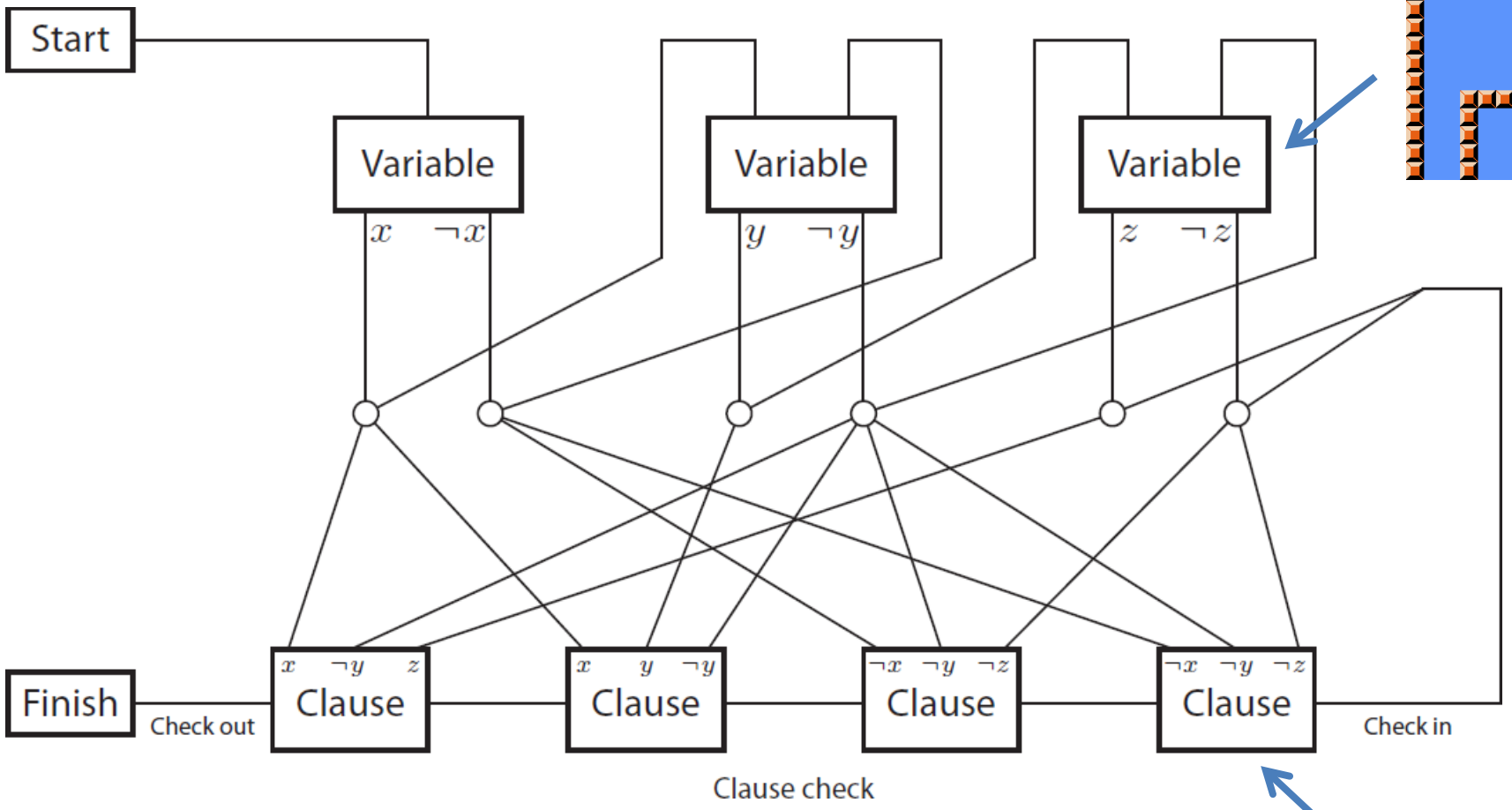
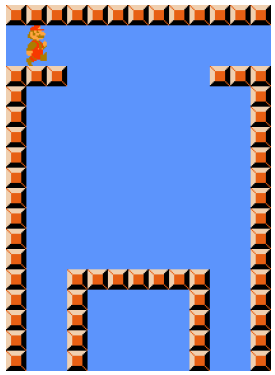
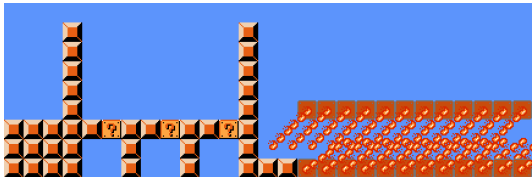
Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]





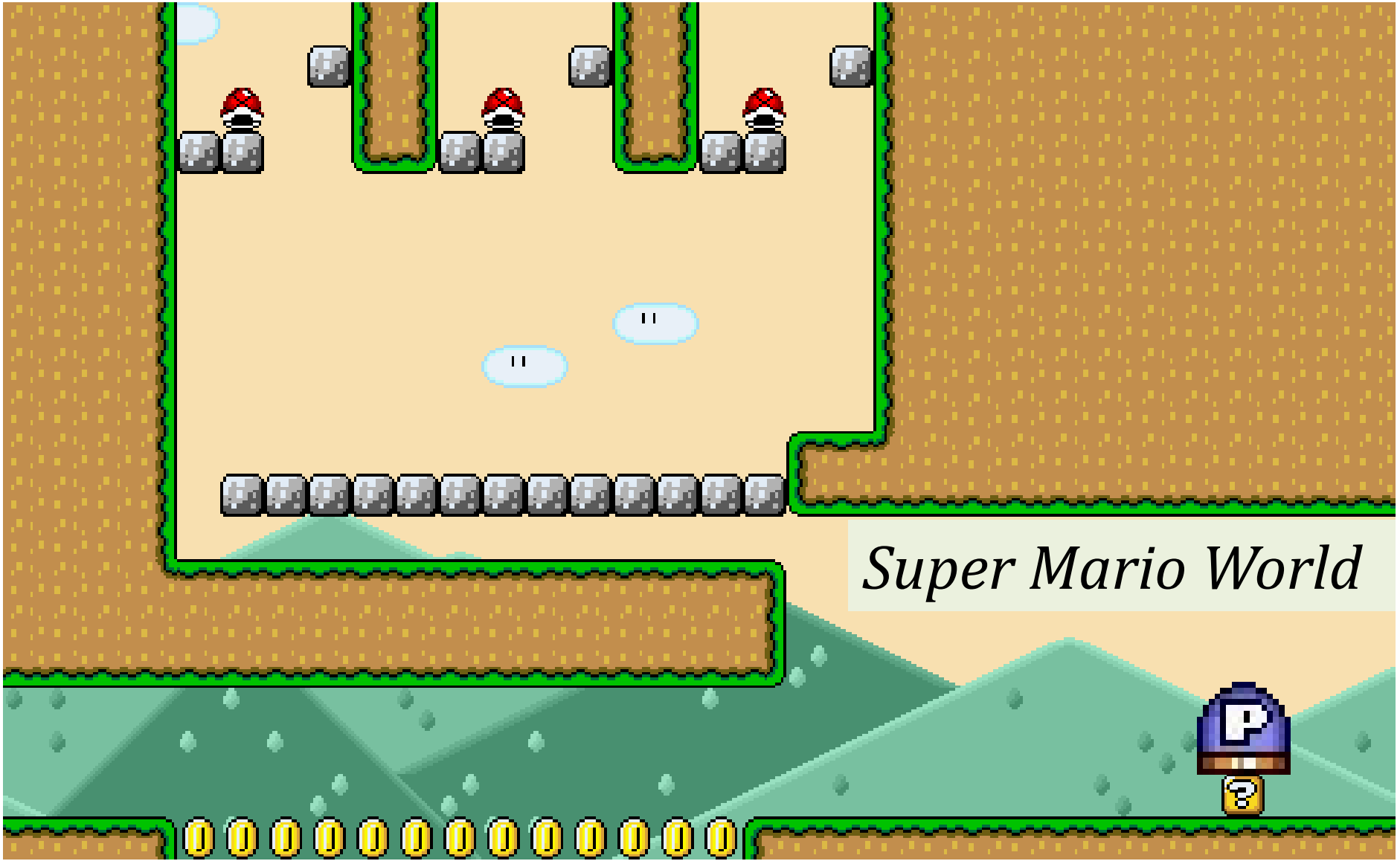


$(x \text{ OR } \neg y \text{ OR } z)$ & $(x \text{ OR } y \text{ OR } \neg y)$ &
$(\neg x \text{ OR } \neg y \text{ OR } \neg z)$ & $(\neg x \text{ OR } \neg y \text{ OR } \neg z)$

Super Mario World is NP-Hard

[Aloupis, Demaine, Guo 2012]

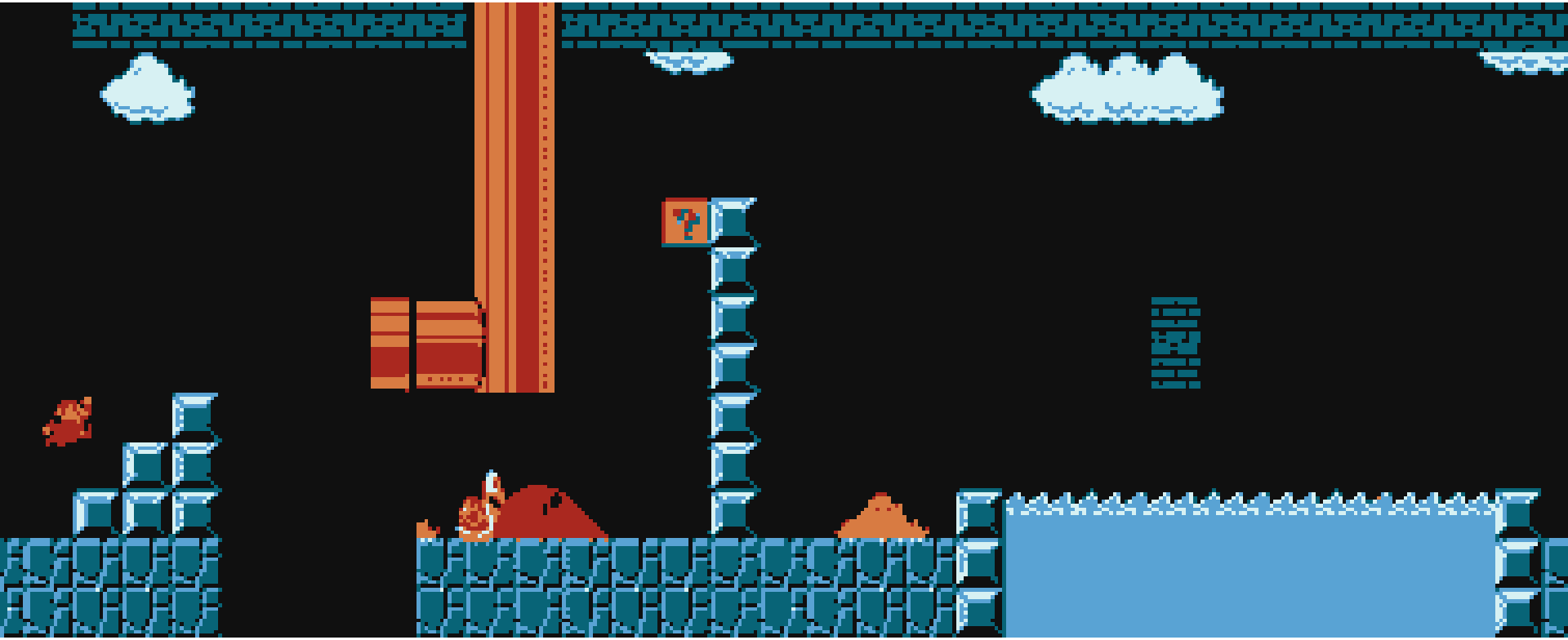
clause



Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

- **Glitch:** Wall jump



Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

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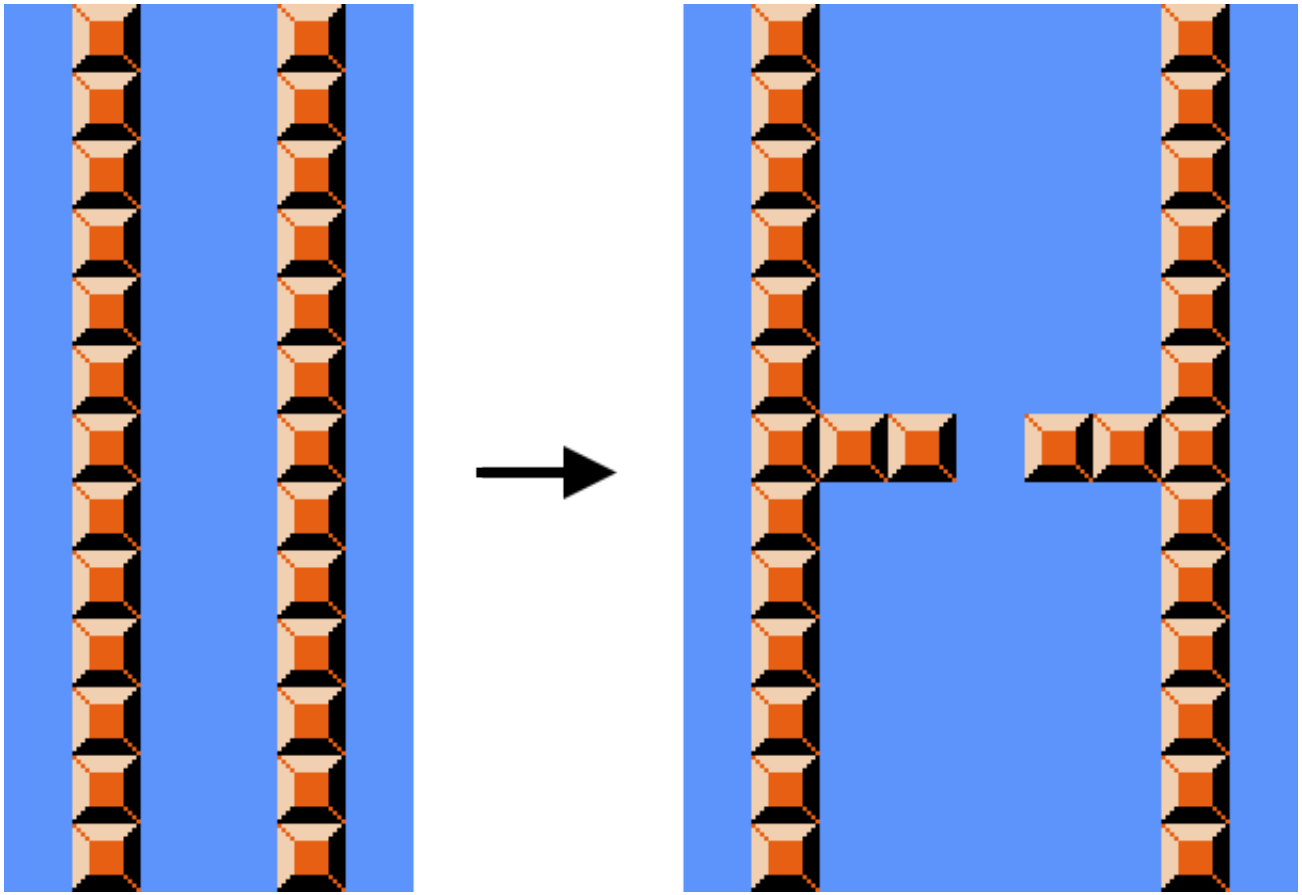
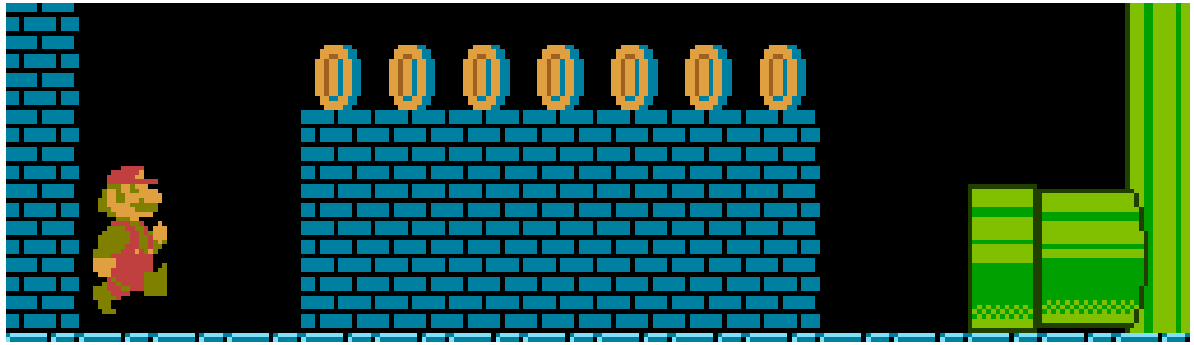
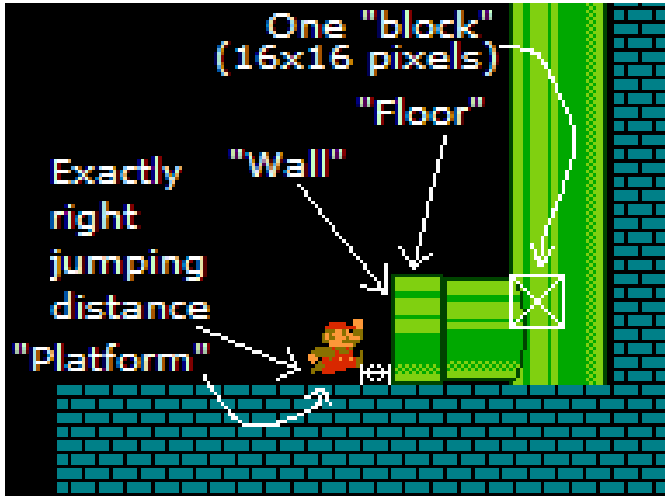


