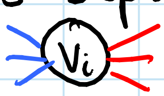
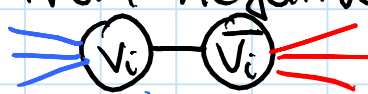
6.890	Lecture 7	Sept. 24, 2014

Planar 3SAT:

[Lichtenstein – SICOMP 1982]

- NP-hard special case of 3SAT
- variable-clause bipartite graph is planar
  ↳ edge (v_i, c_j) whenever v_i or $\overline{v_i}$ is in c_j

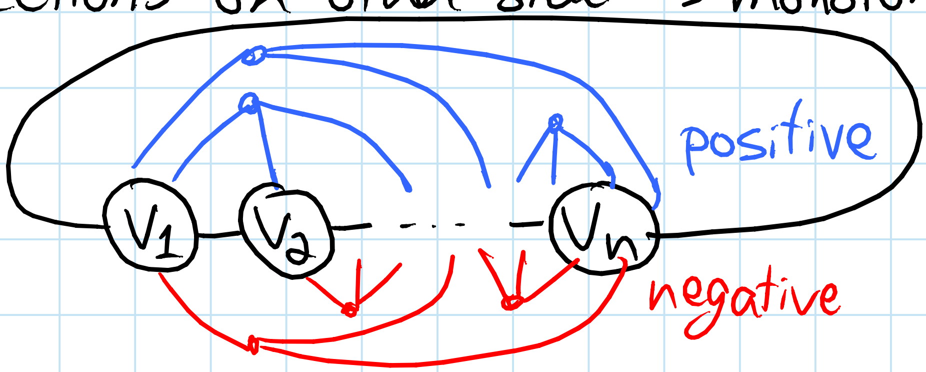
+ remains planar after connecting variables in a cycle: $v_1 \to v_2 \to \cdots \to v_n \to v_1$
  - OR after connecting variables & clauses in a cycle [Dyer & Frieze 1986]

+ remains planar if we require v_i's positive connections separated from negative connections
  i.e. split  into  [Lichtenstein]

  positive connections negative connections

  sometimes called "strongly planar"

+ remains planar if we require all positive connections on one side of cycle & negative connections on other side ⇒ monotone 3SAT



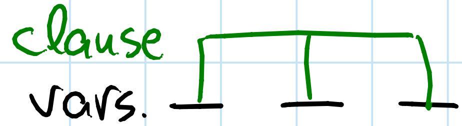
[de Berg & Khosravi – COCOON 2010]

- also if ≤ 2 occurrences of each literal & ≤ 3 occ. of each var. [Brunner, Chung, Coulombe, Demaine, Gomez, Lynch – 2023]
- reductions from 3SAT

(essentially Lichtenstein 1982)

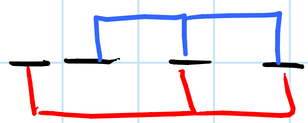
Planar rectilinear 3SAT: [Knuth & Raghunathan 1992]

- variable = horizontal segment on x axis
- clause = horizontal segment (off x axis) + 3 vertical connections to variables
- no crossings/overlap (other than connections)



Planar monotone rectilinear 3SAT: as above

+ monotone 3SAT: each clause all positive or all negative
+ positive clauses above x axis
+ negative clauses below x axis



[de Berg & Khosravi – COCOON 2010]

- reduction from planar rectilinear 3SAT

Careful:

- if all clauses on one side of variable cycle (above x axis in planar rectilinear 3SAT) then ∈P via tree dynamic program

⇒ if clauses also connected in a path then ∈P (would force clauses on same side)
(wanted this e.g. for Push-1/Nintendo)

Planar 1-in-3SAT: [Dyer & Frieze 1986]
- NP-hard special case of 1-in-3SAT
- variable-clause bipartite graph is planar
+ remains planar after connecting variables in a cycle: $v_1 \to v_2 \to \cdots \to v_n \to v_1$
  - OR after connecting variables & clauses in a cycle

Reduction from Planar 3SAT:
- clause gadget

Planar positive 1-in-E3SAT: no negations
→ exactly 3 distinct variables/clause
also [Laroche 1993] ← [Mulzer & Rote - J.ACM 2008]
+ remains planar after connecting variables a cycle: $v_1 \to v_2 \to \cdots \to v_n \to v_1$

Rectilinear ... :
- variable = horizontal segment on x axis
- clause = horizontal segment (off x axis) + 3 vertical connections to variables

Reduction from Planar 3SAT:
- equal & not-equal gadgets
- remove negations
- expand clauses (2 cases: u=0 or 1)

Careful: Planar NAE 3SAT is polynomial!

[Moret – SIGACT News 1988]

Reduction to Planar Max Cut: 2-color vertices of planar graph to maximize red-blue edges

↳ ∈ P [Orlova & Dorfman 1972] [Hadlock – SICOMP 1975]

(in dual, red-blue edges are non-doubled edges in Chinese Postman problem)

- variable gadget / wire
- NAE clause

Planar X3C: [Dyer & Frieze 1986]

- bipartite graph of elements vs. 3-sets is planar
- reduction from planar 1-in-3SAT

Planar 3DM: [Dyer & Frieze 1986]

- special case where elements are 3-colored & each 3-set is trichromatic
+ remains planar if elements connected in cycle
- reduction from planar 1-in-3SAT

Planar vertex cover: [Lichtenstein 1982]

- given a planar graph
- choose k vertices to hit all edges

- reduction from planar 3SAT
  - variable gadget: even cycle
  - clause gadget: triangle
- maximum degree 3

Planar (directed) Hamiltonian cycle: [Lichtenstein 1982]

- reduction from planar 3SAT
  - visit cycle through variables
  - variable gadget = ladder
  - clause gadget
  - can't jump var. → clause → other var.
- same reduction claimed for undirected

Shakashaka [Guten 2008; Nikoli 2012–]

- reduction from Planar 3SAT

Flattening fixed-angle chains:

- reduction from Partition [Soss & Toussaint 2000]
- reduction from planar monotone rectilinear 3SAT [Demaine & Eisenstat 2011]