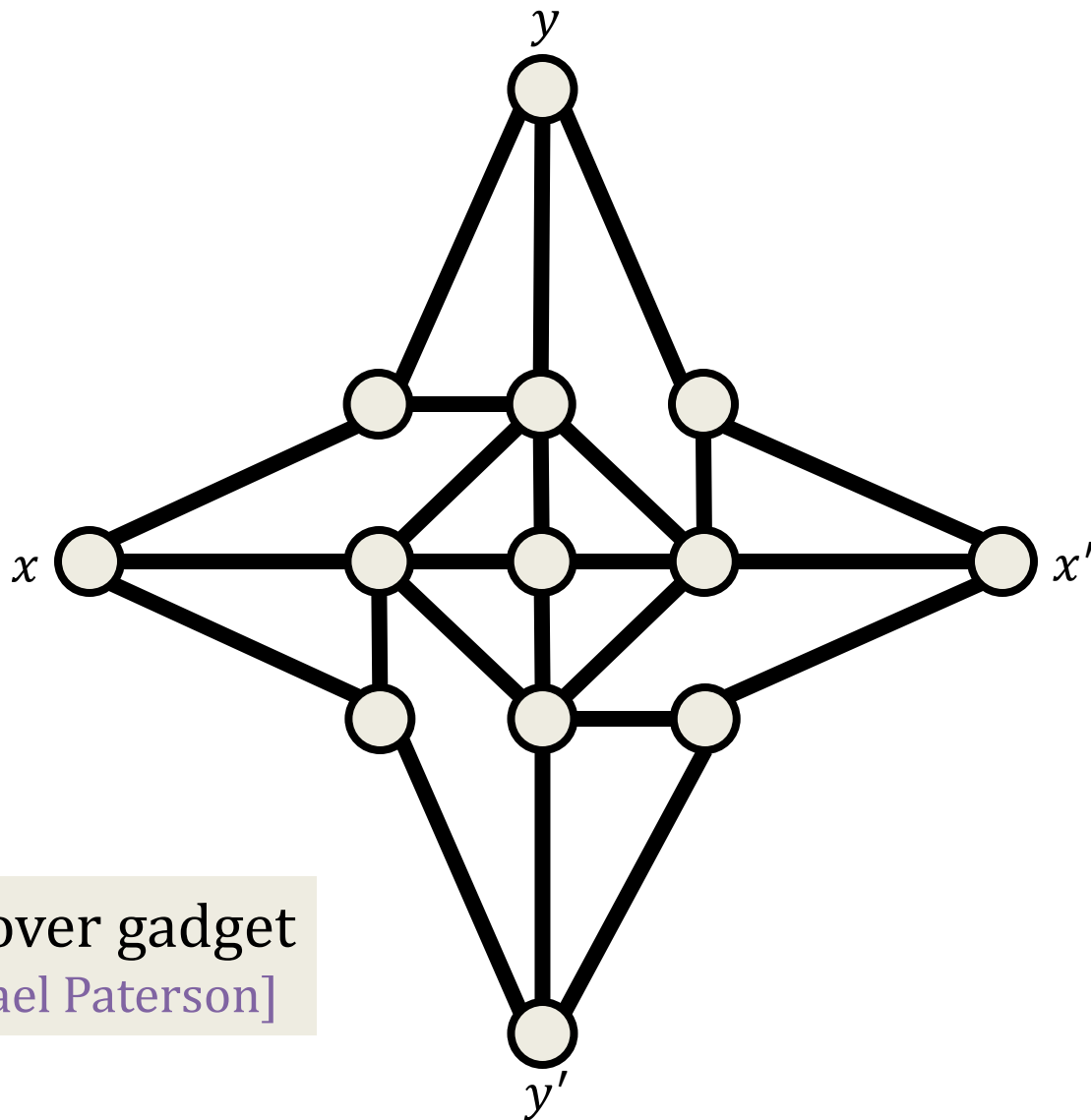
[Garey, Johnson, Stockmeyer 1976]





Planar 3-Coloring

[Garey, Johnson, Stockmeyer 1976]

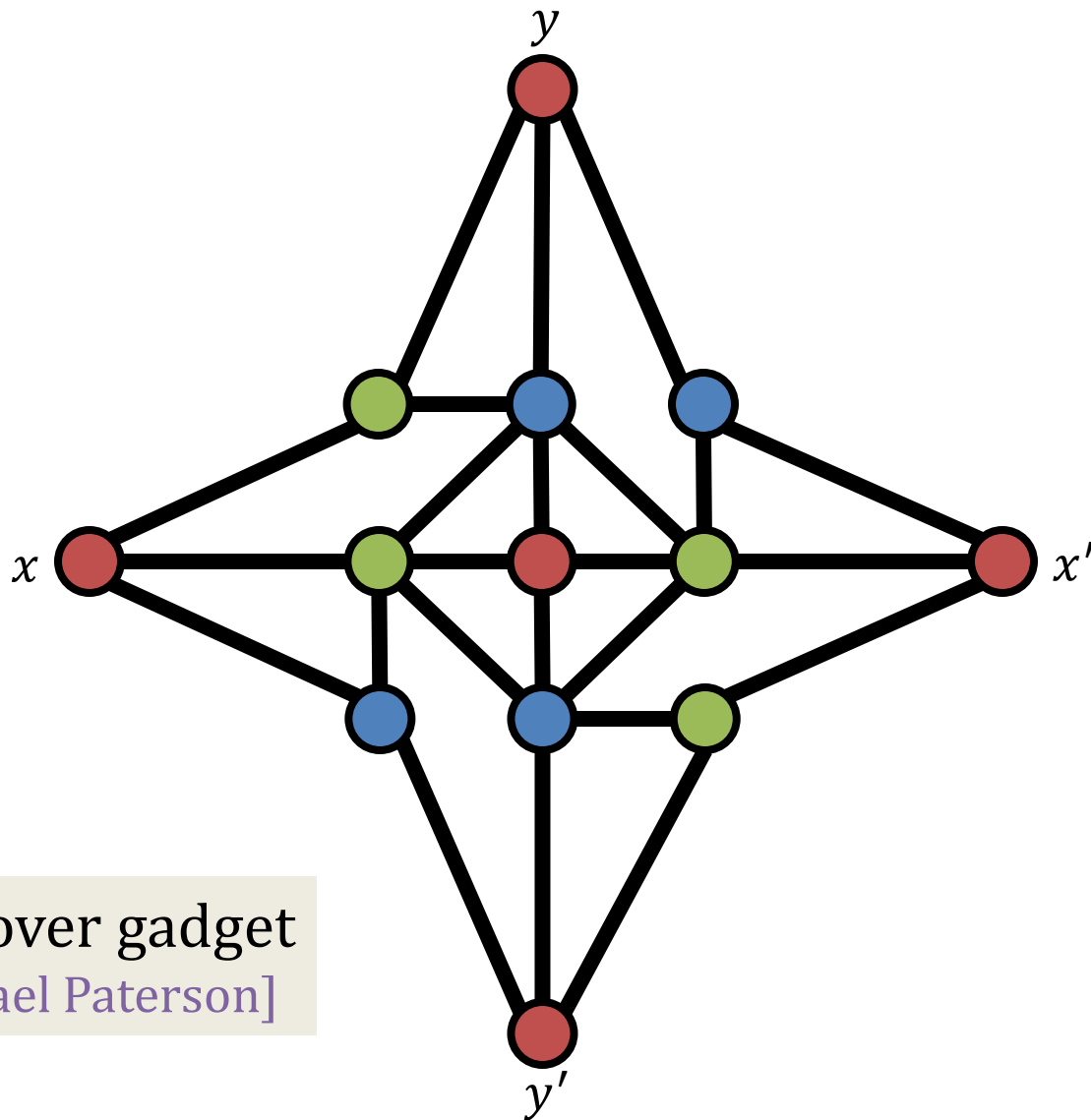


crossover gadget
[Michael Paterson]



Planar 3-Coloring

[Garey, Johnson, Stockmeyer 1976]

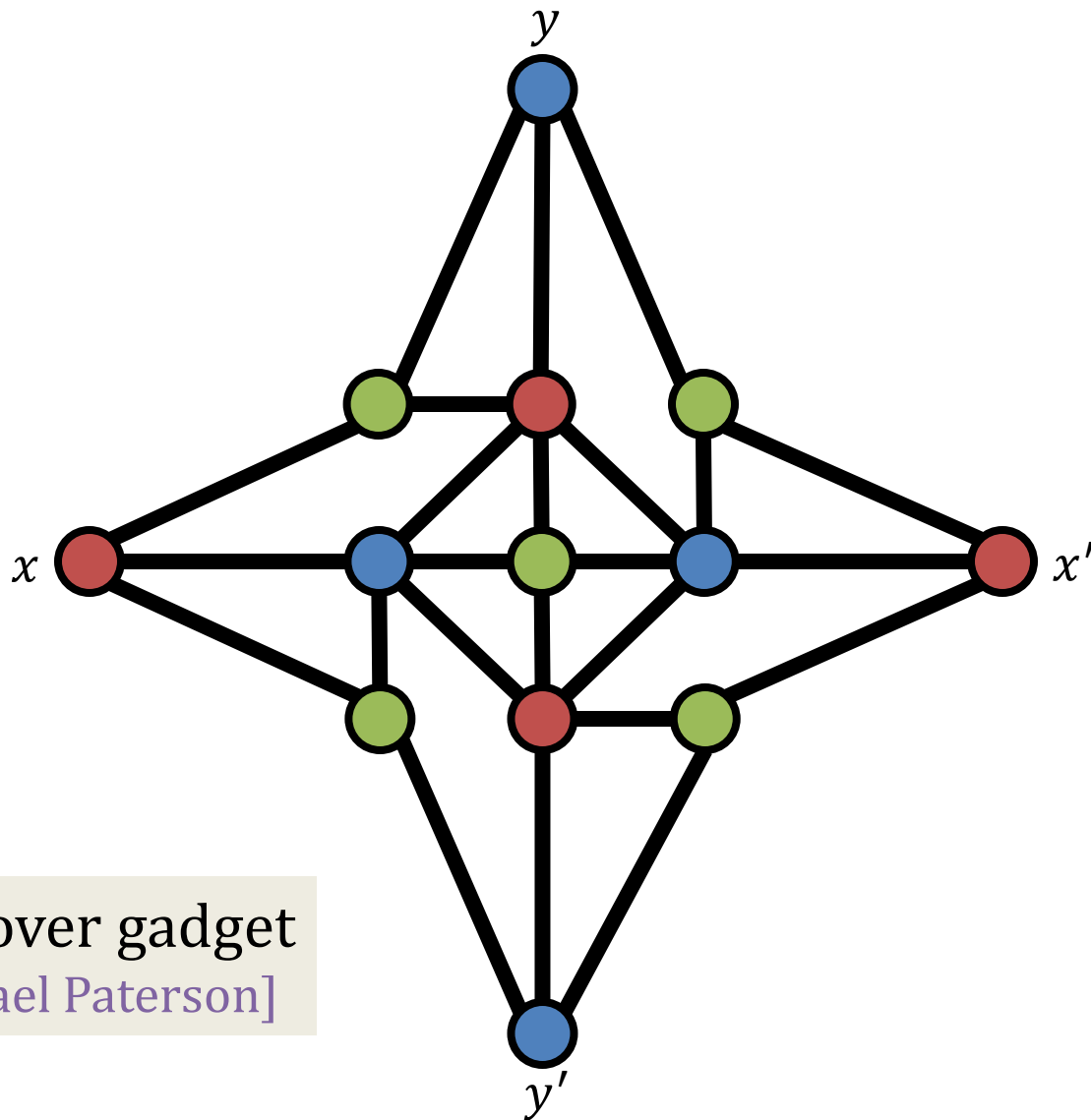


crossover gadget
[Michael Paterson]