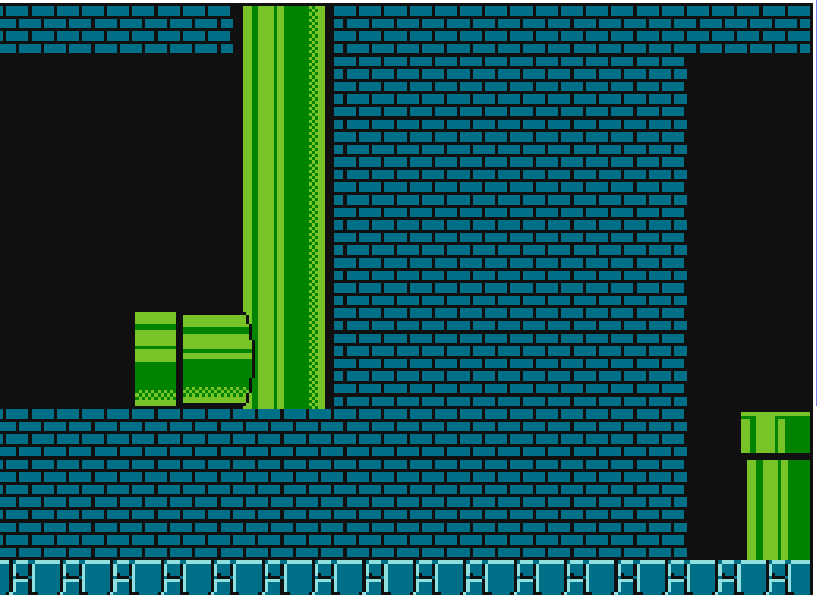
Super Mario Bros. is NP-Hard

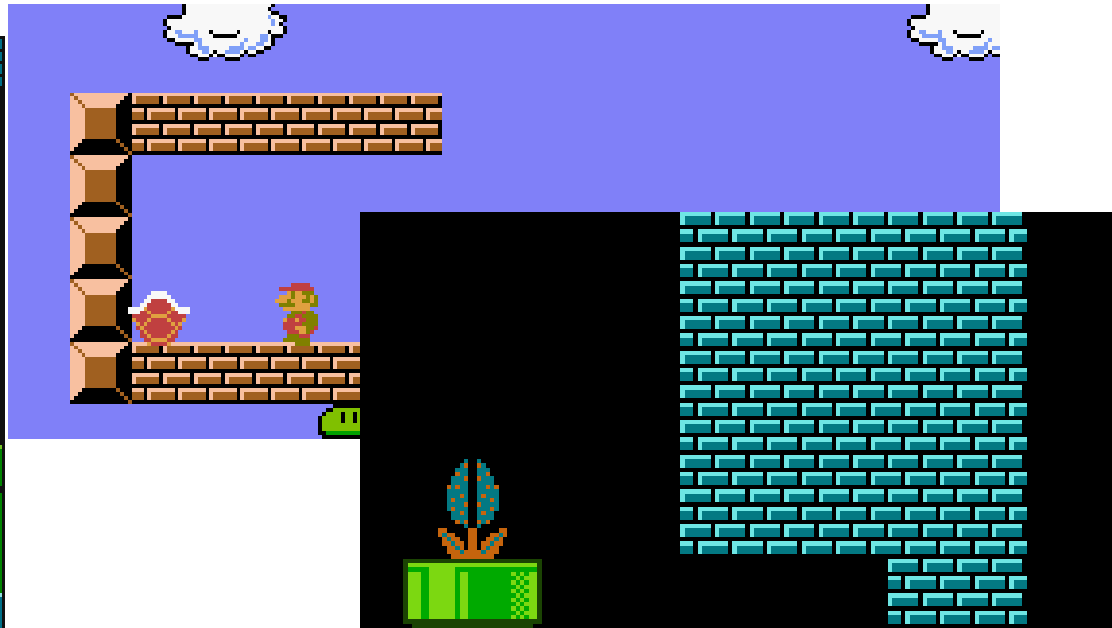
[Aloupis, Demaine, Guo, Viglietta 2014]

- **Glitch:** Jump through walls





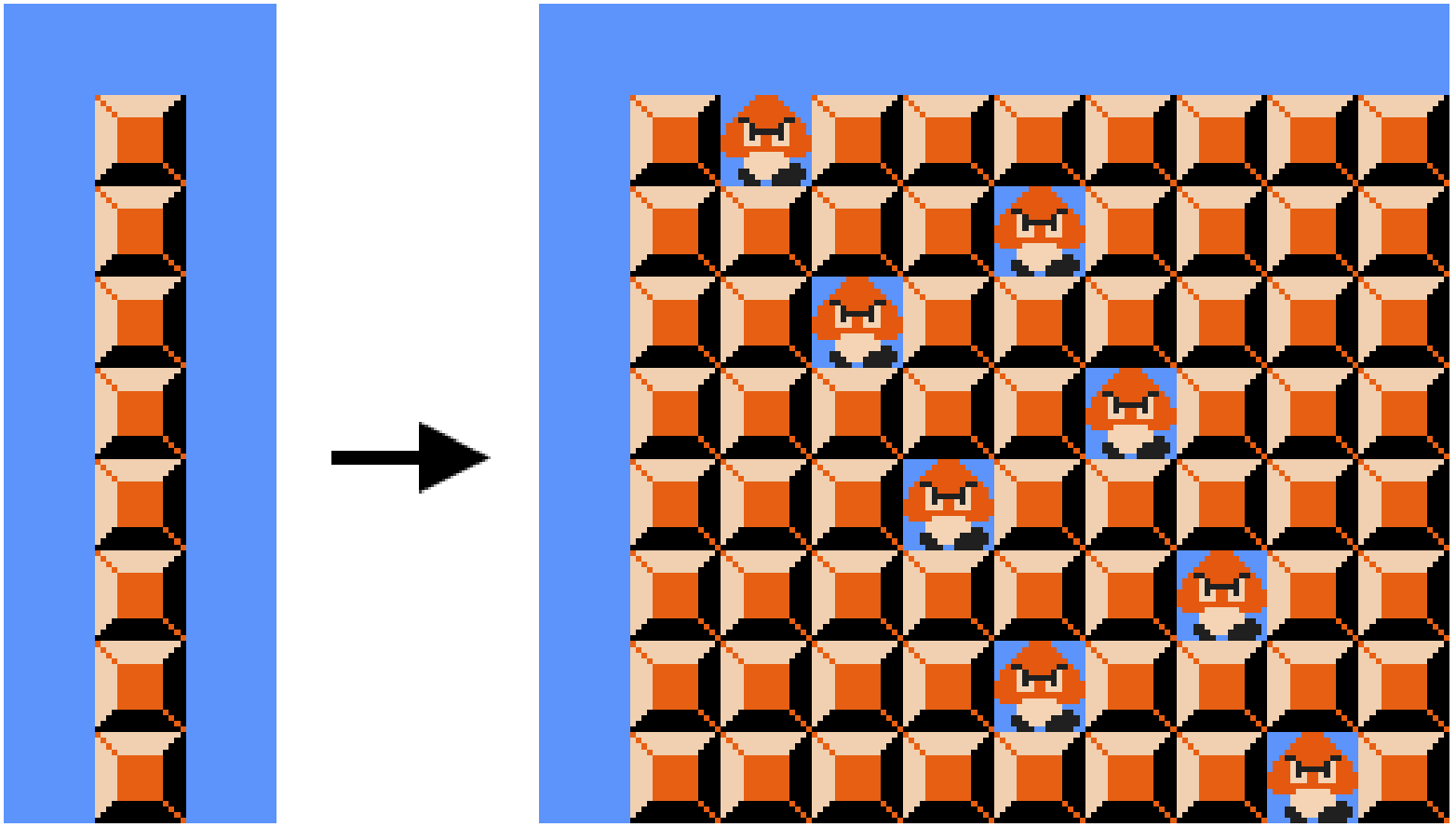




Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

- **Glitch:** Jump through walls



Legend of Zelda Block Pushing

The Legend of Zelda (NES) Walkthrough Part 9 – Level 7 Demon Dungeon
by Zeldajiggmin http://youtu.be/rxjeGXhwkqI

Legend of Zelda Block Pushing

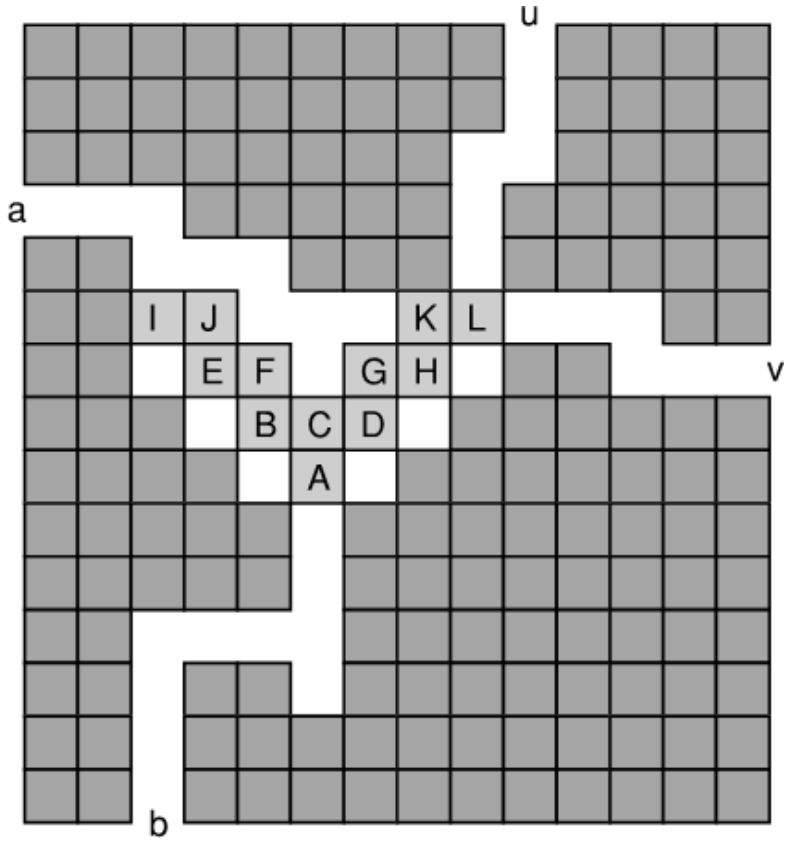
Zelda Link to the Past (Blind) Episode 11: The Hardest Block Pushing Puzzle Ever
by LiamSixx http://youtu.be/B_CUC6ByaSI

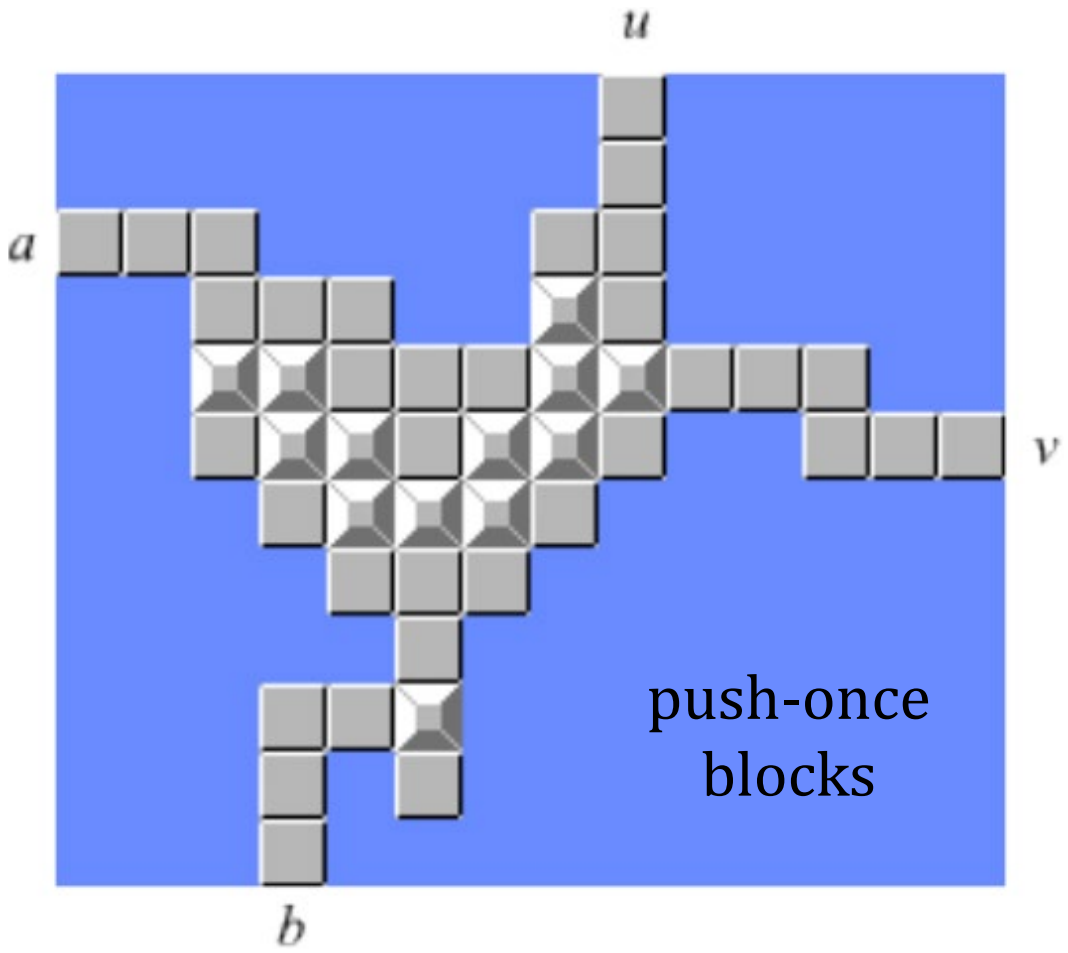
Legend of Zelda Block Pushing

Let's Play Zelda: A Link to the Past #12 – Hookshot
by newfiebangaa http://youtu.be/ZznLKBYcvc0

Legend of Zelda is NP-hard

[Aloupis, Demaine, Guo 2012]





Legend of Zelda Hookshot

Let's Play Zelda: A Link to the Past #13 – Evil Popcorn Chicken & #15 – Flame On
by newfiebangaa http://youtu.be/6i_YGCy5krM & http://youtu.be/ezsLn3_KcGs

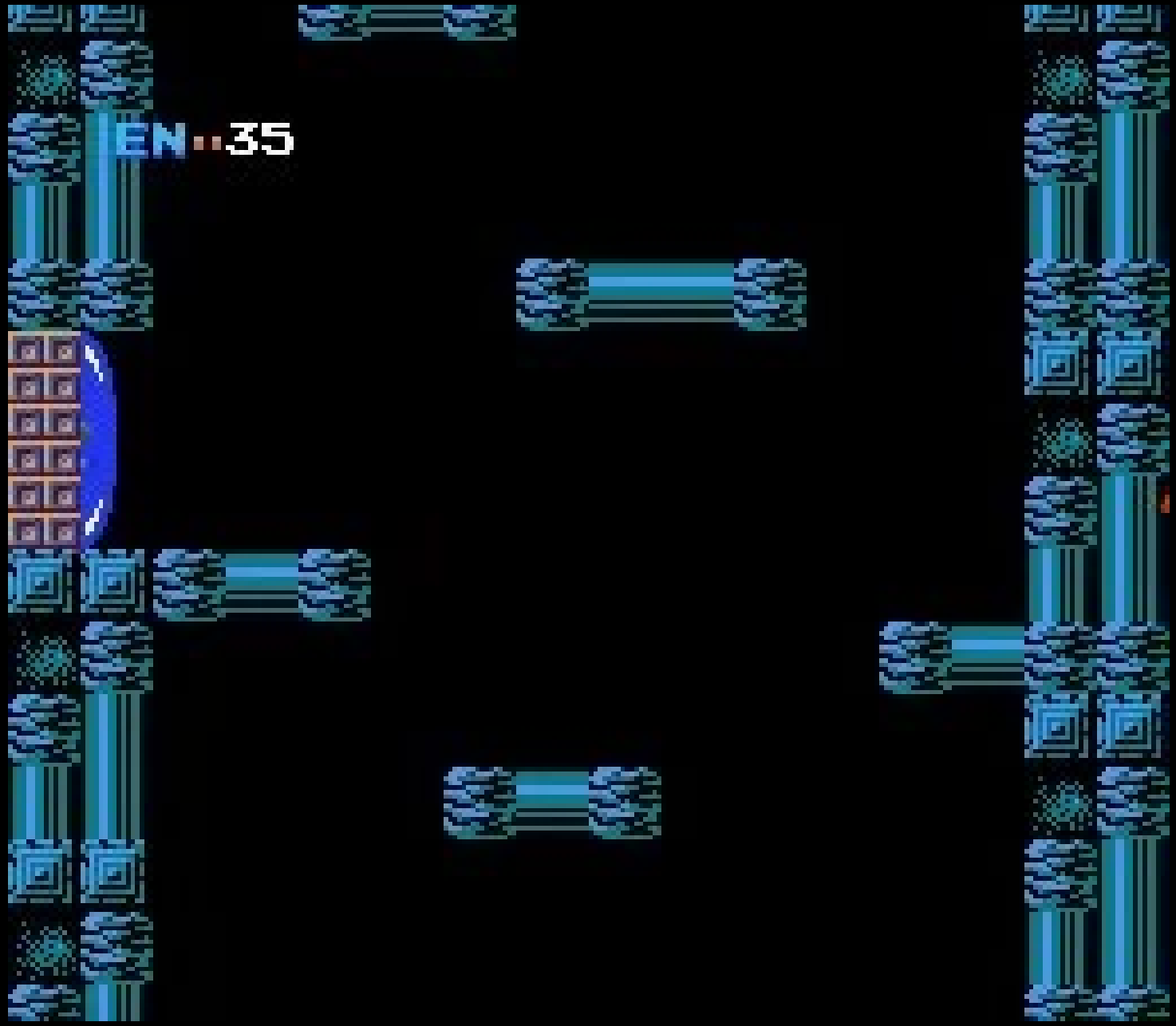
Legend of Zelda is NP-hard

[Aloupis, Demaine, Guo 2012]

A Link to the Past

variable clause crossover

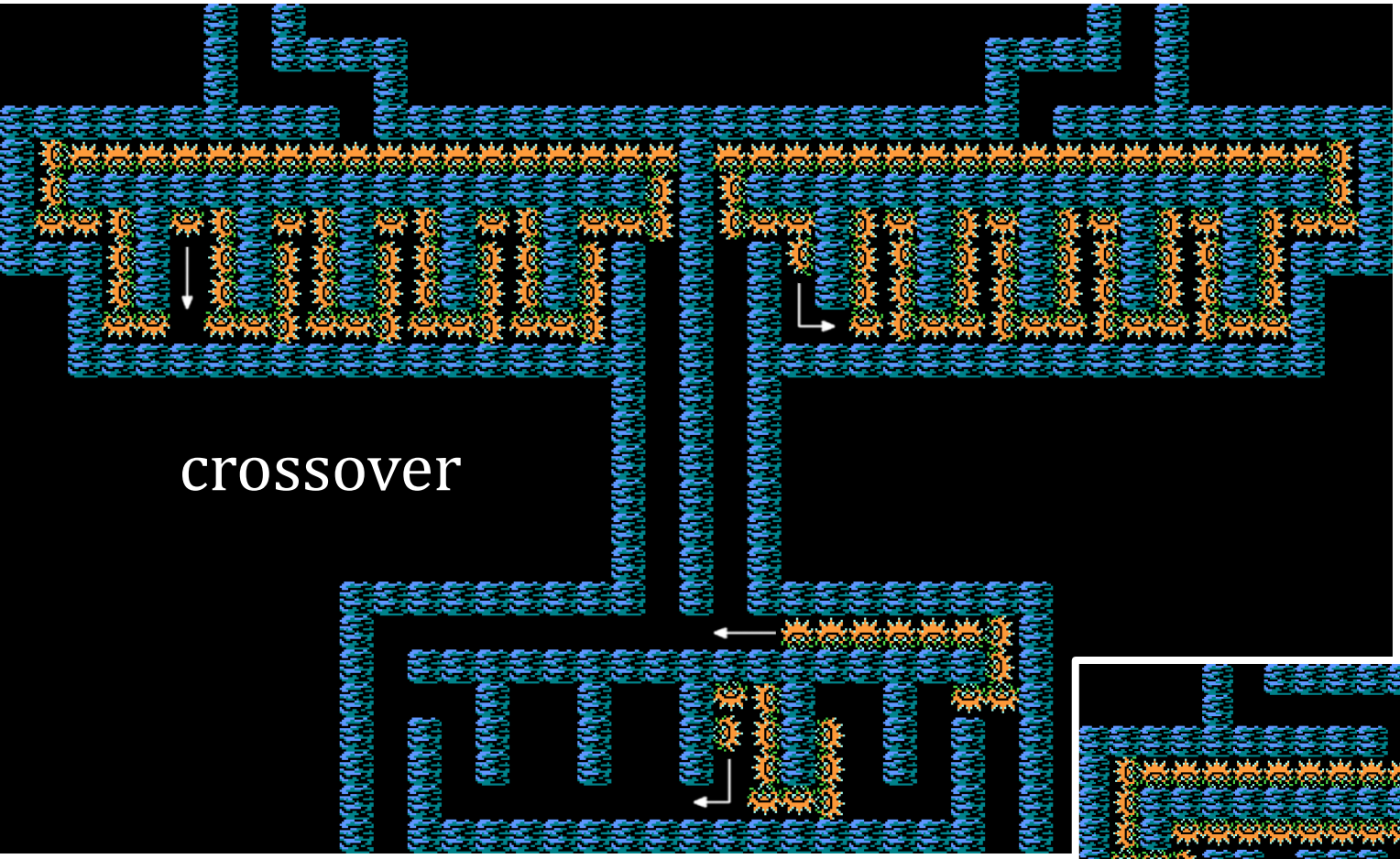
Metroid



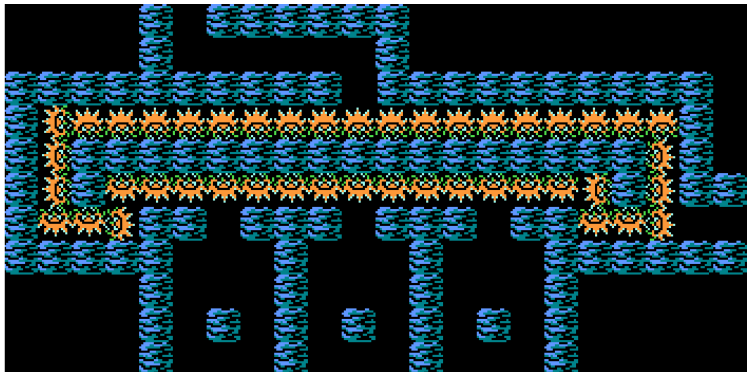
Metroid NES 100% Walkthrough Part 1 Alternate Route
by Timothy Cookson http://youtu.be/INkHYcWvQag

Metroid is NP-complete

[Aloupis, Demaine, Guo, Viglietta 2014]



clause



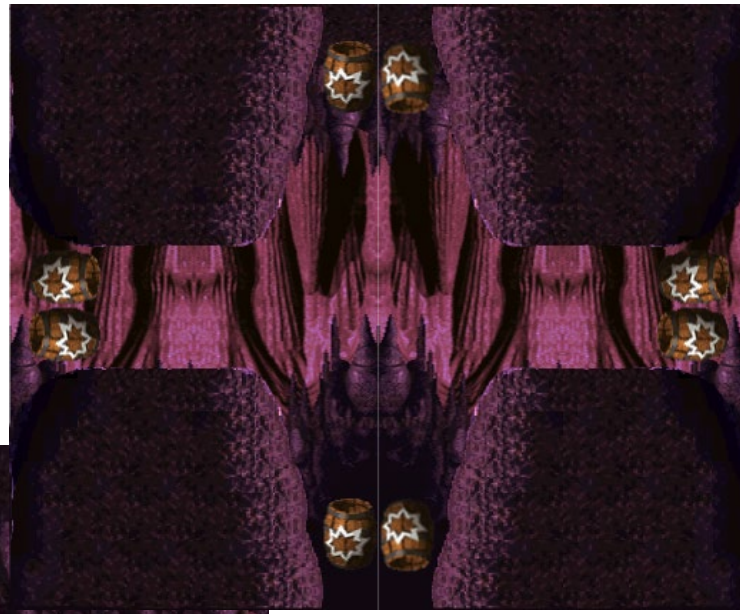
Donkey Kong Country

Metroid NES 100% Walkthrough Part 1 Alternate Route
by Timothy Cookson http://youtu.be/INkHYcWvQag

Donkey Kong Country is NP-hard

[Aloupis, Demaine, Guo 2012]

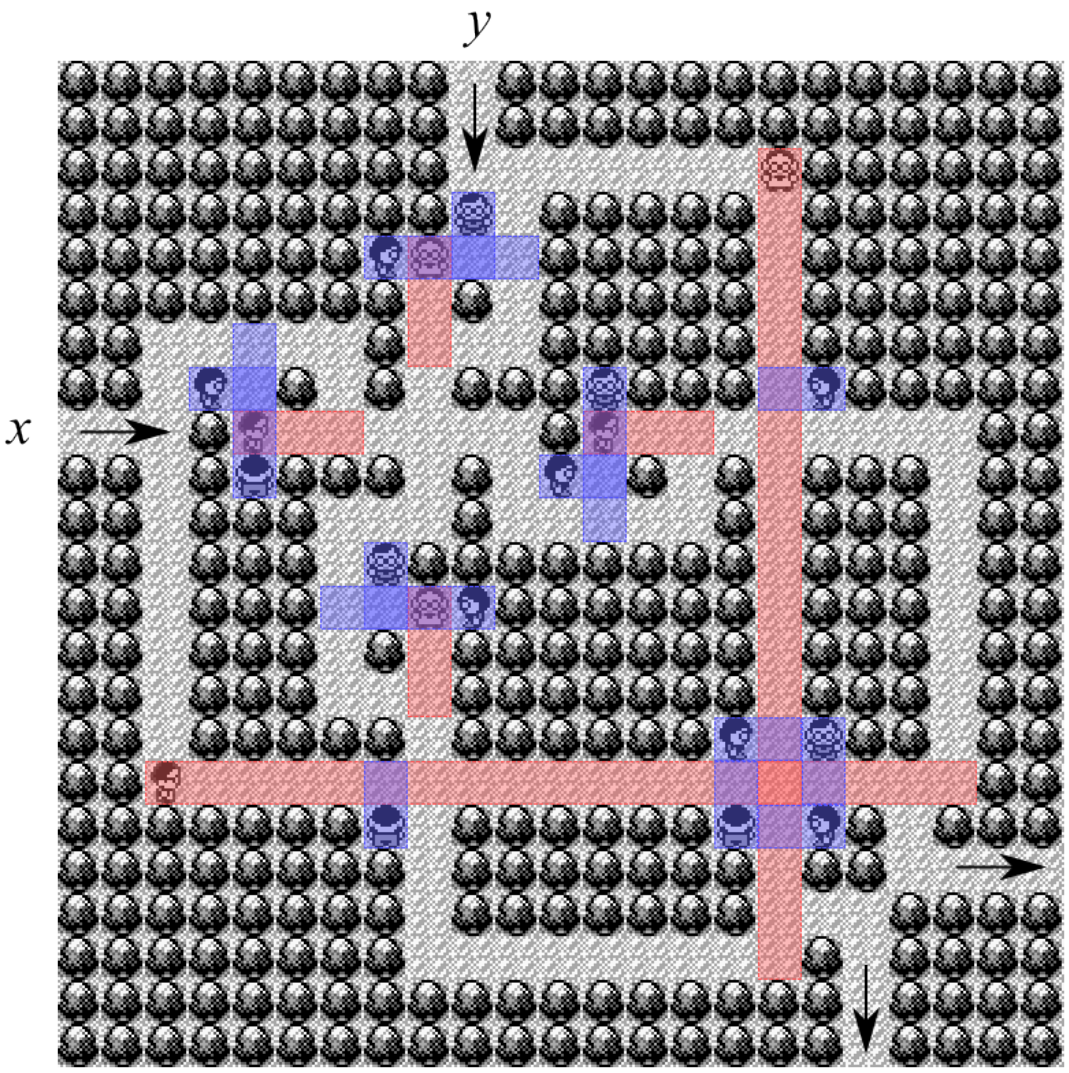
clause



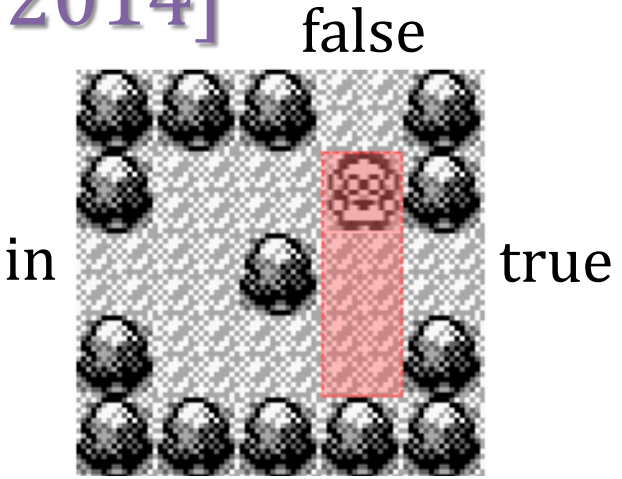
crossover

Pokémon is NP-Hard

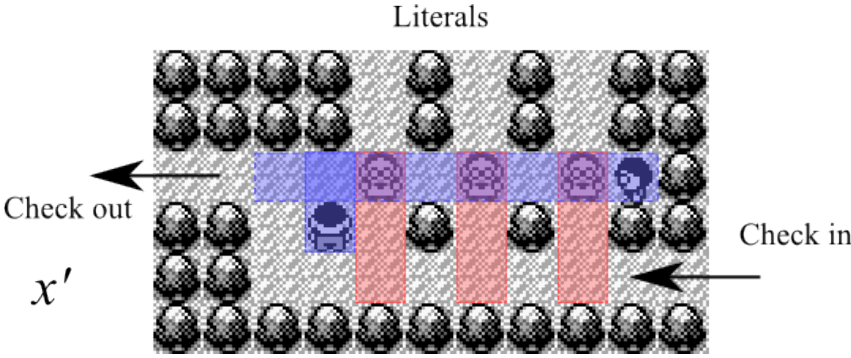
[Aloupis, Demaine, Guo, Viglietta 2014]



crossover



variable



clause

Pokémon is NP-Hard

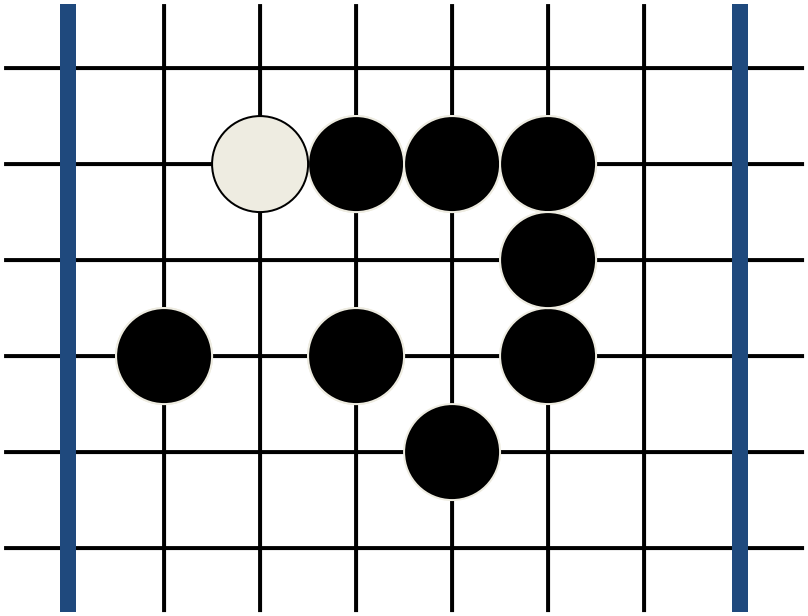
[Aloupis, Demaine, Guo, Viglietta 2014]