Planar 3-Coloring

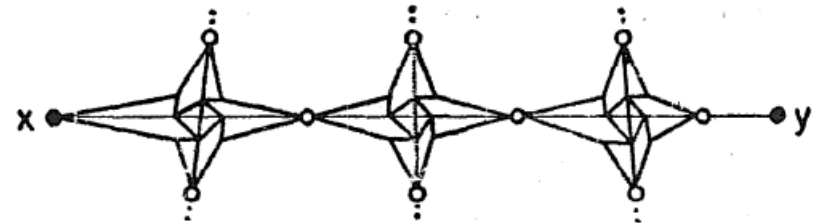
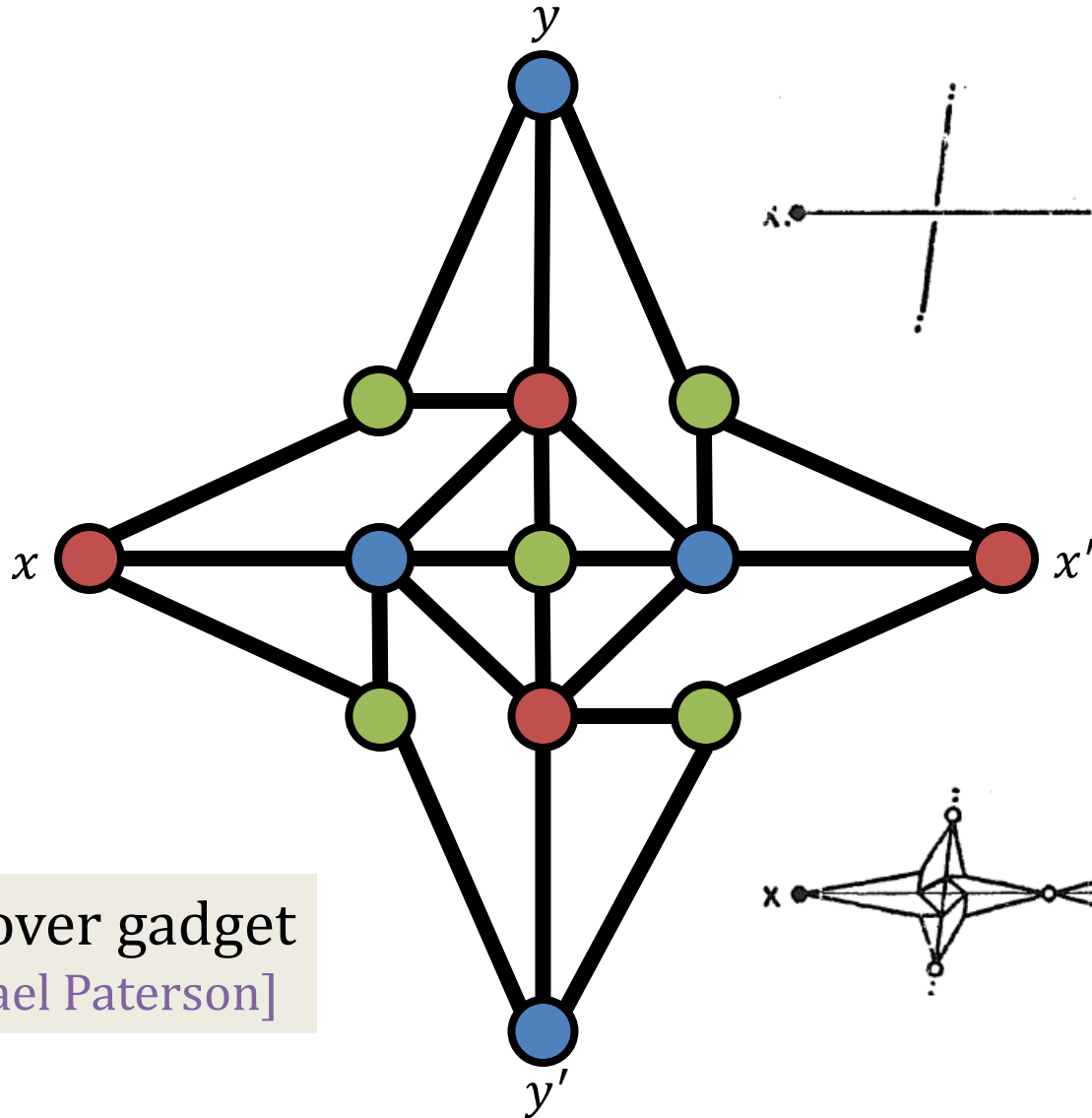
[Garey, Johnson, Stockmeyer 1976]



crossover gadget
[Michael Paterson]

Planar 3-Coloring

[Garey, Johnson, Stockmeyer 1976]

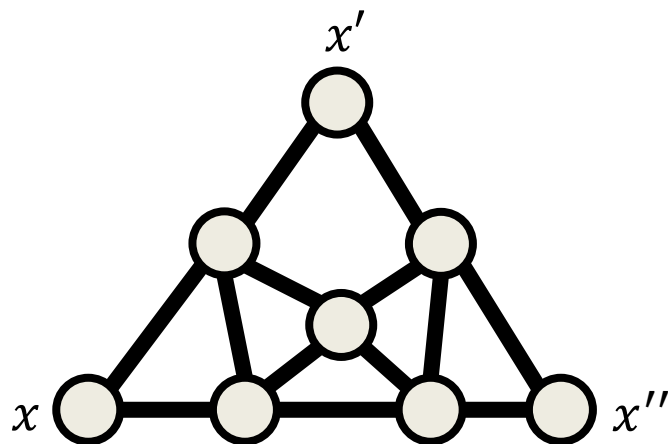


crossover gadget
[Michael Paterson]

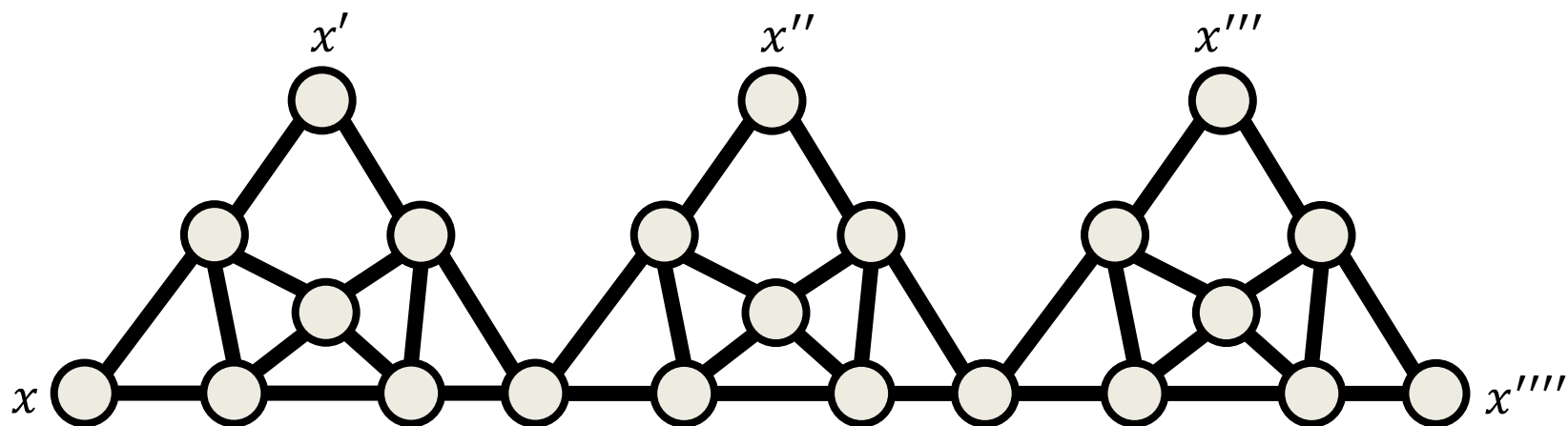


Planar 3-Coloring, Max Degree 4

[Garey, Johnson, Stockmeyer 1976]



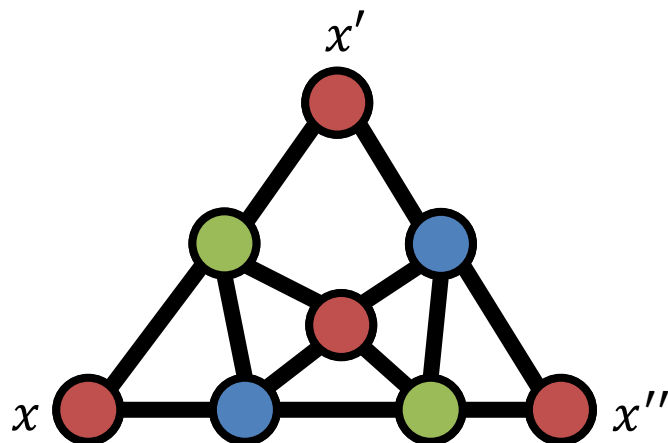
high-degree gadget



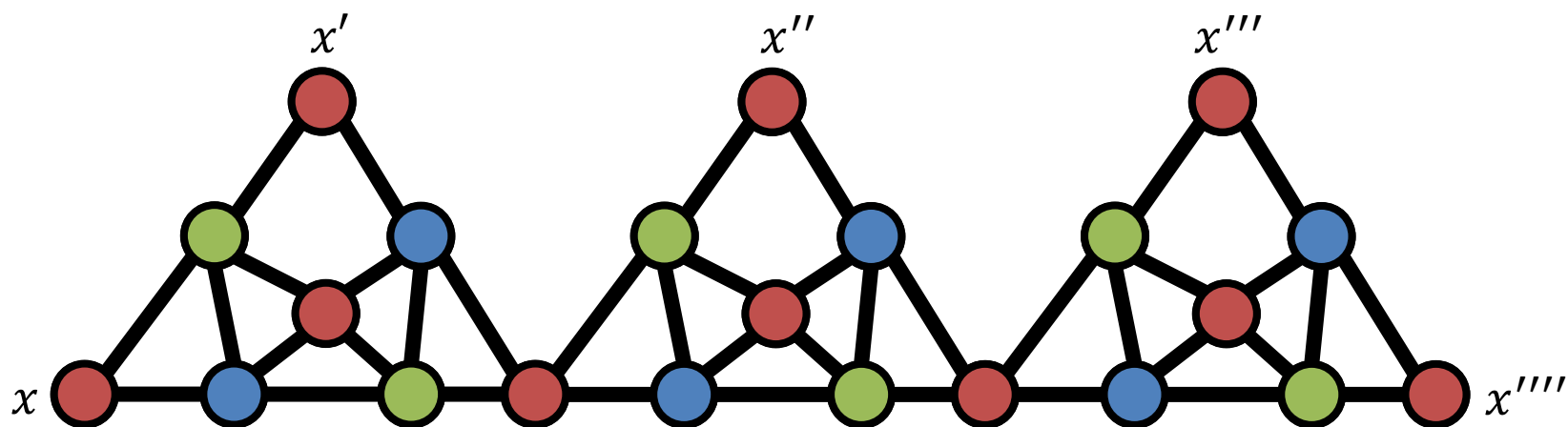


Planar 3-Coloring, Max Degree 4

[Garey, Johnson, Stockmeyer 1976]