"Weak Trainers each hold a **Level 100 Electrode** with maximum **Speed** and equipped with only the **Self Destruct** move. Strong Trainers each hold two **Snorlaxes**, with Speed of 30. The player has no items, and only one Pokémon in his team. For Generation I and II games (**Red/Blue/Yellow** and **Gold/Silver/Crystal** versions respectively), the player holds a **Gastly** which has learned Self Destruct using **TM36**, and its **PP** for its other moves have all been expended, so it can only use Self Destruct in battle. When the player encounters a weak Trainer, the enemy Electrode will move first and use Self Destruct, which deals no damage to Gastly since Self Destruct is a **Normal type attack** and Gastly is **Ghost type**, so the weak Trainer immediately loses. When the player encounters a strong Trainer, Gastly moves first and uses Self Destruct, causing the player to lose (even if it defeats the enemy Snorlax, the opponent holds another one). This implementation only works in Generations I and II since TM36 exists only in **Generation I** and the **Time Capsule** feature in **Generation II** allows a Gastly with Self Destruct to be traded from Generation I to Generation II. In **Generations III, IV, and V**, Gastly can be replaced by **Duskull**, which is allowed to learn the move **Memento**, which serves the same purpose as Self Destruct, via **breeding**."

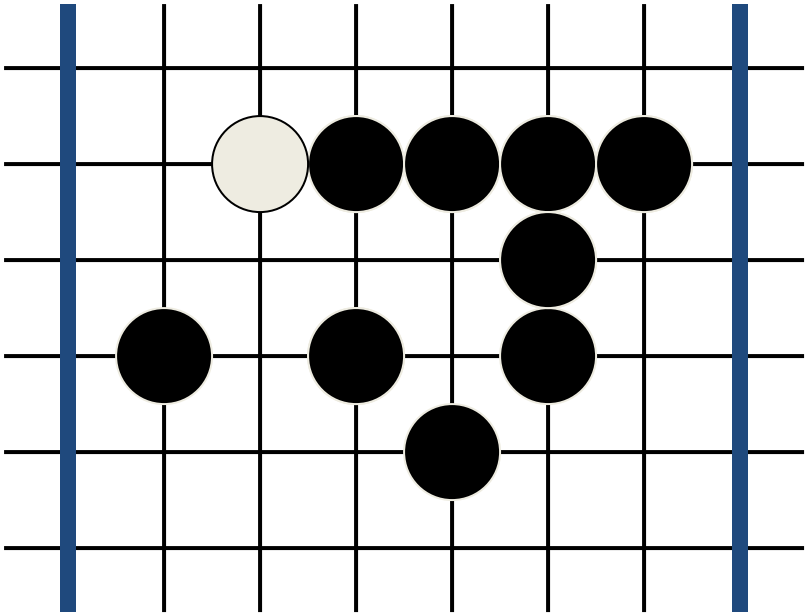
Conway's Phutball (Philosopher's Football)

photo by Rina Sergeeva, 2012

Conway's Phutball (Philosopher's Football)



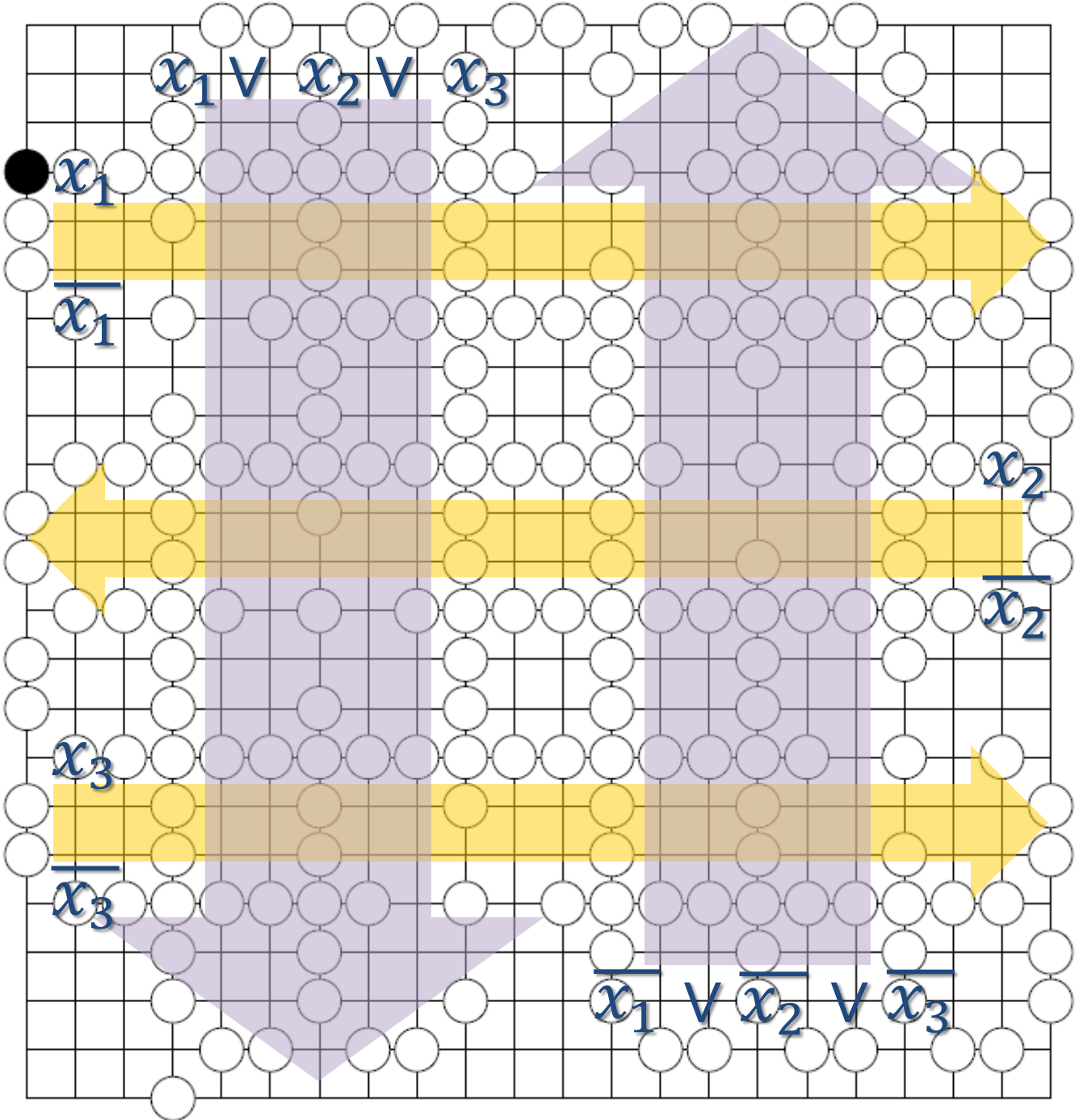
Conway's Phutball (Philosopher's Football)



Phutball

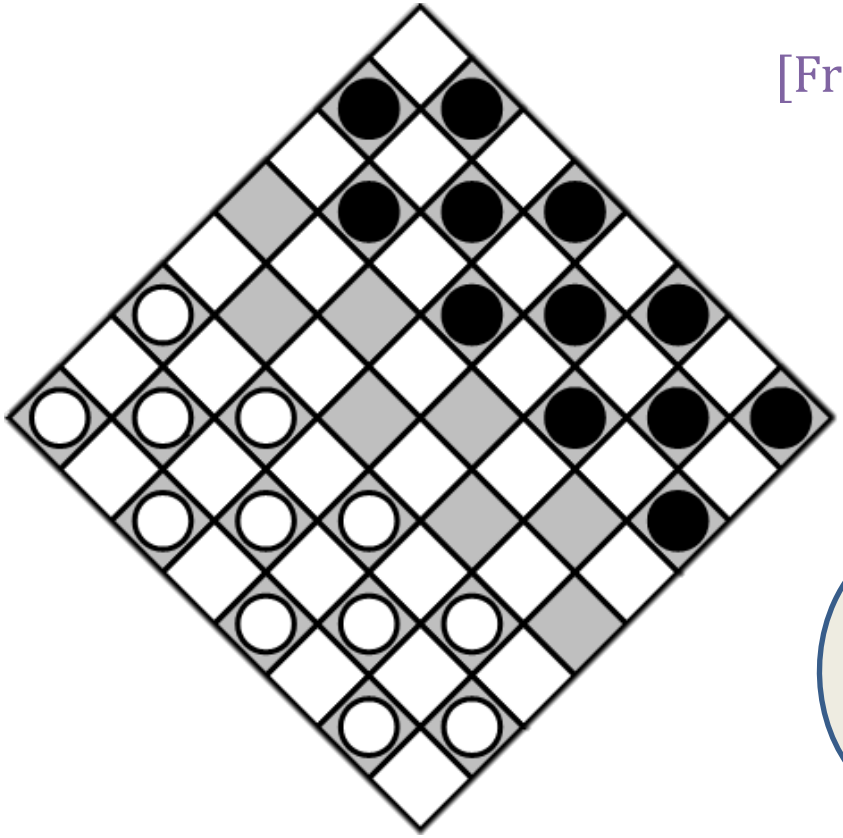
[Conway]

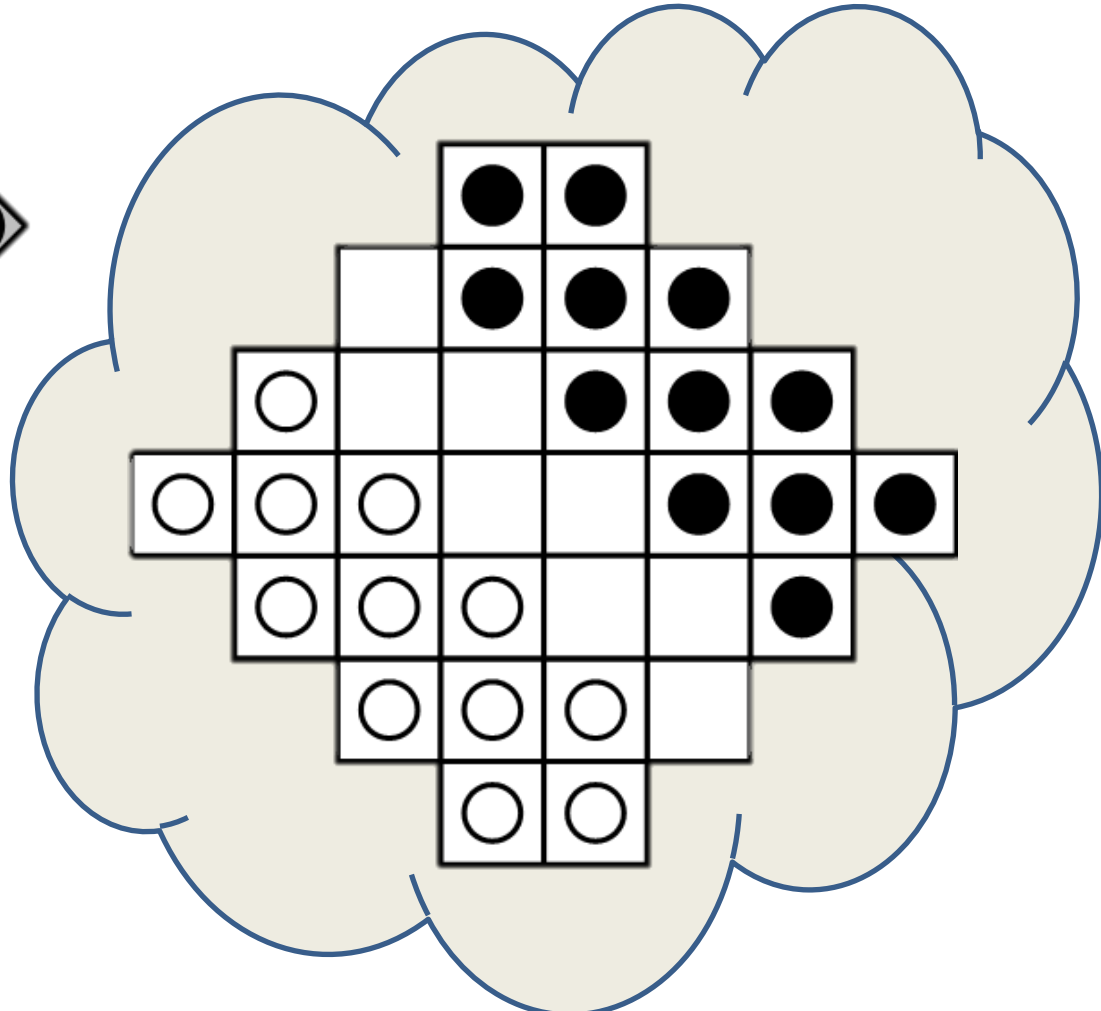
- Mate in 1 is NP-complete [Demaine, Demaine, Eppstein 2000]
- PSPACE-hard [Dereniowski 2009]
- EXPTIME-complete?



Checkers Mate-in-1 is Polynomial

[Fraenkel, Garey, Johnson, Schaefer, Yesha 1978]





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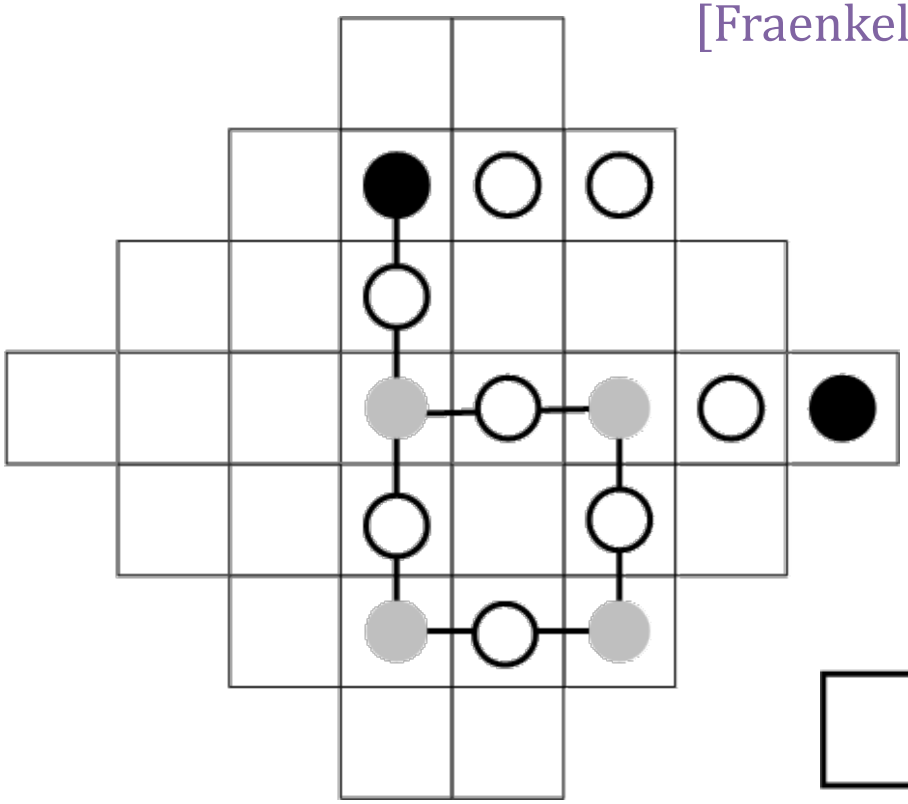
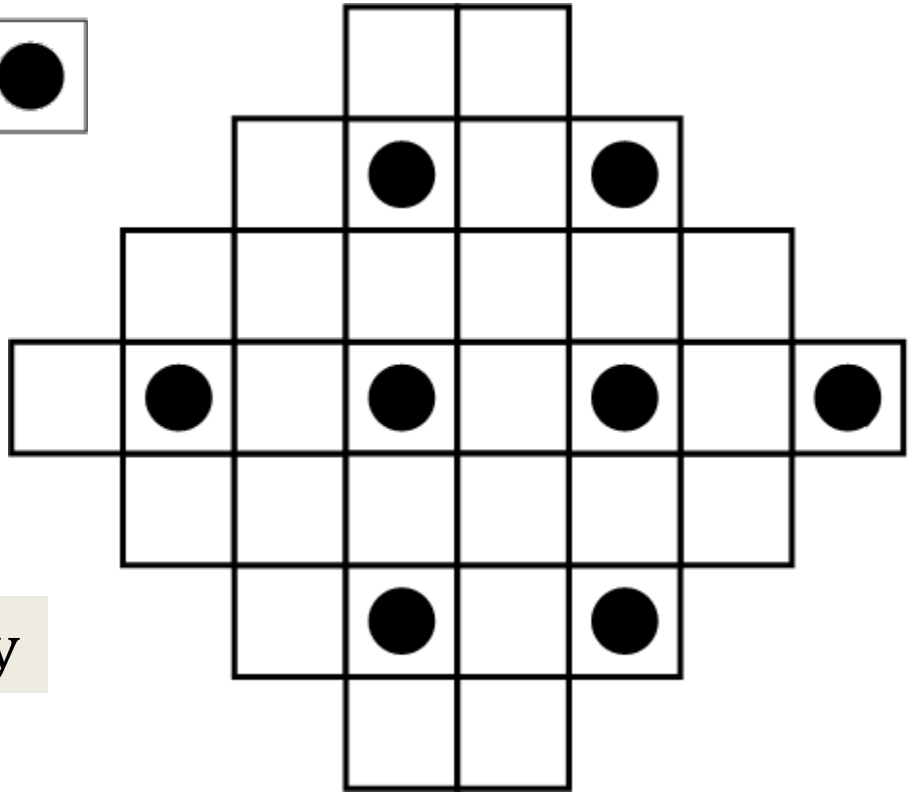


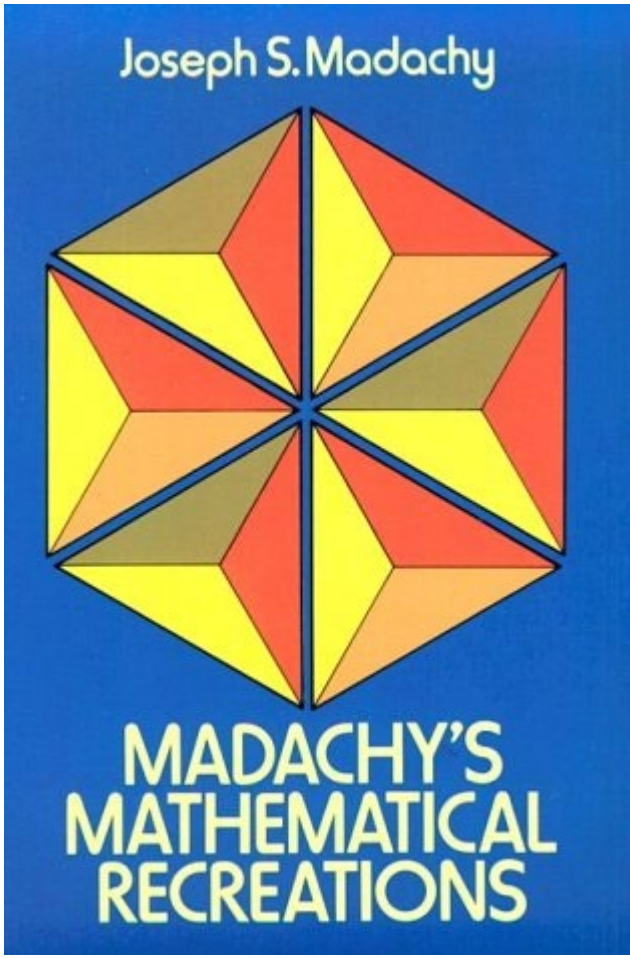


jumps preserve x & y parity

Cryptarithms / Alphametics

```
  SEND
+ MORE
--------
 MONEY
```

S=9, E=5, N=6, D=7,
M=1, O=0, R=8, Y=2



1979, pp. 178–200

Cryptarithms are Strongly NP-complete

[Eppstein 1987]

C = carry of $2y_i \in \{0,1\}$

$$\begin{array}{r} 0\ p\ 0 \\ 0\ p\ 0 \\ \hline 1\ q\ 0 \end{array}$$

variable gadget

$$\begin{array}{r} d_i\ 0\ \ 1\ y_i\ 0\ c_i\ y_i\ 0\ b_i\ y_i\ 0\ a_i\ 0 \\ e_i\ 0\ d_i\ y_i\ 0\ c_i\ y_i\ 0\ b_i\ y_i\ 0\ a_i\ 0 \\ \hline \overline{v}_i\ 0\ e_i\ z_i\ 0\ d_i\ z_i\ 0\ v_i\ z_i\ 0\ b_i\ 0 \end{array}$$

$0 = 0$
$1 = 1$

$$v_i = 2 \cdot 2a_i + C \equiv C \pmod 4$$
$$\overline{v_i} = 2 \cdot (2c_i + C) + 1 + C$$
$$= 4c_i + 3C + 1 \equiv 1 - C \pmod 4$$

clause gadget

$v_a \vee v_b \vee v_c$

$$\begin{array}{r} u_{ab}\ 0\ \ v_a\ 0\ \ 1\ r_i\ 0\ g_i\ w_i\ 0\ f_i\ 0 \\ v_c\ 0\ \ v_b\ 0\ h_i\ r_i\ 0\ g_i\ w_i\ 0\ f_i\ 0 \\ \hline t_i\ 0\ u_{ab}\ 0\ t_i\ s_i\ 0\ h_i\ x_i\ 0\ g_i\ 0 \end{array}$$

$$v_a + v_b + v_c = t_i \in \{1, 2, 3\} \pmod 4$$

Cryptarithms are Strongly NP-complete

[Eppstein 1987, simplified]

C = carry of $2y_i \in \{0,1\}$

$$\begin{array}{l} 0\ p\ 0 \\ \underline{0\ p\ 0} \\ 1\ q\ 0 \end{array}$$

variable gadget

$$\begin{array}{l} b_i\ y_i\ 0\ a_i\ 0 \\ \underline{b_i\ y_i\ 0\ a_i\ 0} \\ v_i\ z_i\ 0\ b_i\ 0 \end{array}$$

$v_i = 2 \cdot 2a_i + C \equiv C \pmod 4$

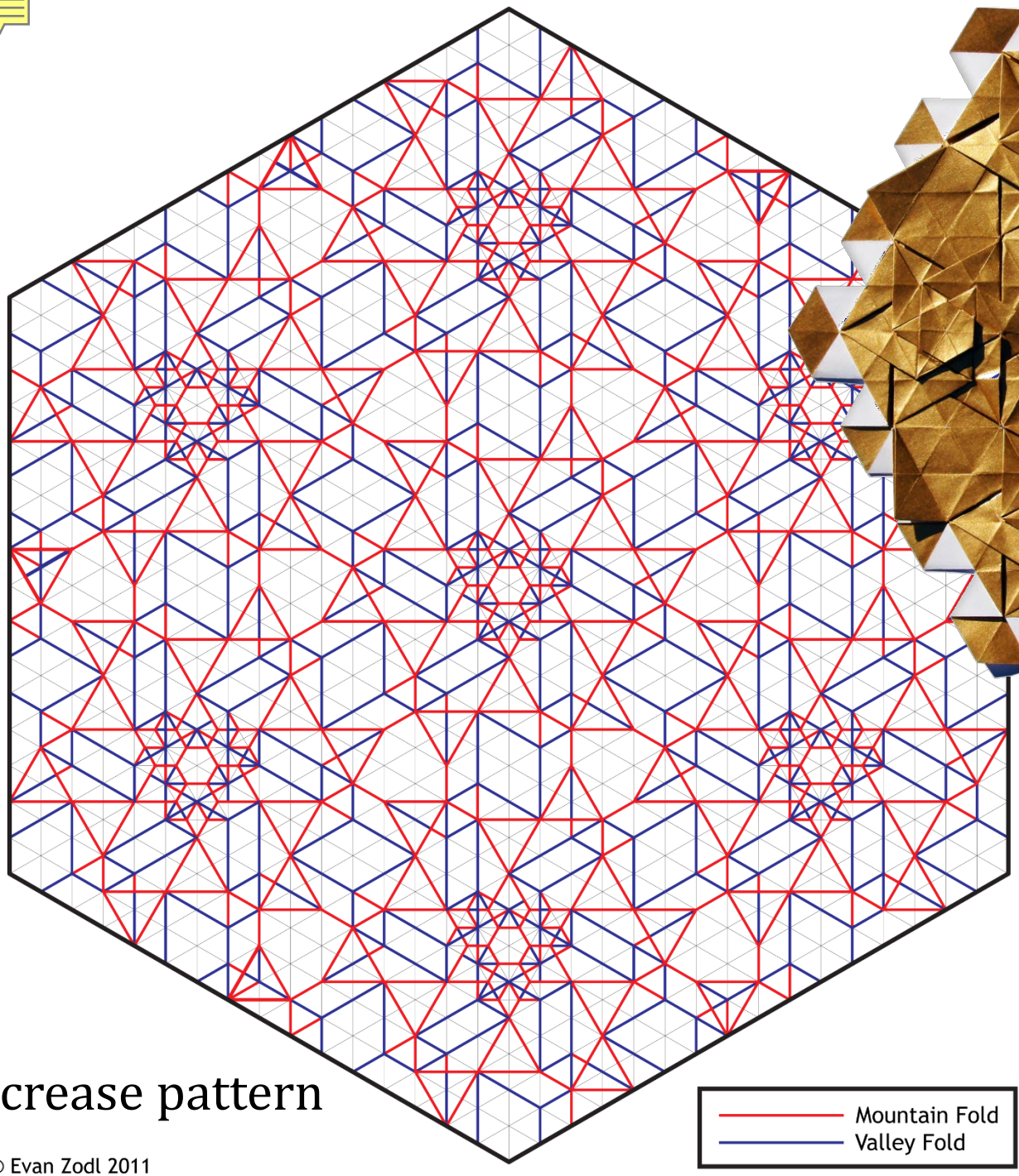
$0 = 0$
$1 = 1$

1-in-3 clause gadget

$v_a \vee v_b \vee v_c$

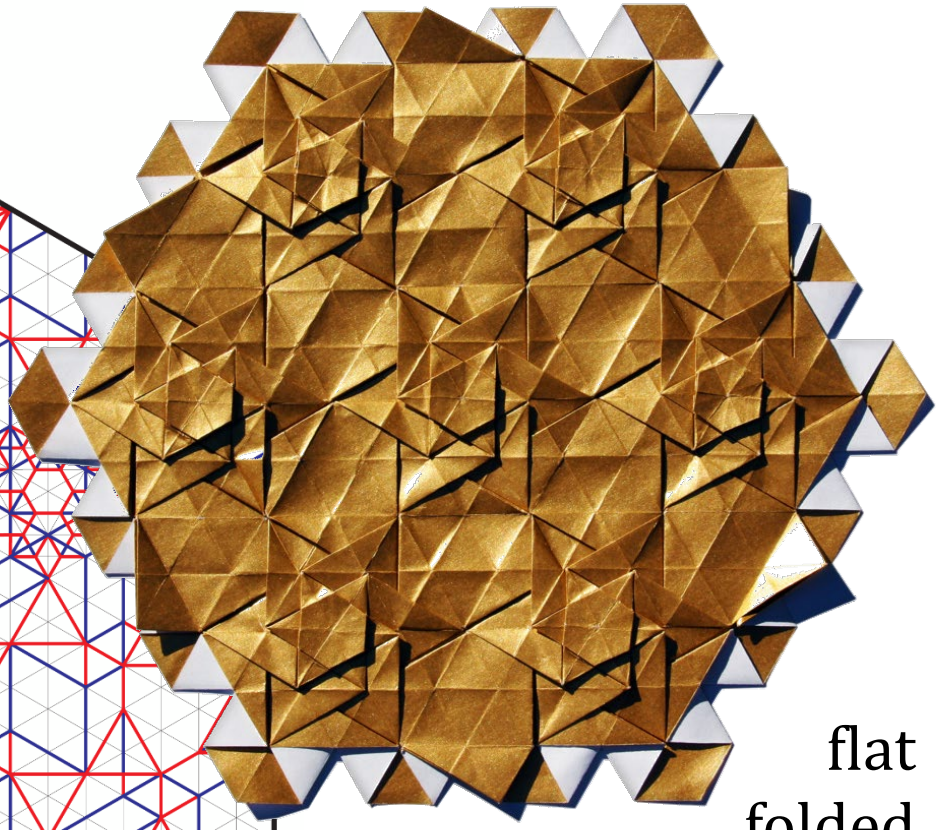
$$\begin{array}{l} u_{ab}\ 0\ \ v_a\ 0\ \ 1\ 0\ g_i\ 0\ f_i\ 0 \\ \underline{v_c\ 0\ \ v_b\ 0\ h_i\ 0\ g_i\ 0\ f_i\ 0} \\ t_i\ 0\ u_{ab}\ 0\ t_i\ 0\ h_i\ 0\ g_i\ 0 \end{array}$$