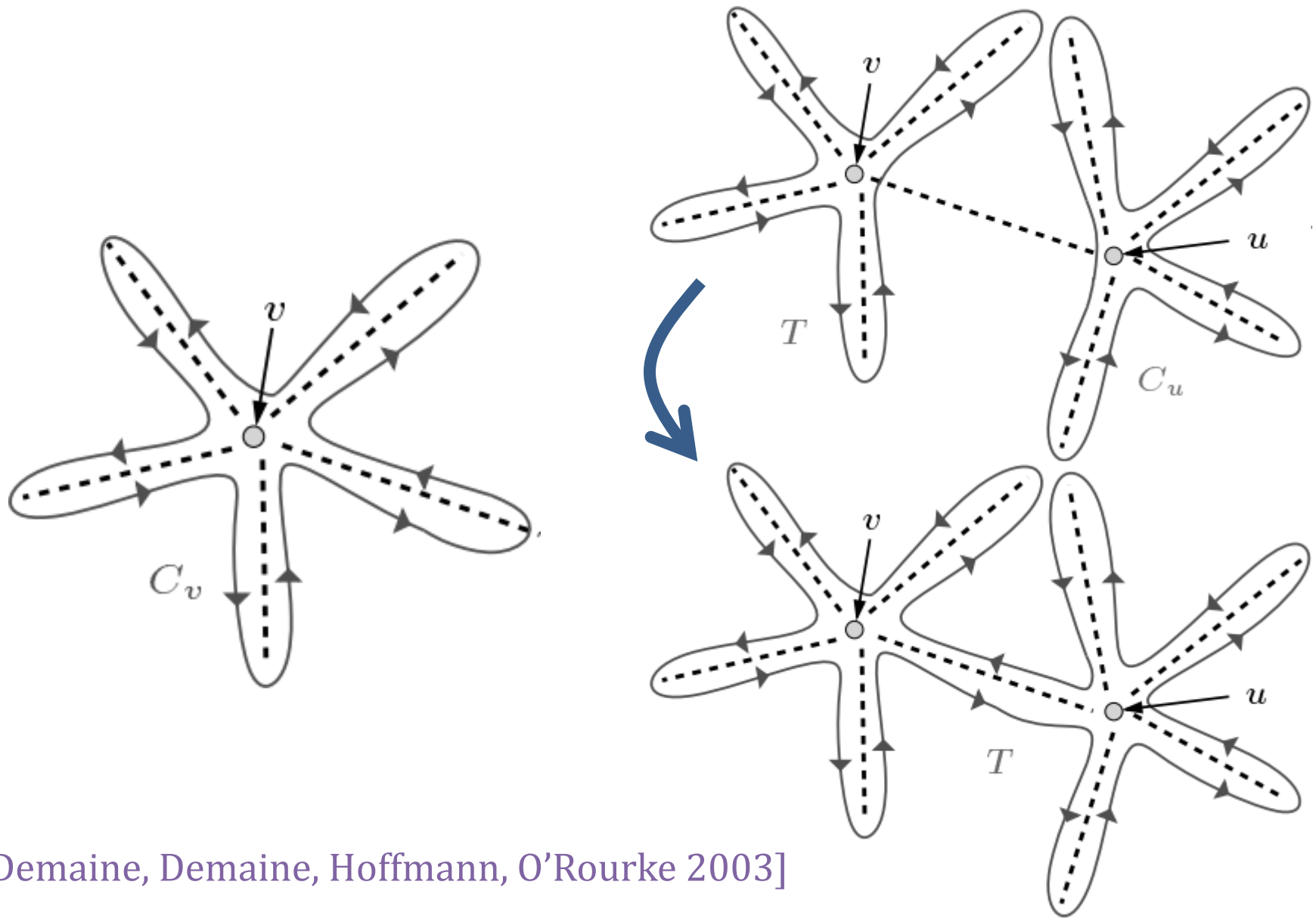
high-degree gadget



Pushing 1×1 Blocks Complexity

Name	Push	Fixed	Slide	Goal	Complexity	Reference
Push- k	$k \geq 1$	no	min	path	NP-hard	D, D, O'Rourke 2000
Push-*	∞	no	min	path	NP-hard	Hoffmann 2000
PushPush- k	$k \geq 1$	no	max	path	PSPACE-complete	D, Hoffmann, Holzer 2004
PushPush-*	∞	no	max	path	NP-hard	Hoffmann 2000
Push-1F	1	yes	min	path	NP-hard	DDO 2000
Push- k F	$k \geq 2$	yes	min	path	PSPACE-complete	D, Hearn, Hoffmann 2002
Push- $*$ F	∞	yes	min	path	PSPACE-complete	Bremner, O'Rourke, Shermer 1994
Push- k X	$k \geq 1$	no	min	simple path	NP-complete	D, Hoffmann 2001
Push- $*$ X	∞	no	min	simple path	NP-complete	Hoffmann 2000
Sokoban	1	yes	min	storage	PSPACE-complete	Culberson 1998

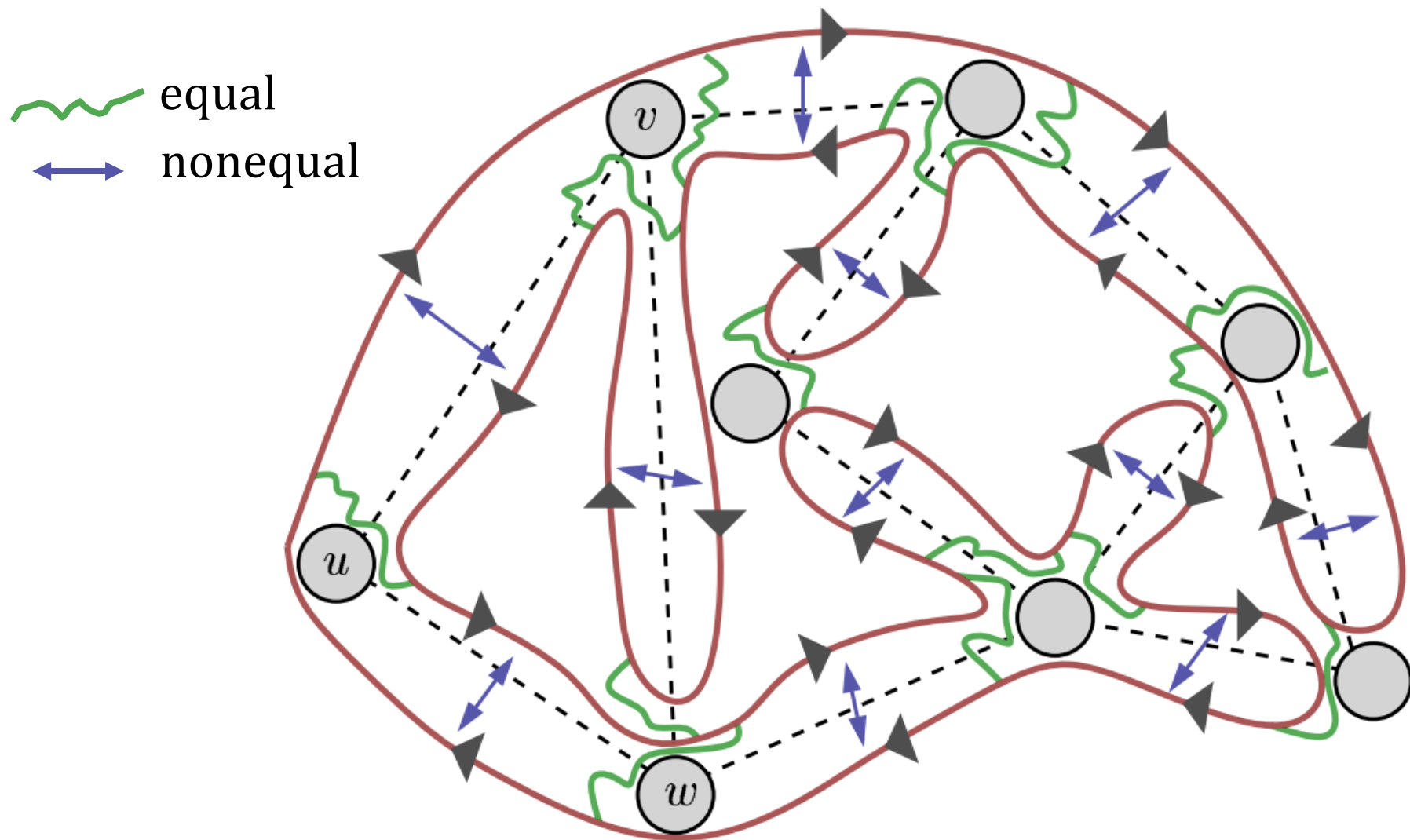
Planar Euler Tours



[Demaine, Demaine, Hoffmann, O'Rourke 2003]

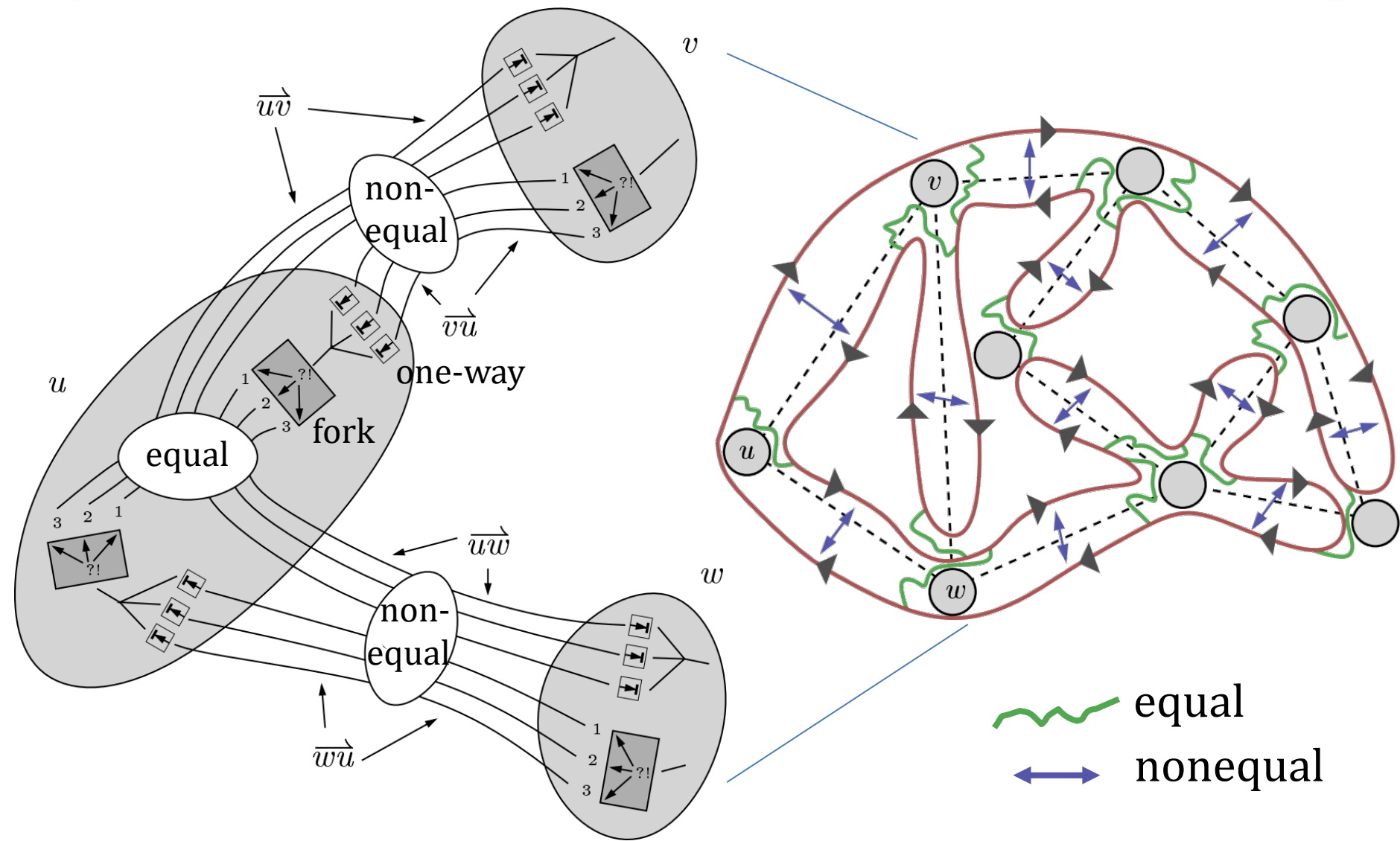
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O'Rourke 2003]



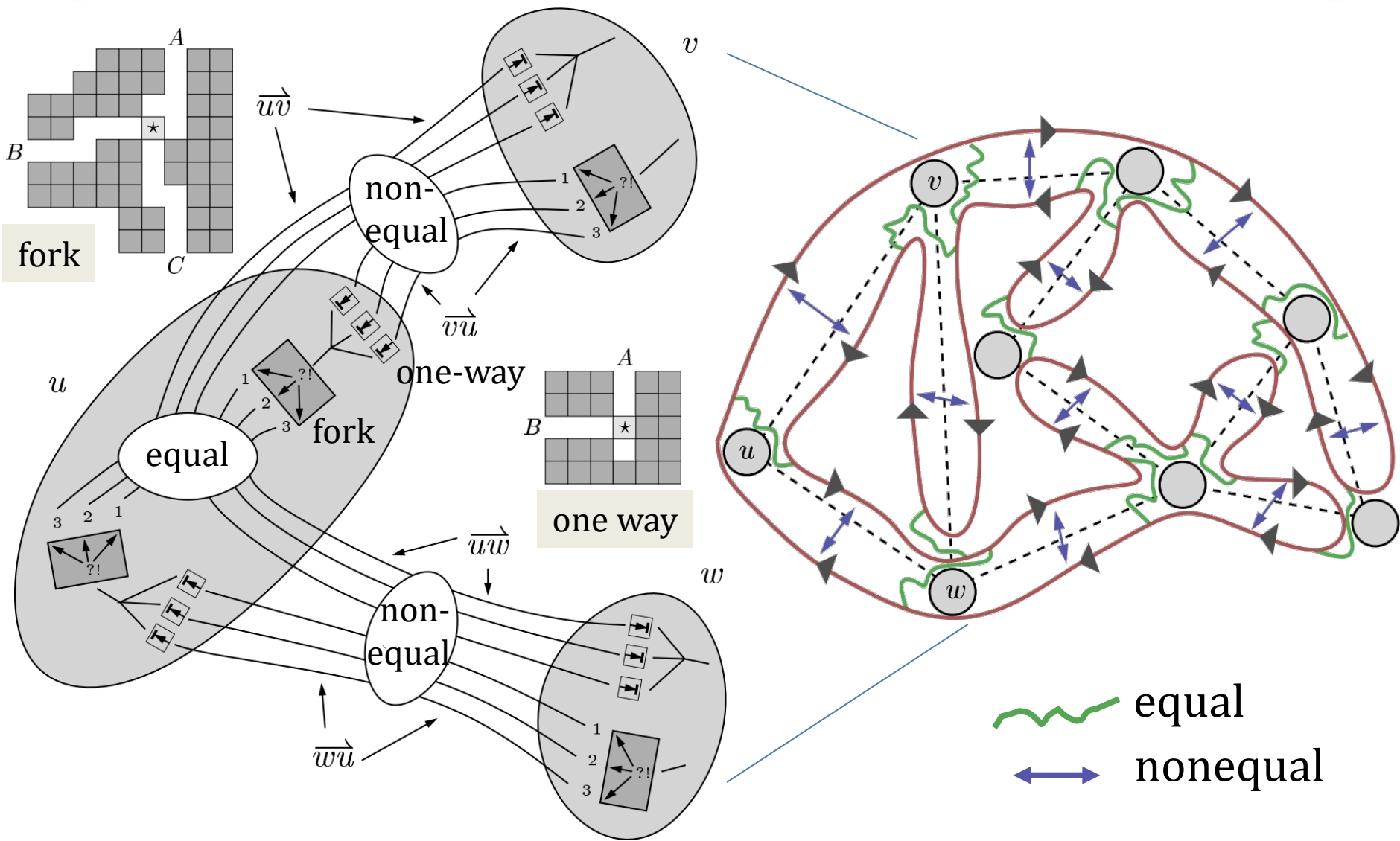
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O'Rourke 2003]



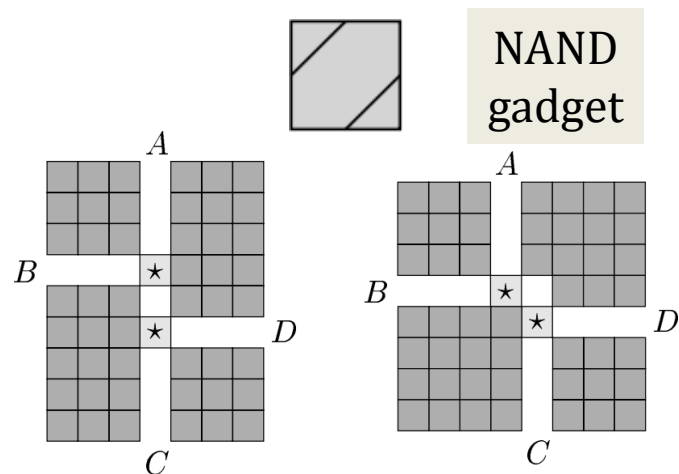
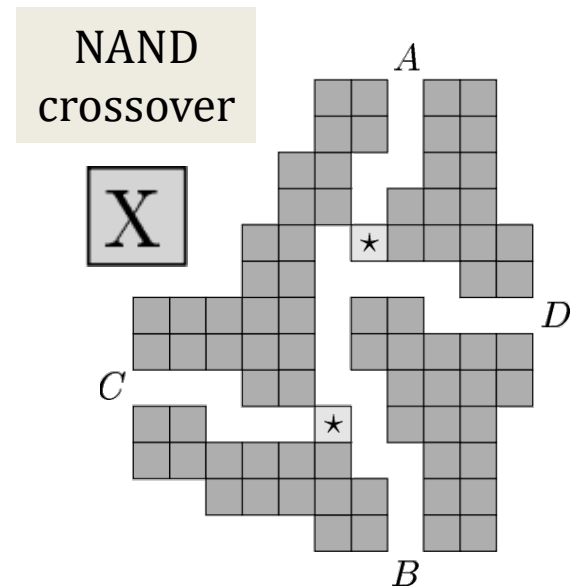
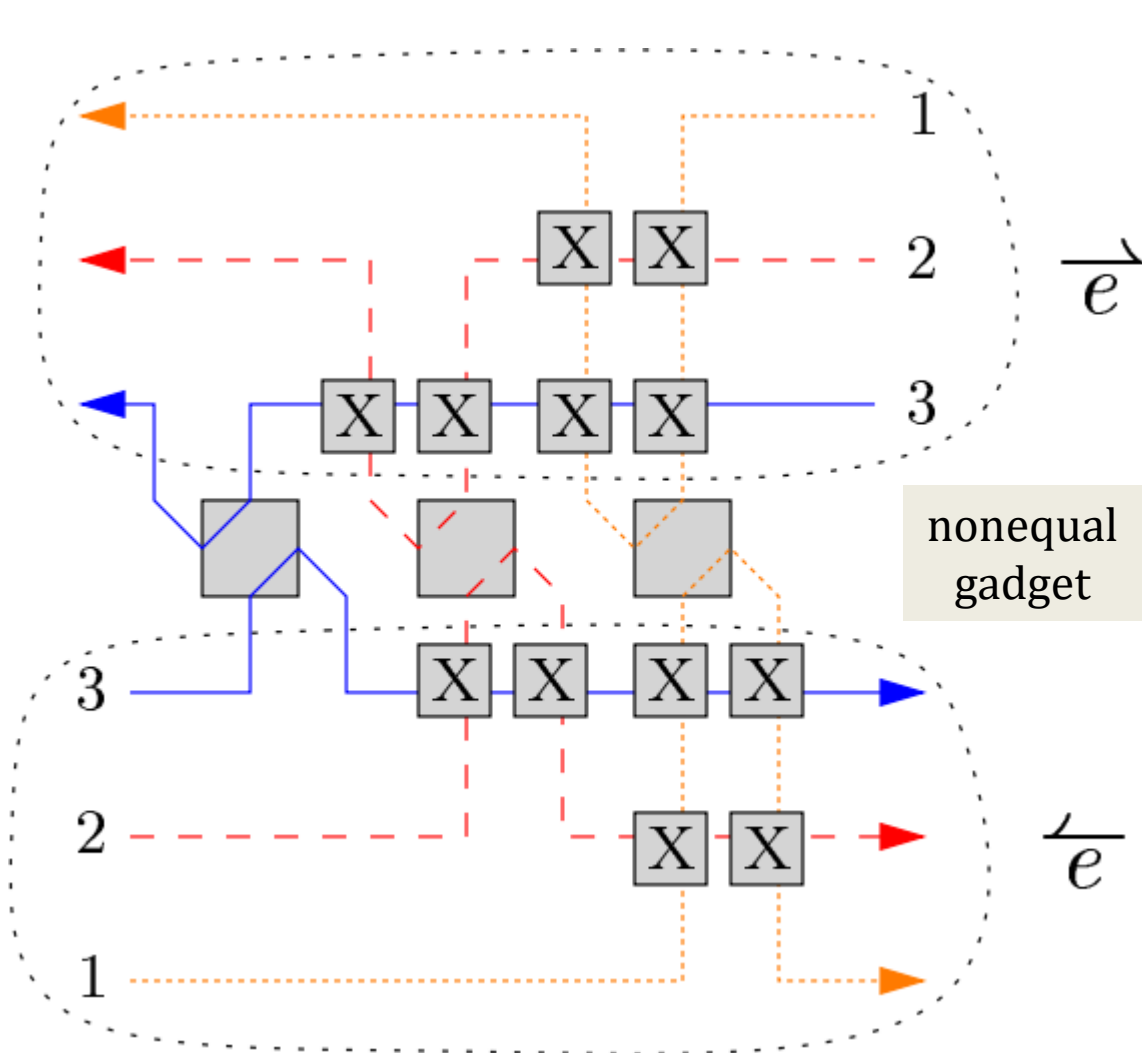
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O'Rourke 2003]