$v_a + v_b + v_c = 4f_i + 1 \equiv 1 \pmod 4$



crease pattern

Mountain Fold
Valley Fold

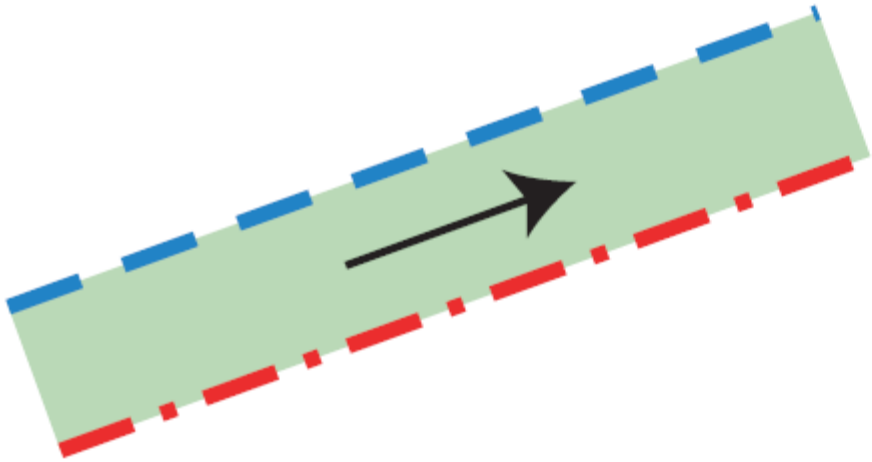


flat
folded
state

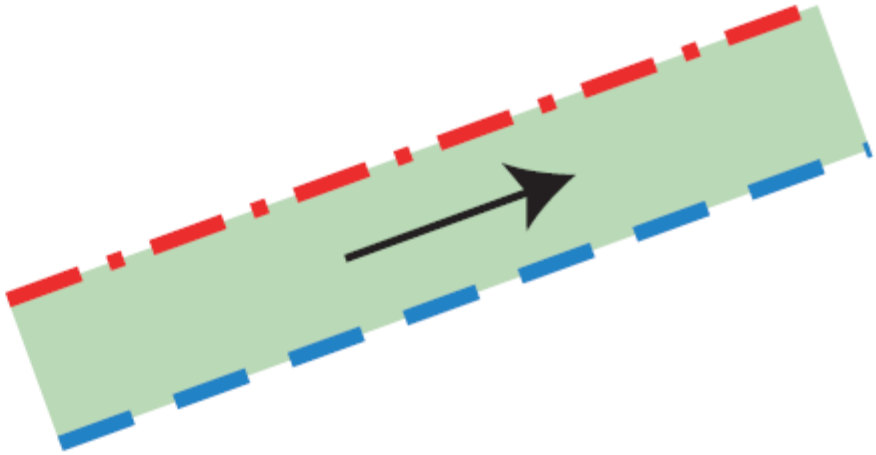
Star Tessellation

Evan Zodl, 2011

wire gadget



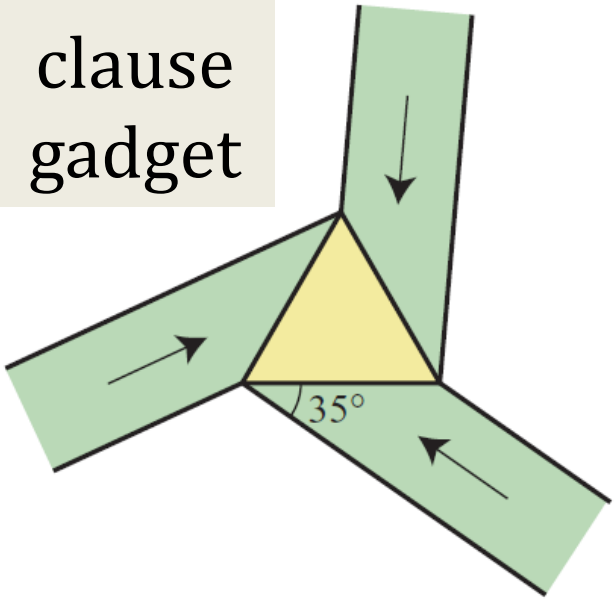
true signal

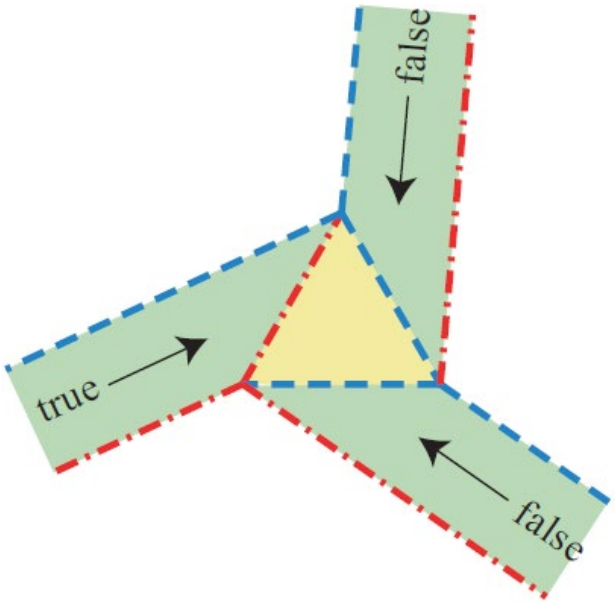


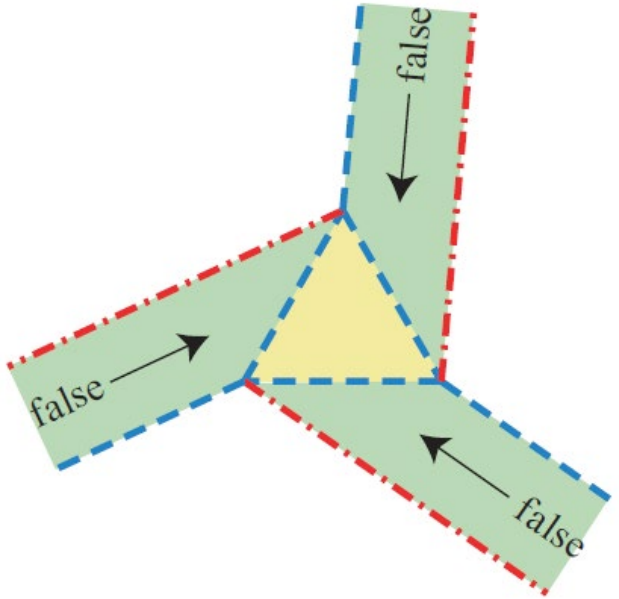
false signal

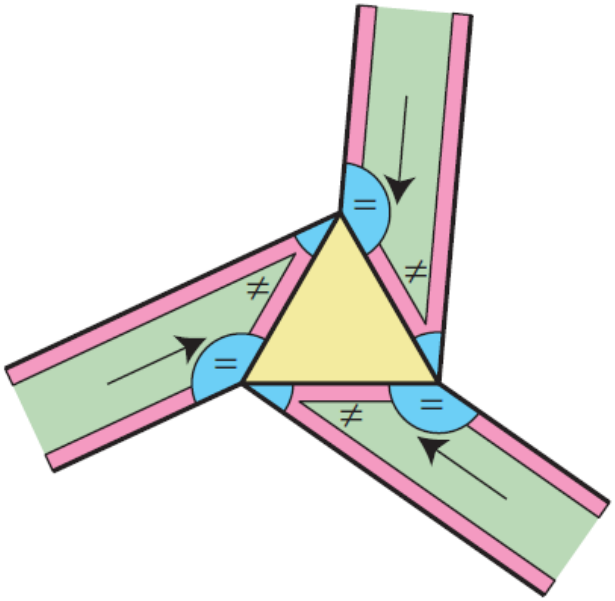
[Bern & Hayes 1996]

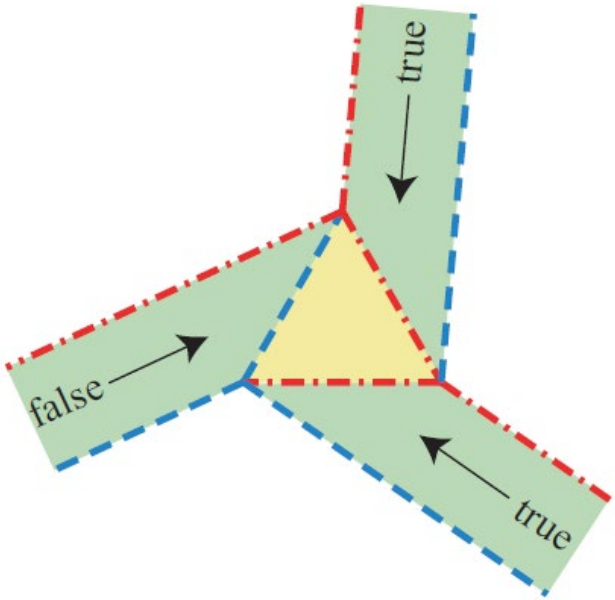
NAE clause gadget

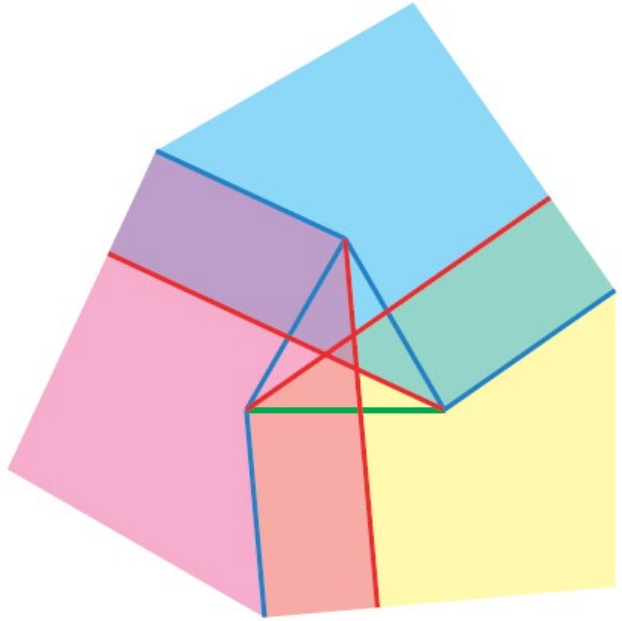






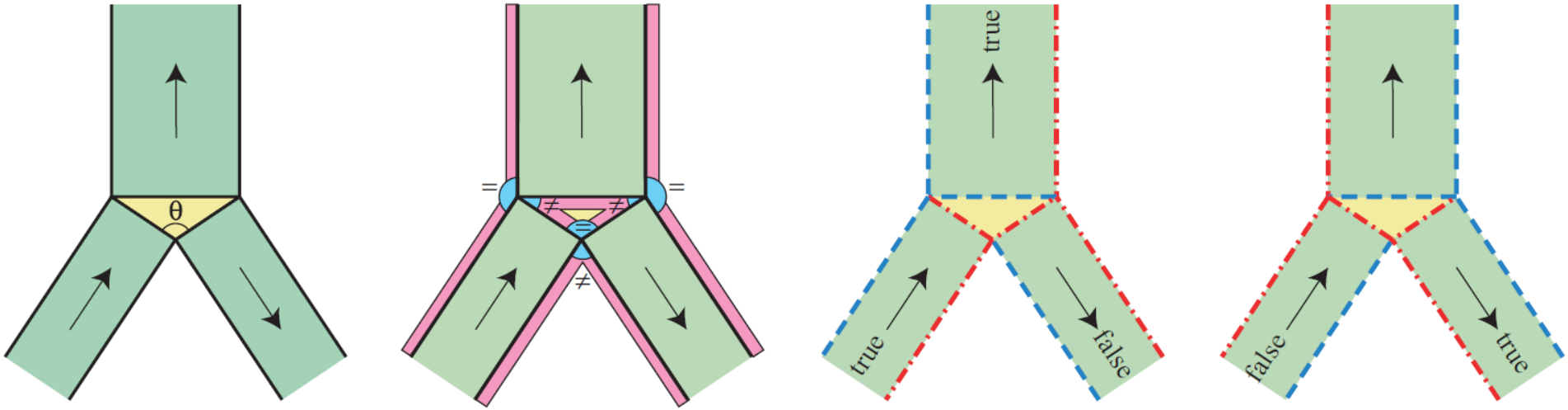




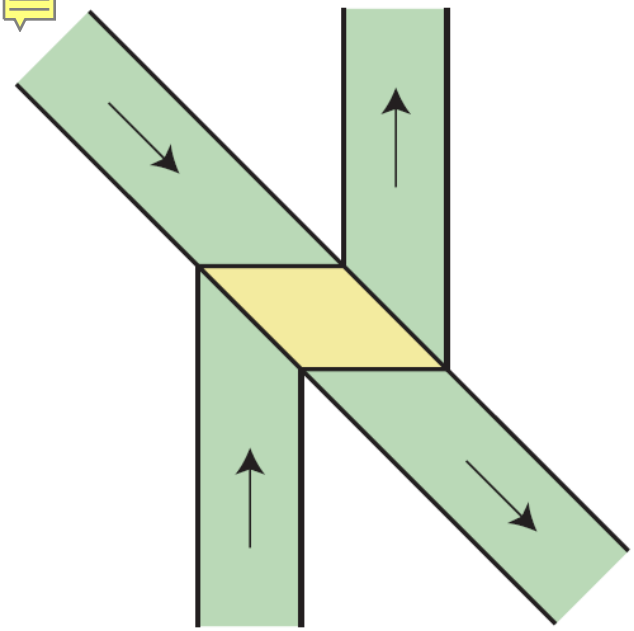


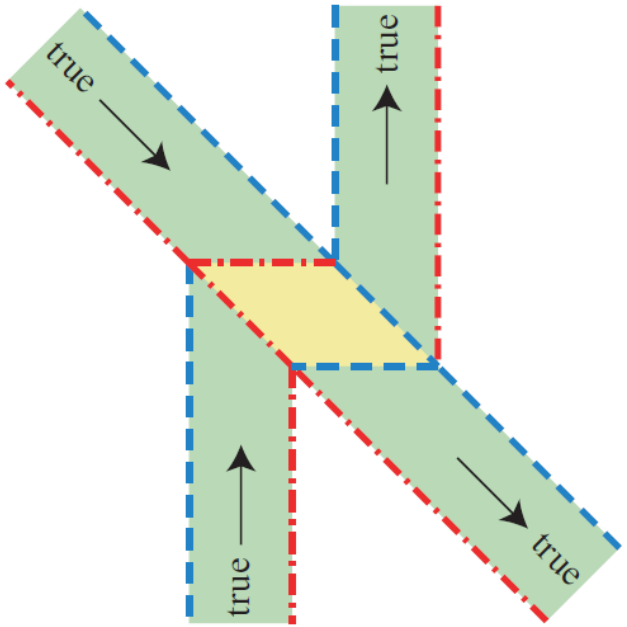
[Bern & Hayes 1996]

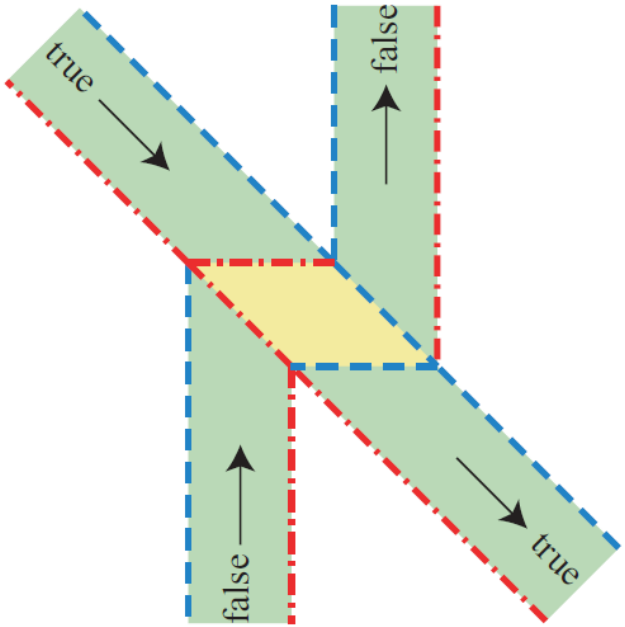
splitter/negation gadget

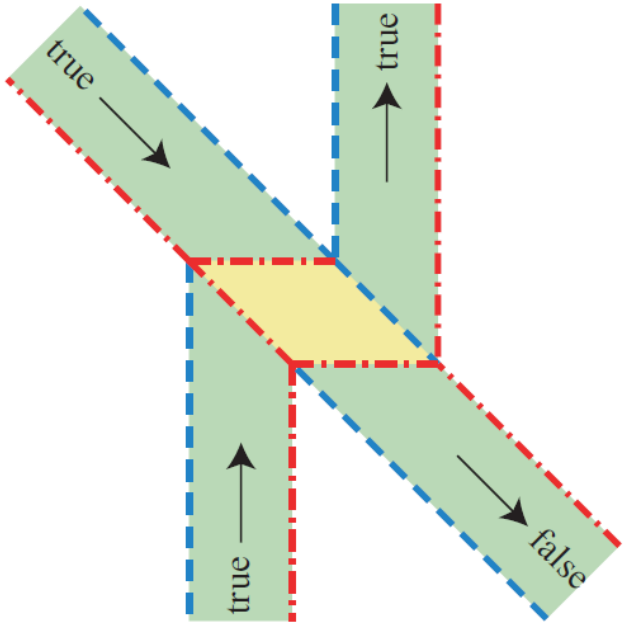


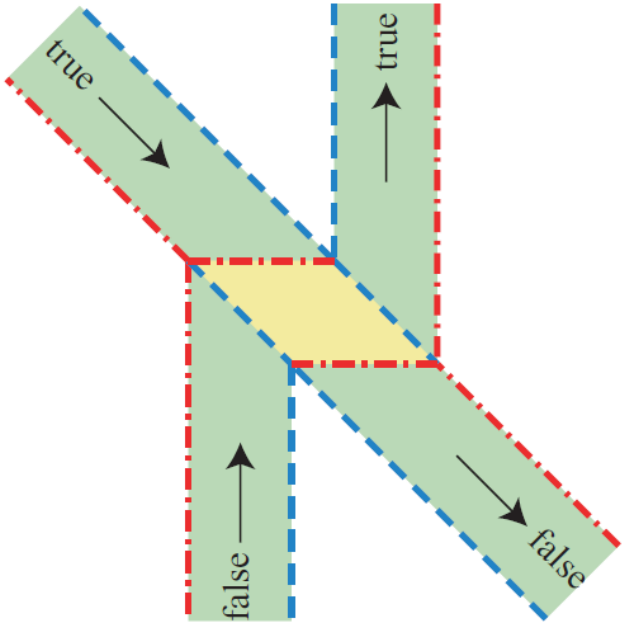
[Bern & Hayes 1996]