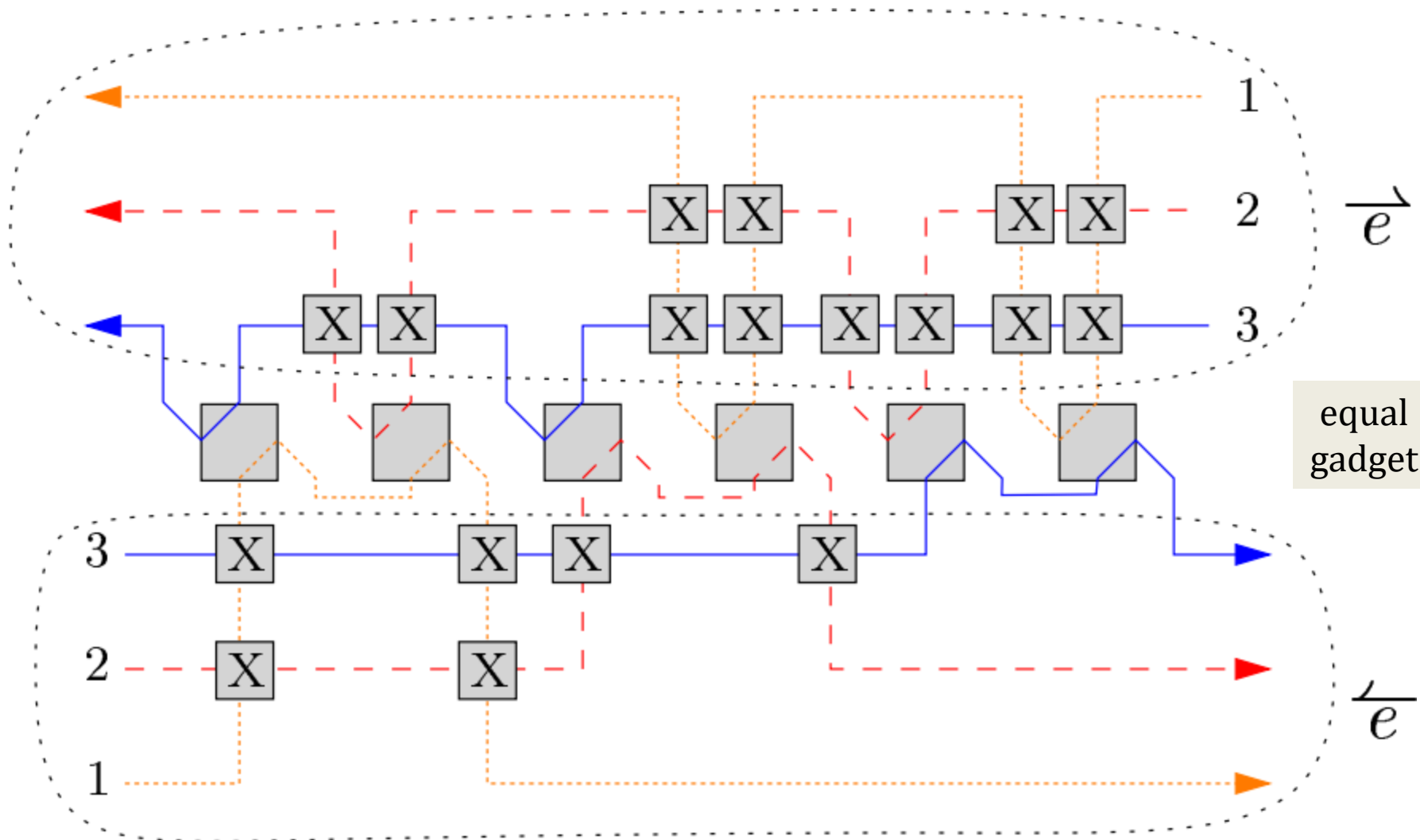
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O'Rourke 2003]



Push-1X is NP-complete

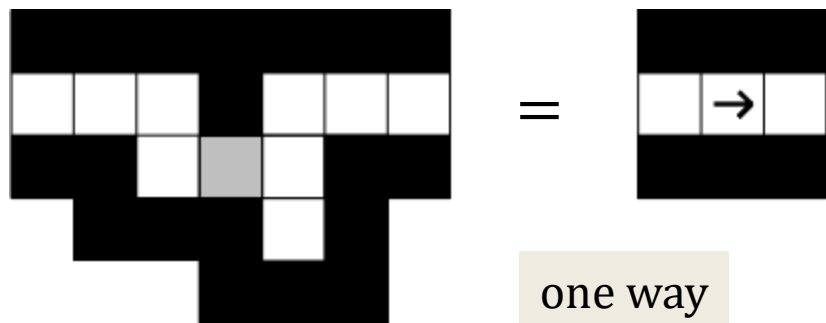
[Demaine, Demaine, Hoffmann, O'Rourke 2003]



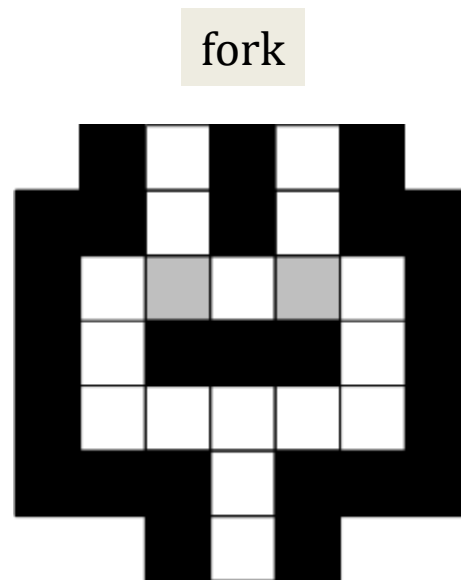


Push-1G is NP-hard

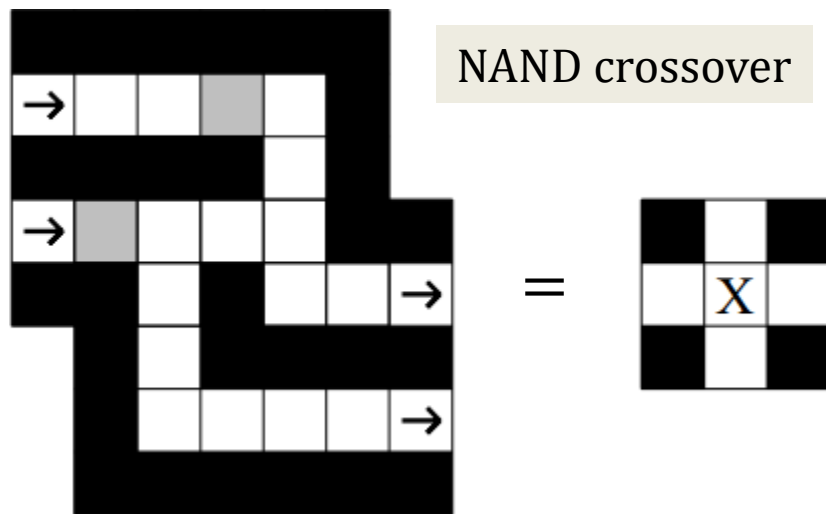
[Friedman 2002]



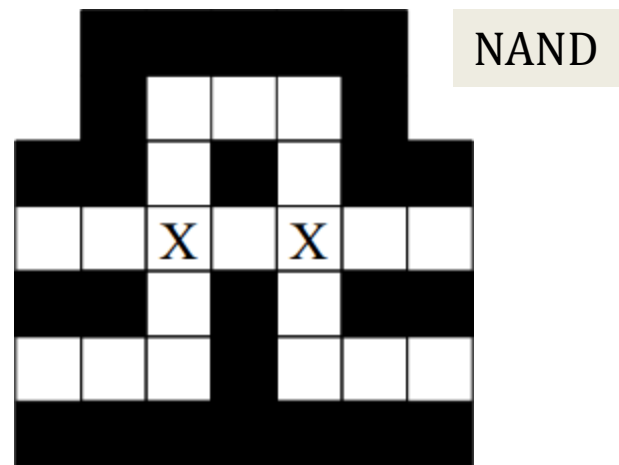
one way



fork



NAND crossover

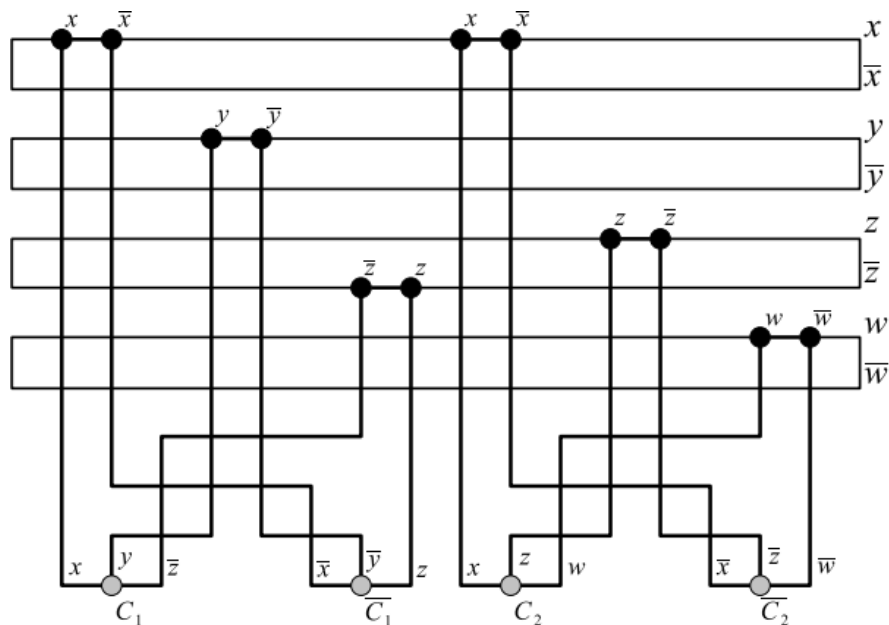


NAND

Graph Orientation

[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]

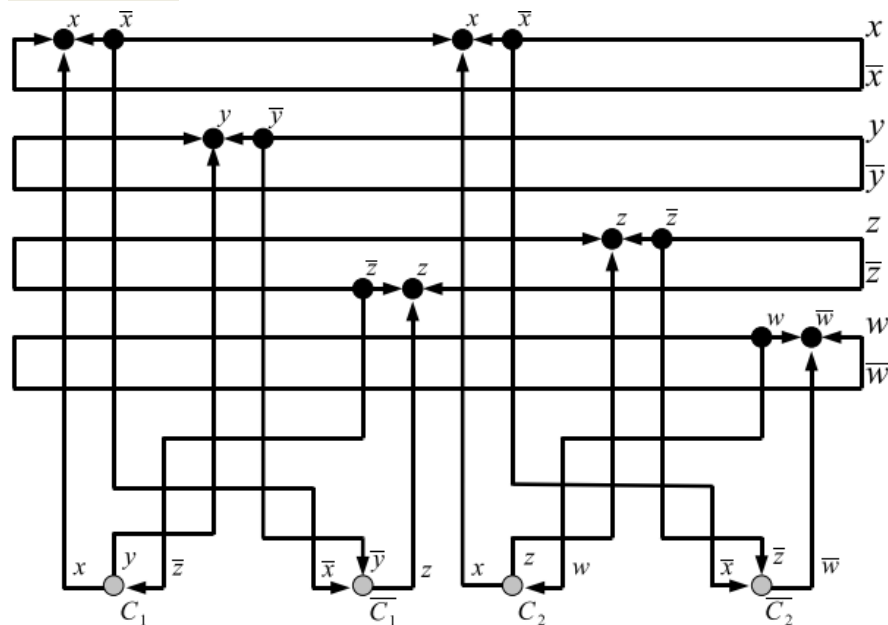
0-or-3



1-in-3

2-in-3

0-or-3



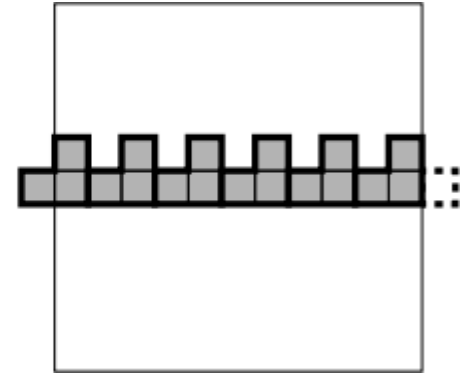
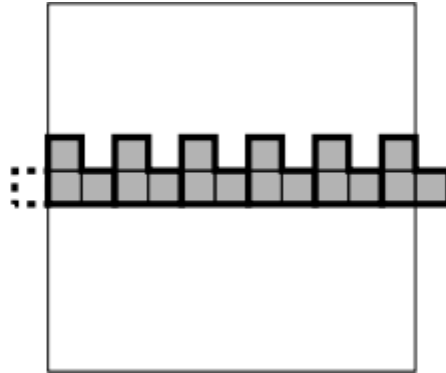
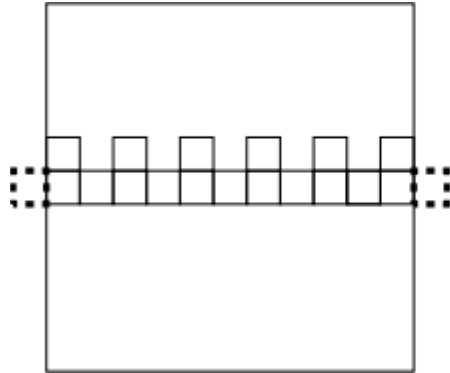
1-in-3