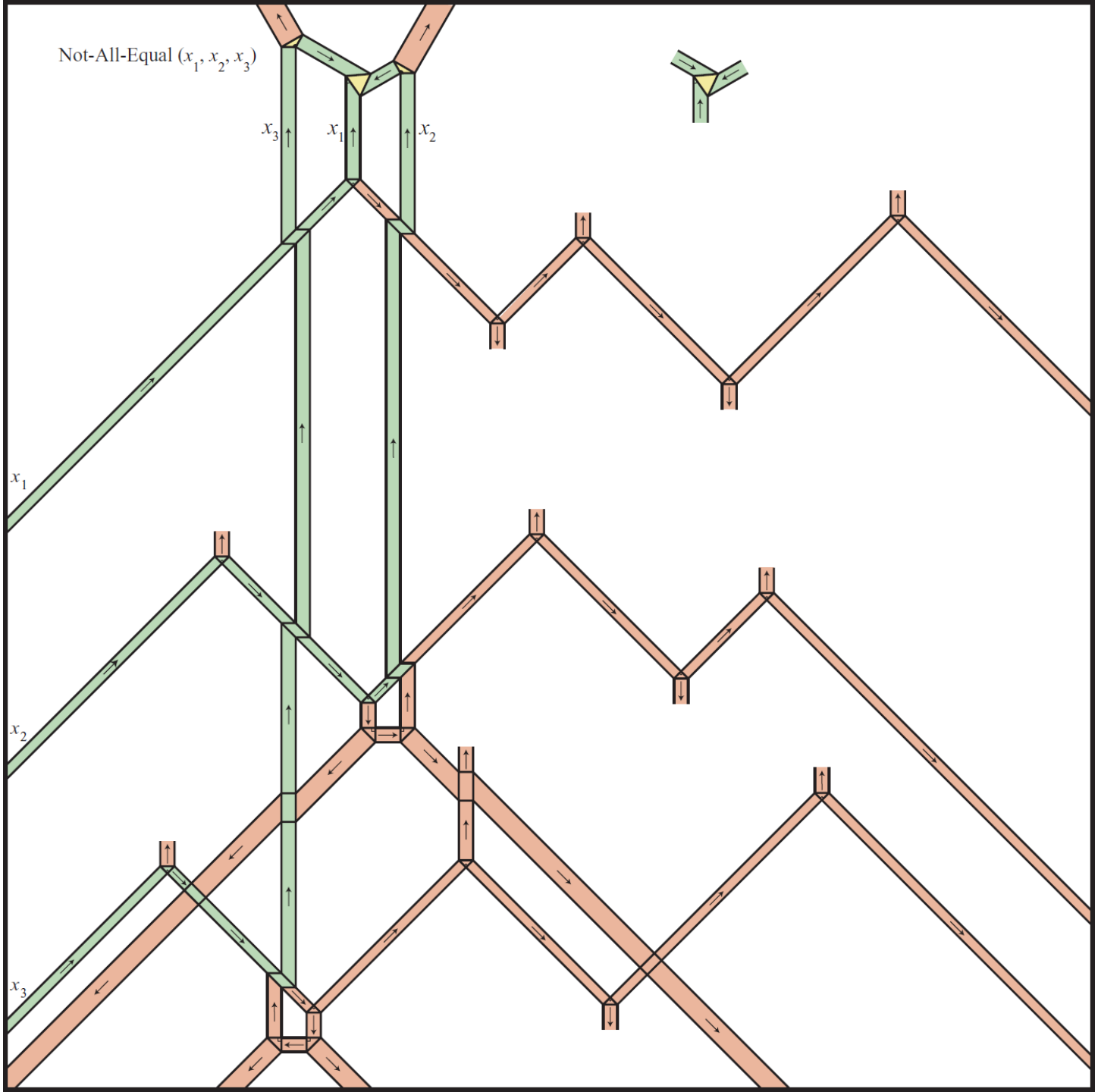






crossover gadget

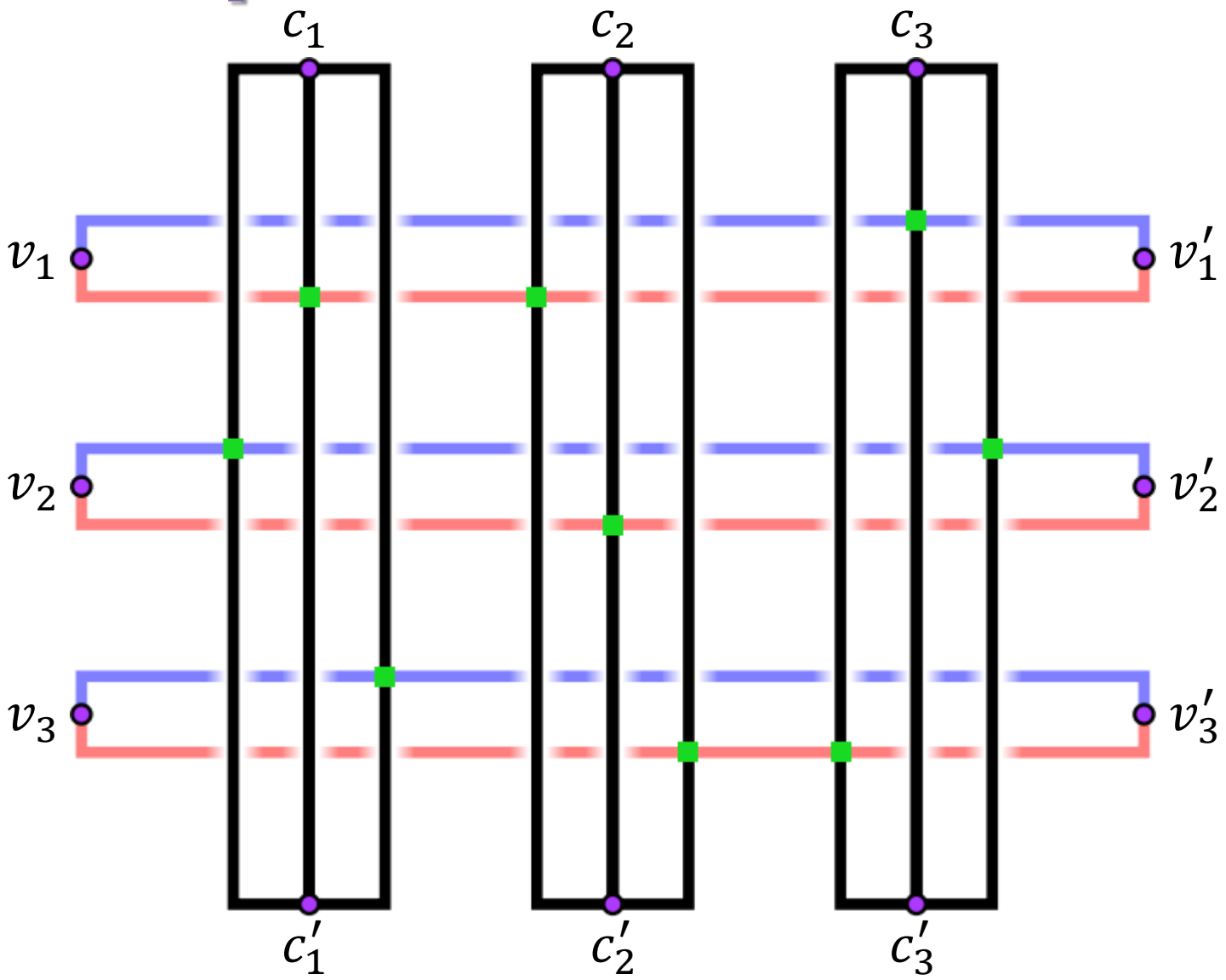
[Bern & Hayes 1996]



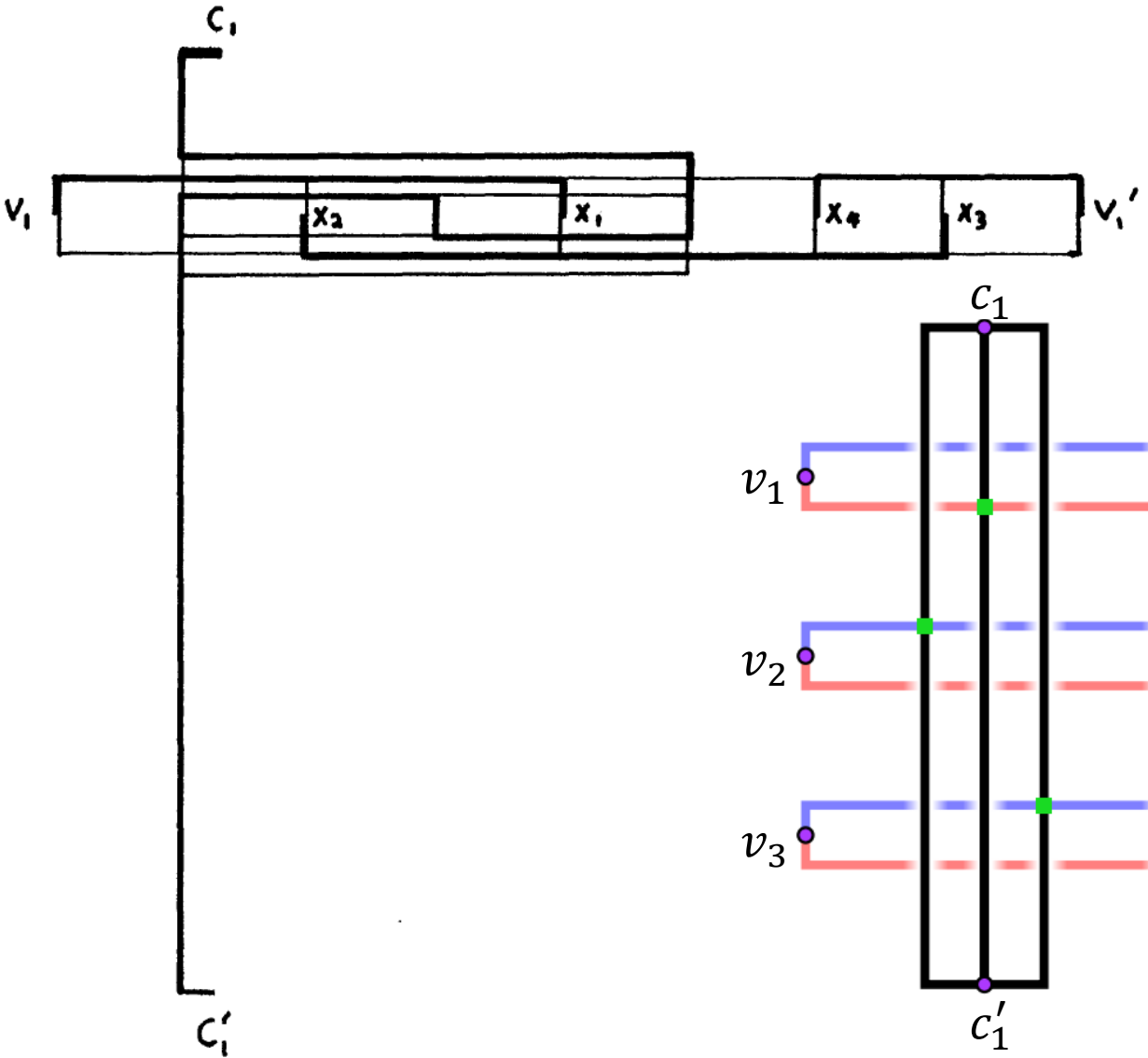
[Bern & Hayes 1996]

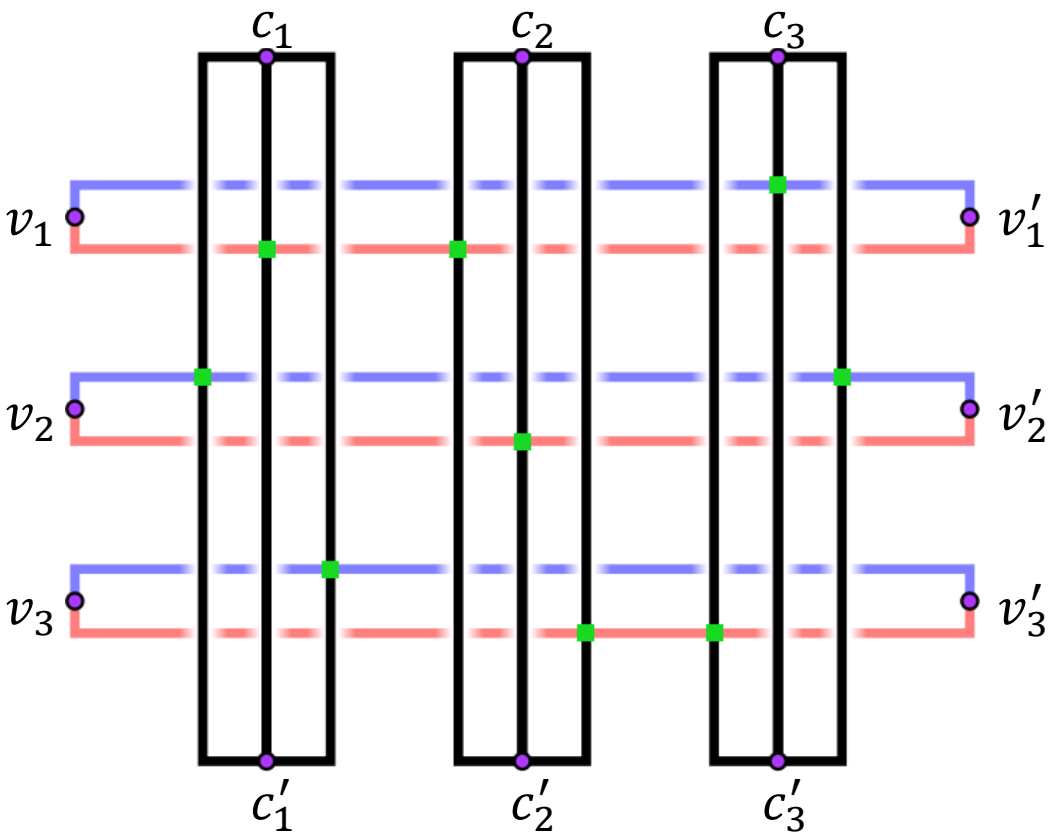
Vertex-Disjoint Paths is NP-complete

[Lynch 1975]



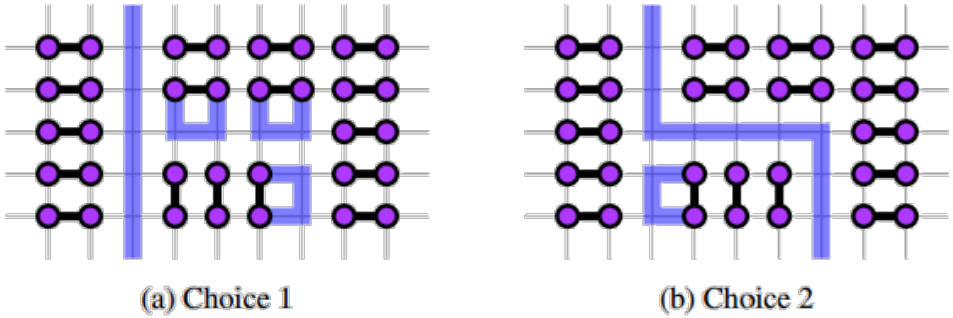
Planar Vertex-Disjoint Paths is NP-complete [Lynch 1975]