2-in-3

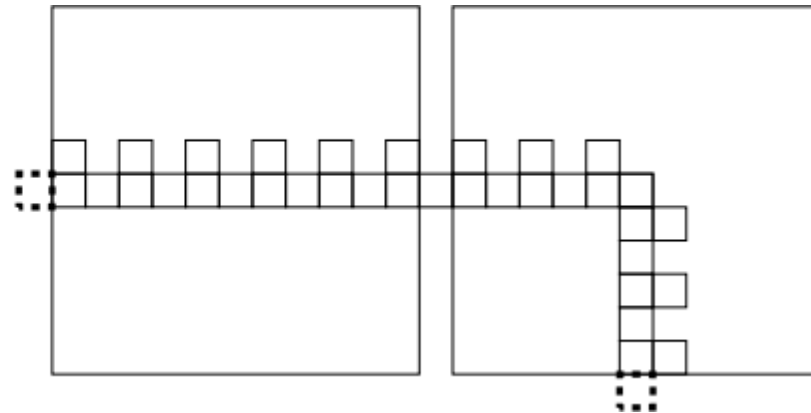
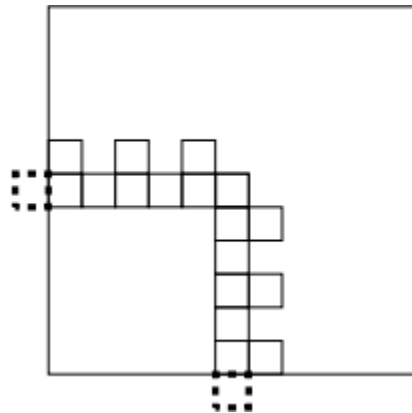


Packing L Trominoes into Polygon

[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]



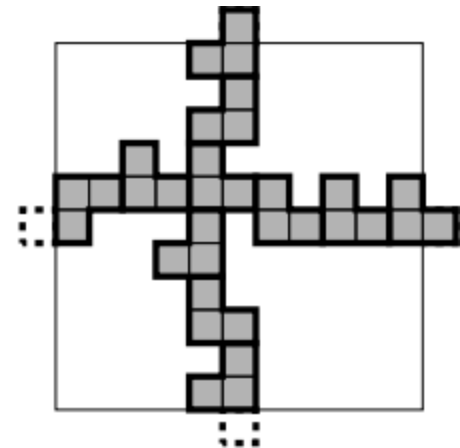
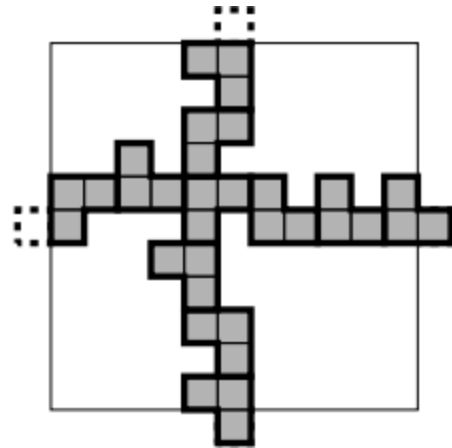
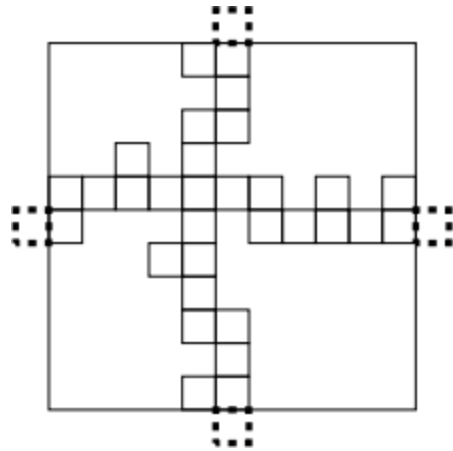
edge gadget



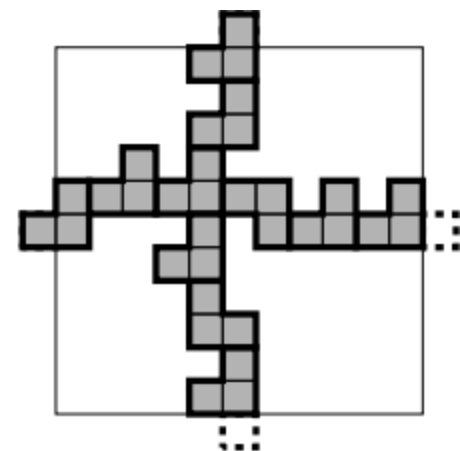
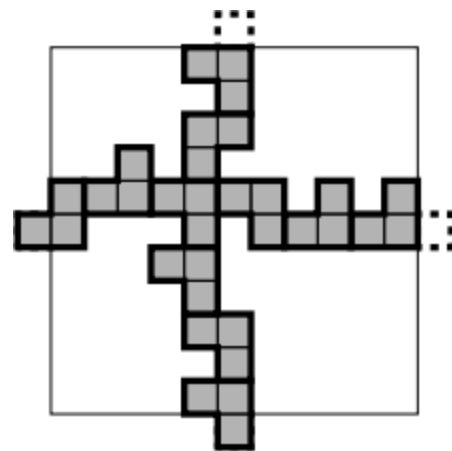


Packing L Trominoes into Polygon

[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]

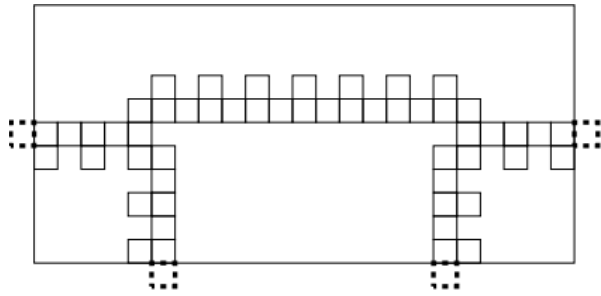


crossover

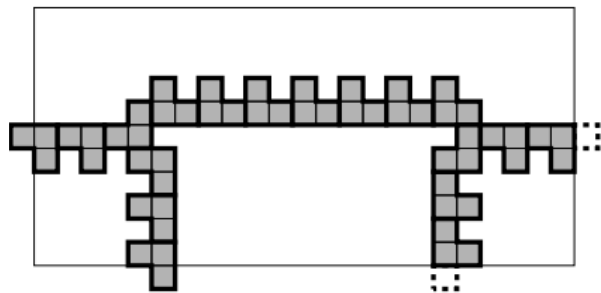
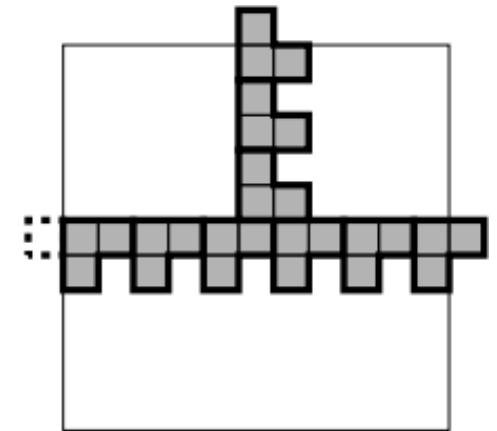
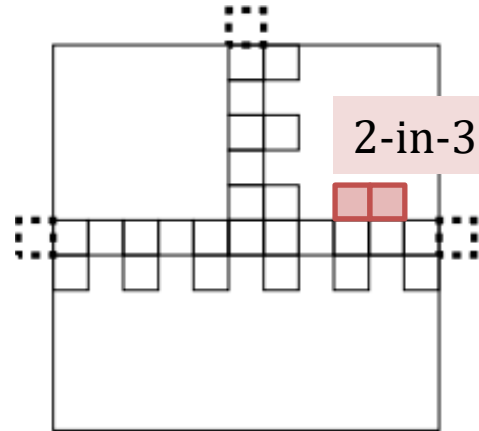


Packing L Trominoes into Polygon

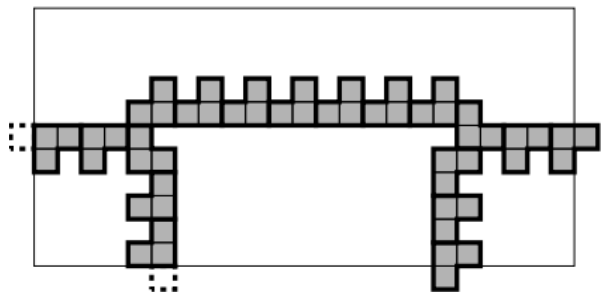
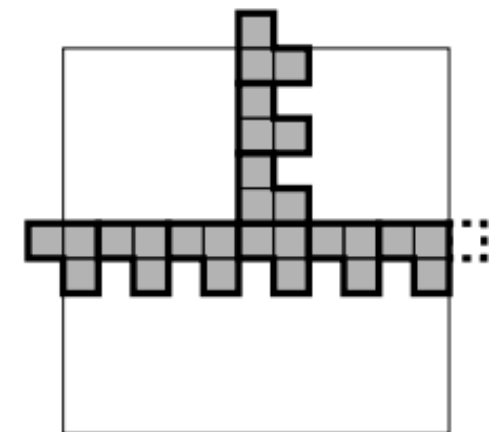
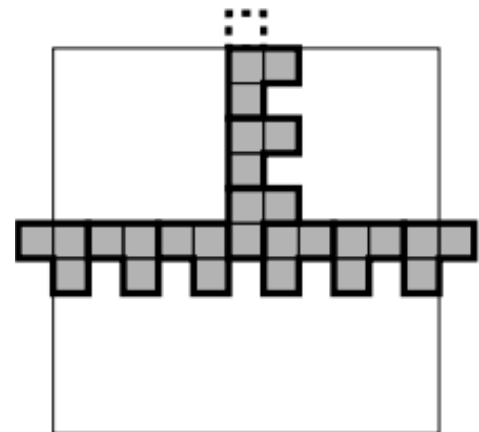
[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]



double 0-or-3

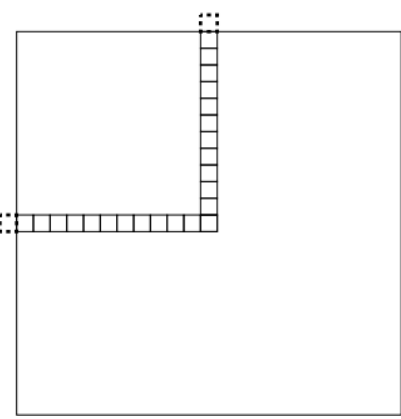
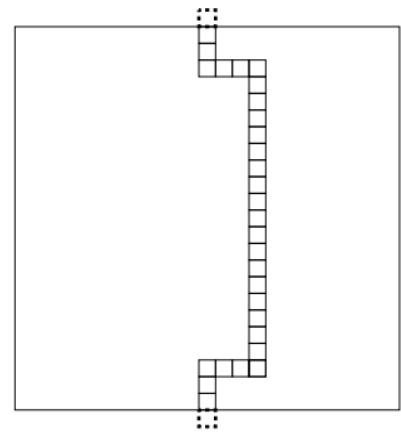


1-in-3

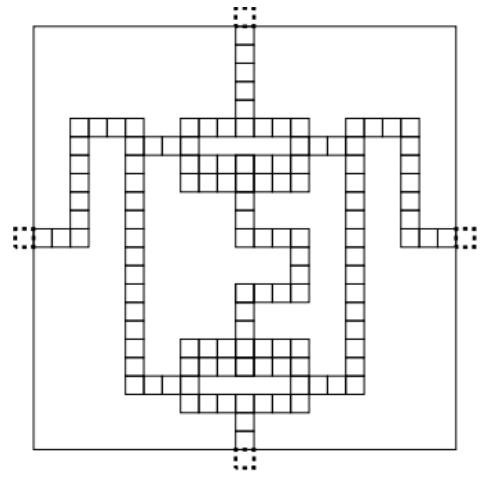


Packing I Trominoes into Polygon

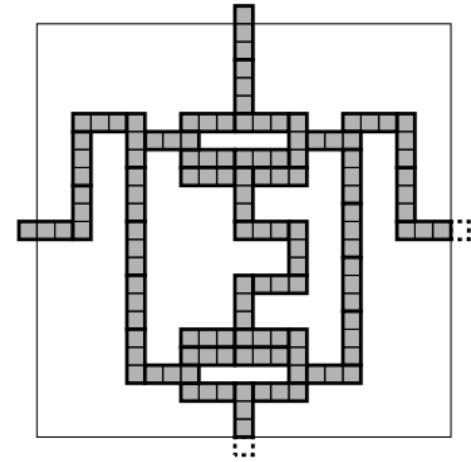
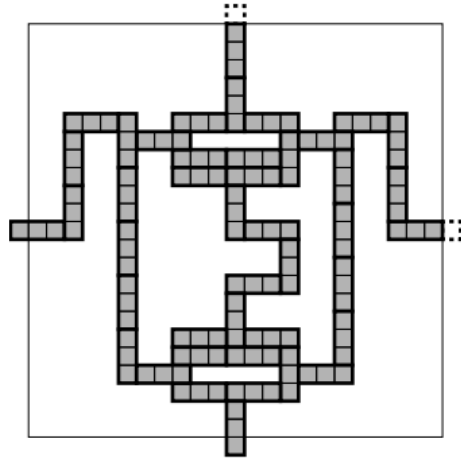
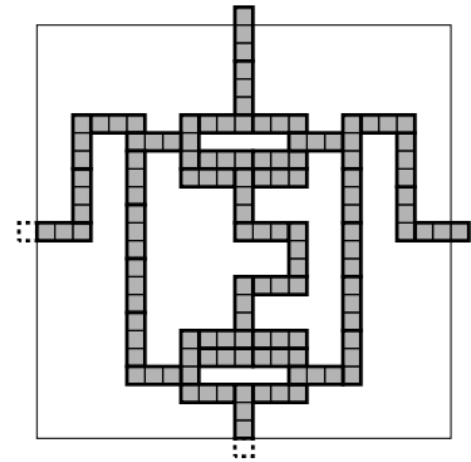
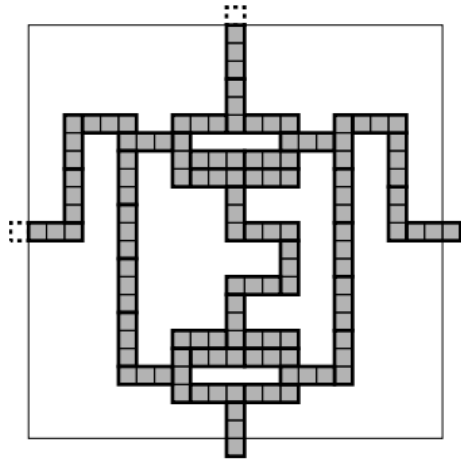
[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]



edge gadget



crossover



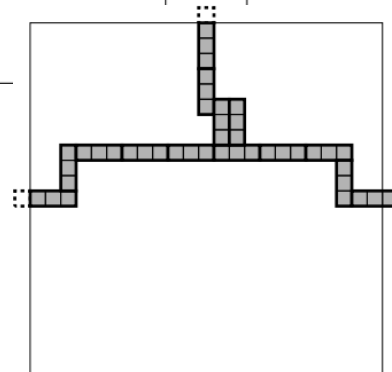
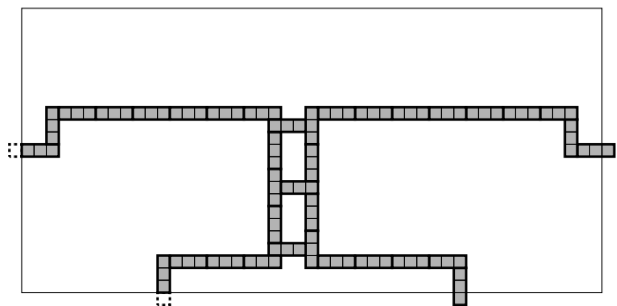
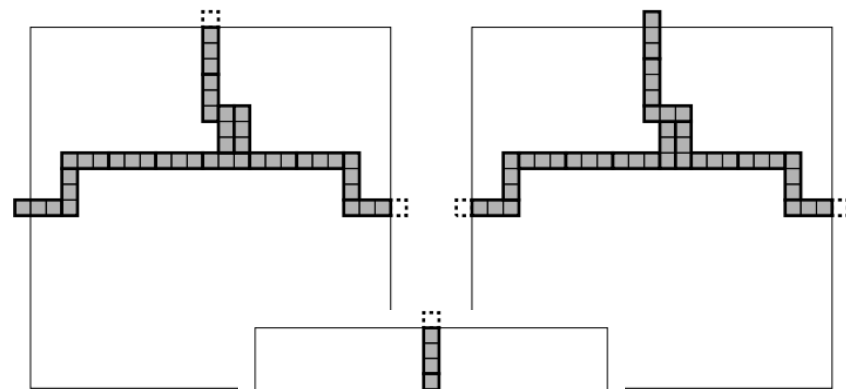
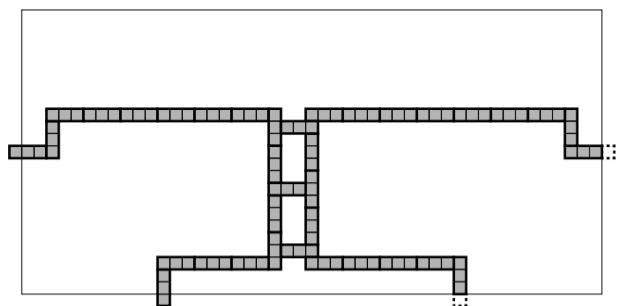
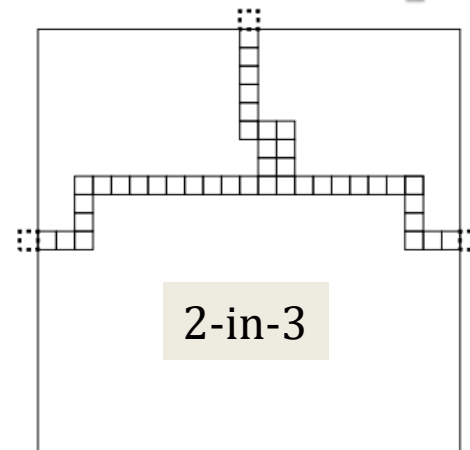
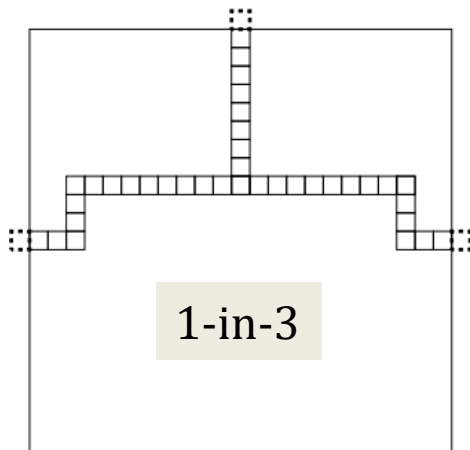
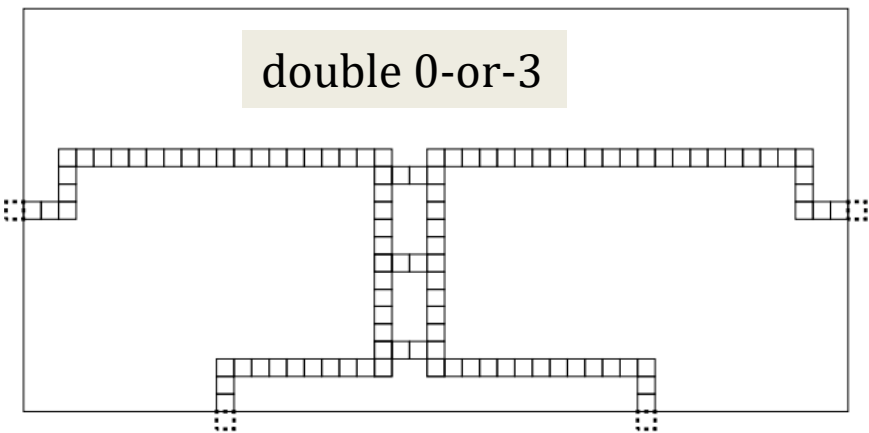
Packing I Trominoes into Polygon

[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]

double 0-or-3

1-in-3

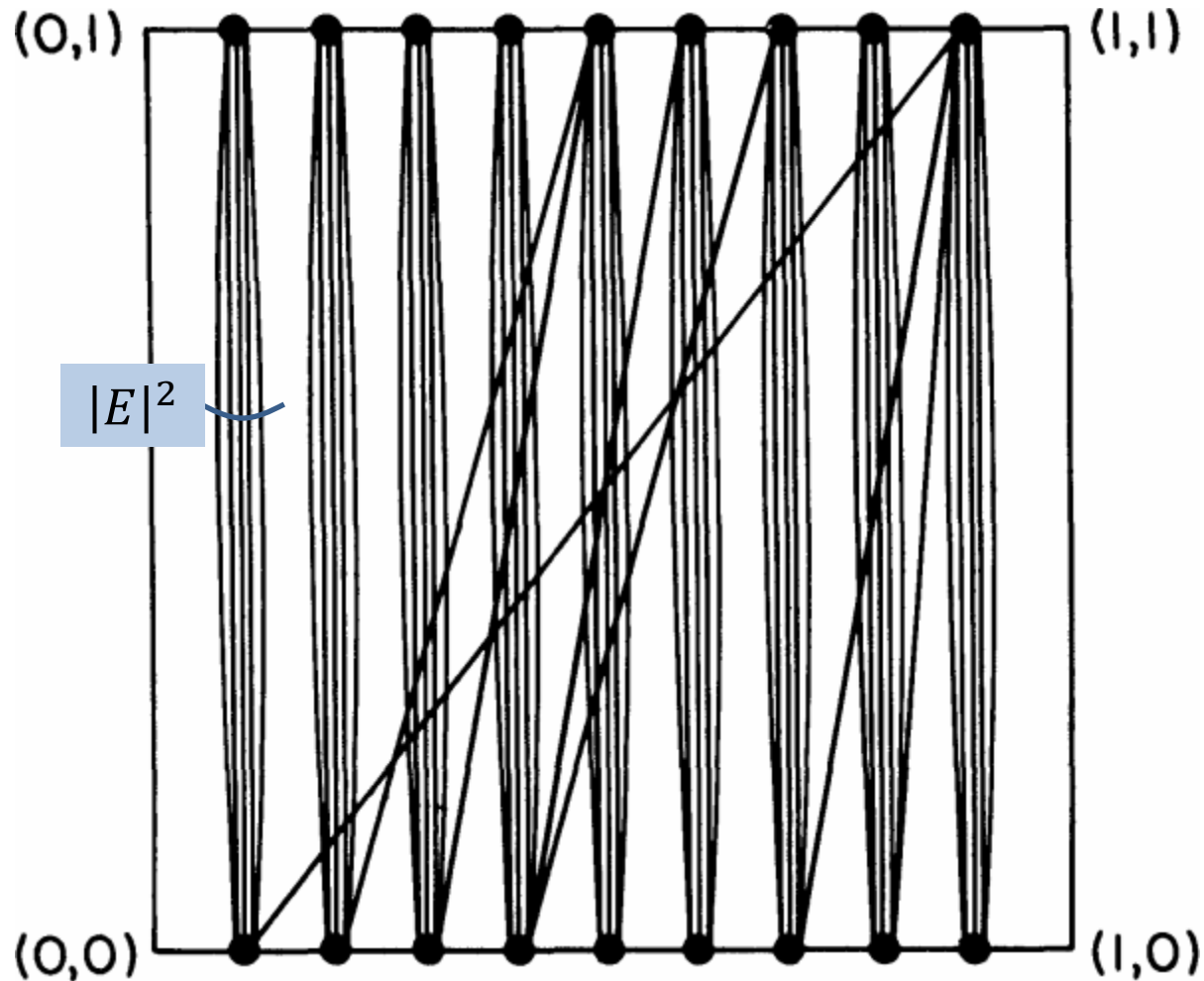
2-in-3



Problem	NP-complete	[Díaz, Petit, Serna 2002]
BANDWIDTH	in general for trees with maximum degree 3 for caterpillars with hair-length ≤ 3 for caterpillars with ≤ 1 hair per backbone vertex for cyclic caterpillars with hair-length 1 for grid graphs and unit disk graphs	[Papadimitriou 1976] [Garey et al. 1978] [Monien 1986] [Monien 1986] [Muradyan 1999] [Díaz et al. 2001a]
MINLA	in general for bipartite graphs	[Garey et al. 1976] [Even and Shiloach 1975]
CUTWIDTH	in general for graphs with maximum degree 3 for planar graphs with maximum degree 3 for grid graphs and unit disk graphs	[Gavril 1977] [Makedon et al. 1985] [Monien and Sudborough 1988] [Díaz et al. 2001a]
MODCUT	for planar graphs with maximum degree 3	[Monien and Sudborough 1988]
VERTSEP	in general for planar graphs with maximum degree 3 for chordal graphs for bipartite graphs for grid graphs and unit disk graphs	[Lengauer 1981] [Monien and Sudborough 1988] [Gustedt 1993] [Goldberg et al. 1995] [Díaz et al. 2001a]
SUMCUT	in general for cobipartite graphs	[Díaz et al. 1991] [Lin and Yuan 1994b] [Golovach 1997] [Yuan et al. 1998]
EDGEBIS	in general for graphs with maximum degree 3 for graphs with maximum degree bounded for d -regular graphs	[Garey et al. 1976] [MacGregor 1978] [MacGregor 1978] [Bui et al. 1987]

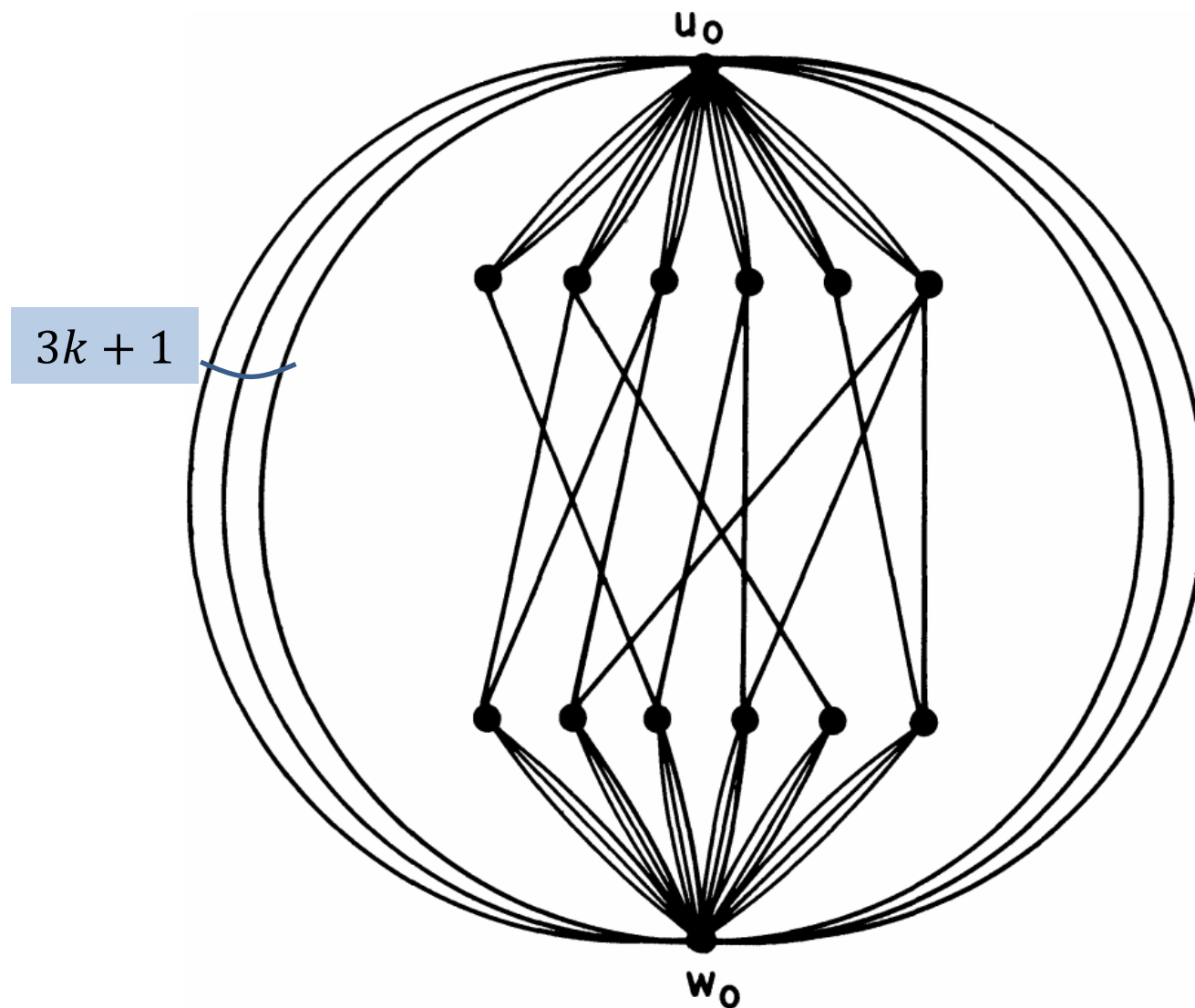
Bipartite Crossing Number

[Garey & Johnson 1983]



Crossing Number is NP-Complete

[Garey & Johnson 1983]

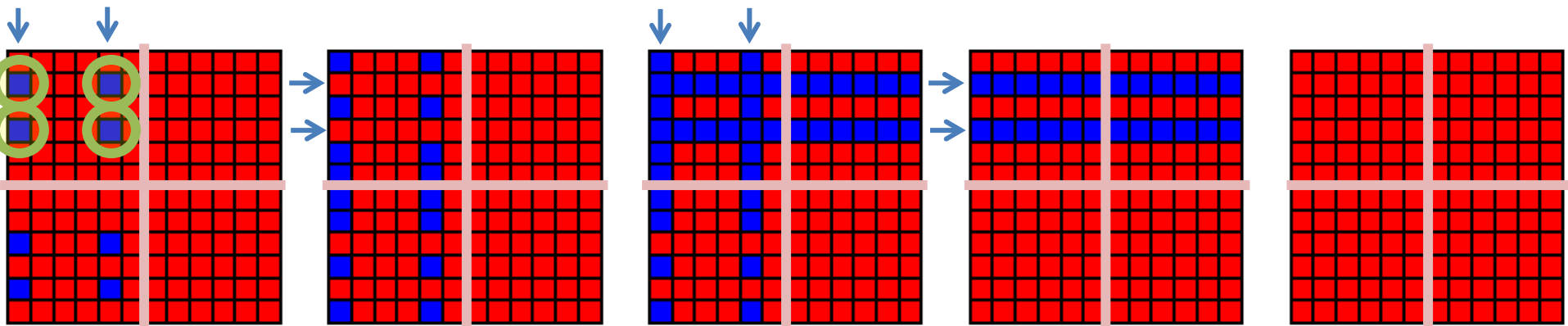
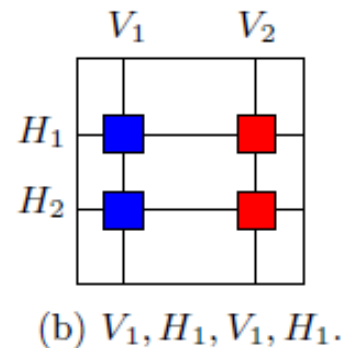




How To Solve Rubik's Cube Faster

[Demaine, Demaine, Eisenstat, Lubiw, Winslow 2011]




- Kill $\Theta(\log n)$ birds with $\Theta(1)$ stones
- Look for cubies arranged in a grid that have the same solution sequence
 - $X \times Y$ grid can be solved in $\Theta(X + Y)$ moves instead of the usual $\Theta(X \cdot Y)$ moves
 - Can always find $\Theta(\log n)$ -factor savings like this



Optimal Rubik's Cube Solutions

[Demaine, Demaine, Eisenstat, Lubiw, Winslow 2011]

- NP-hard to solve a specified subset of $n \times n \times 1$ “Rubik's Square” using fewest possible moves

-  important & solved
-  important & unsolved
-  unimportant / don't care / chameleon

first x_2 between first x_1 & first x_3

- Open: NP-hard if all cubies are important?
[Erickson 2010]