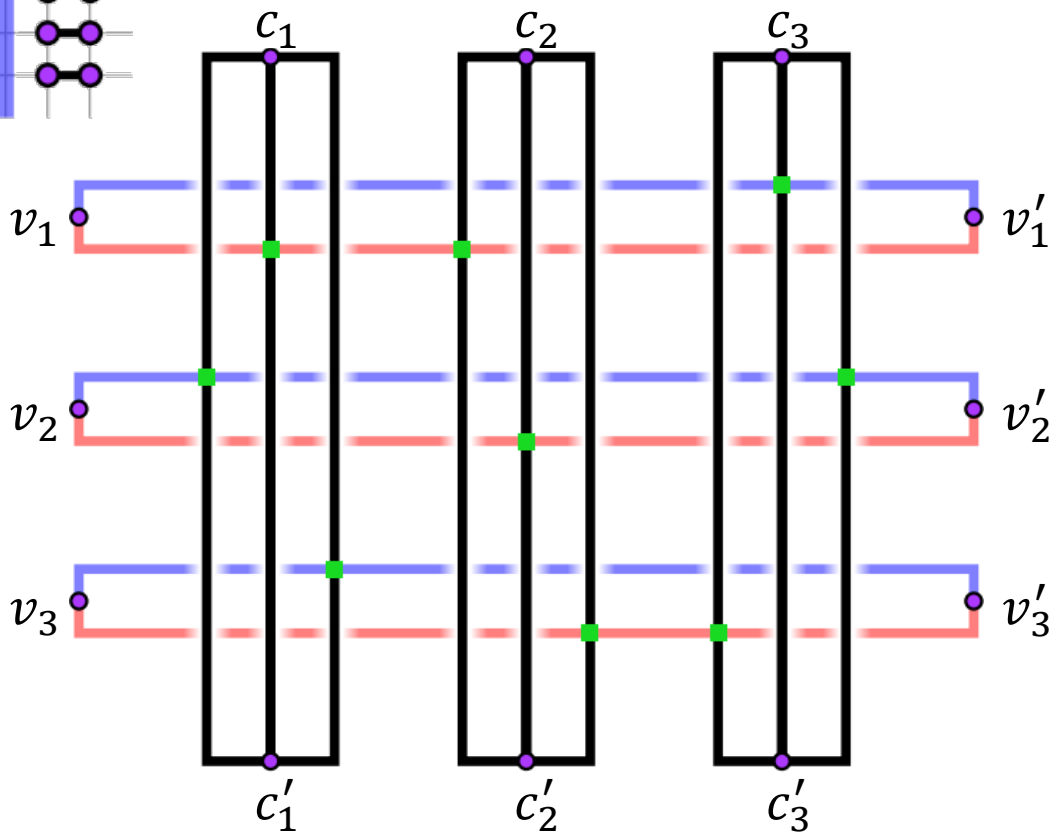
Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O'Brien, Reidl, Sánchez Villaamil, Sullivan 2014]



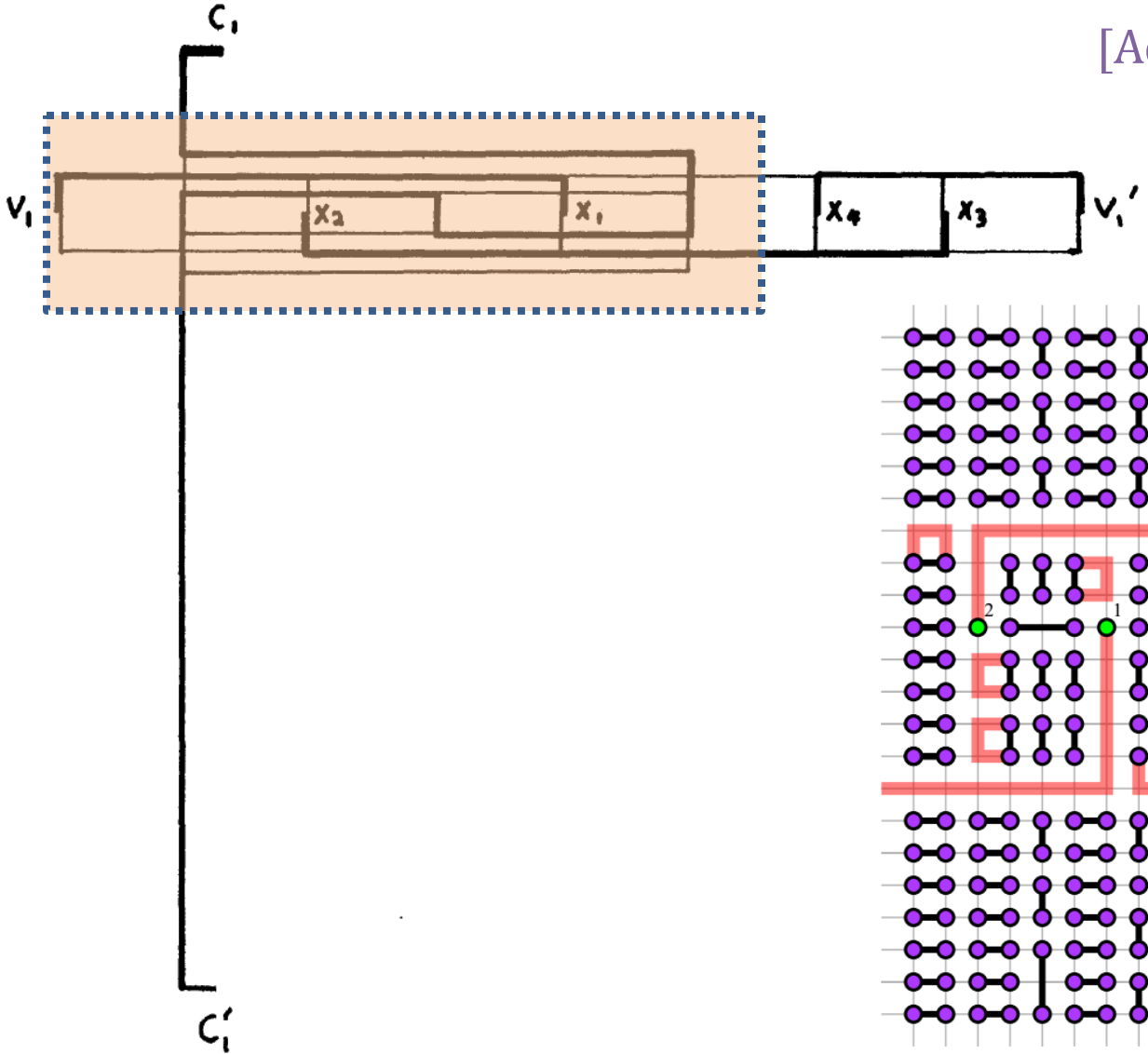
(a) Choice 1 (b) Choice 2

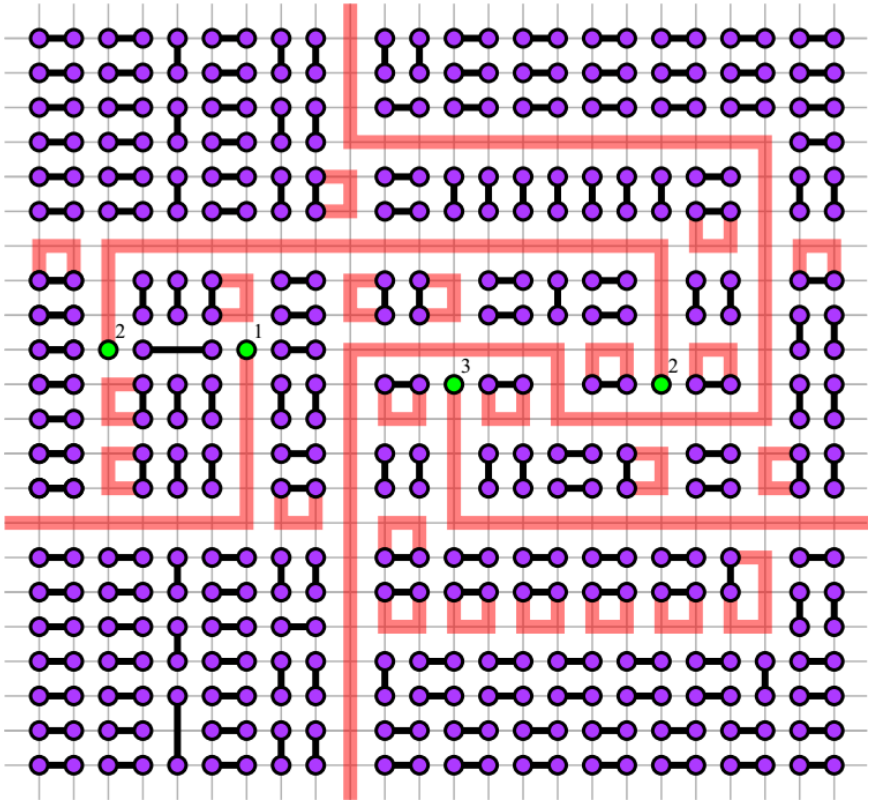
Fig. 7: Split gadget.



Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O'Brien, Reidl, Sánchez Villaamil, Sullivan 2014]

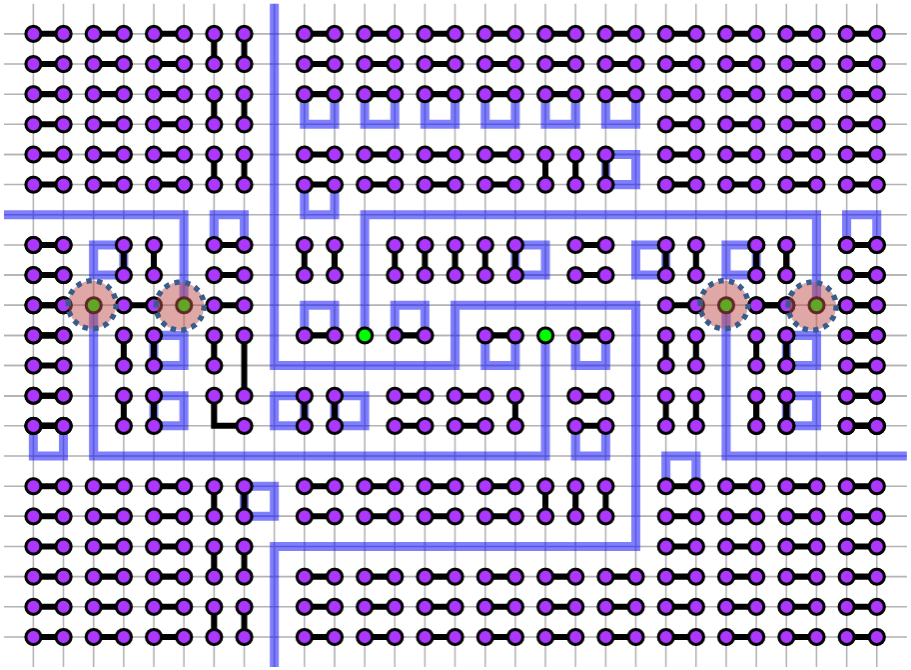


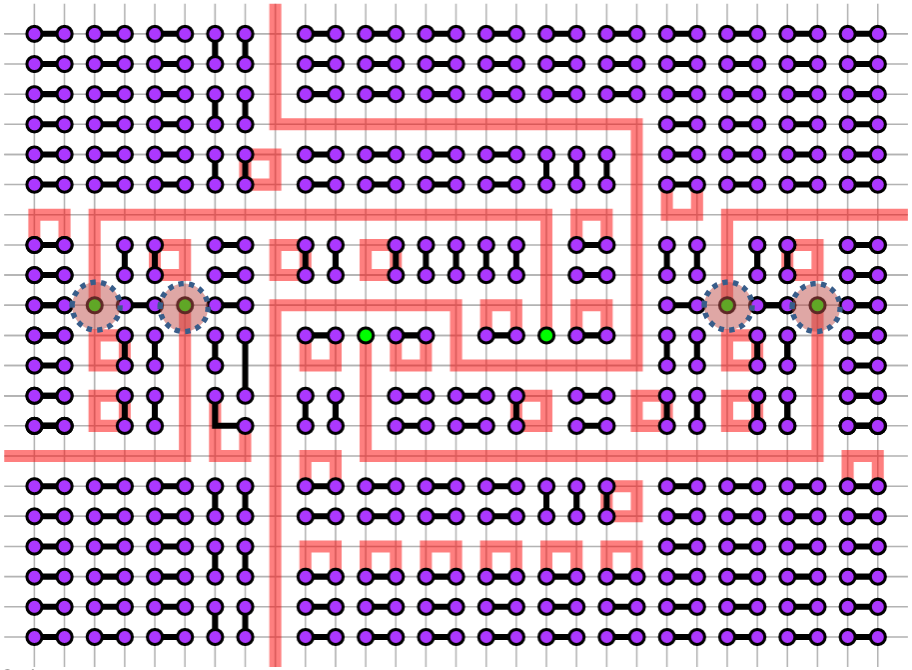


Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O'Brien, Reidl, Sánchez Villaamil, Sullivan 2014]

Issue 1: Empty space parity



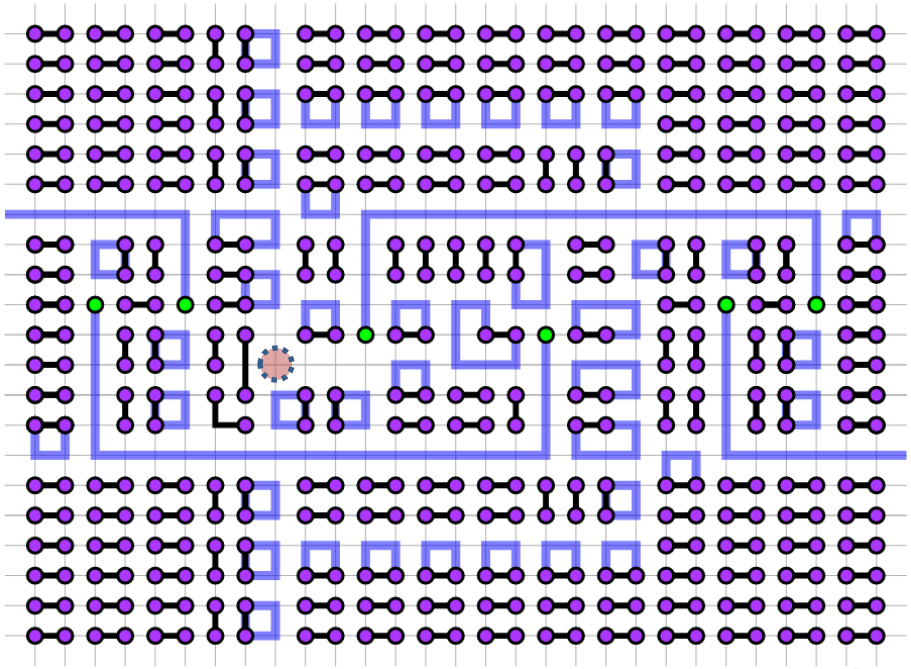


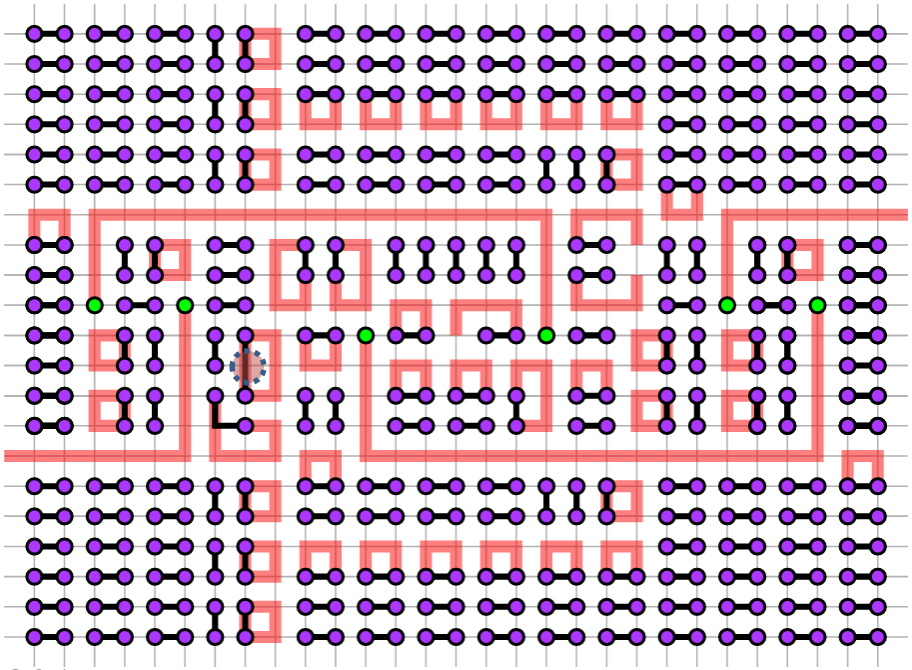
May 2014

Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O'Brien, Reidl, Sánchez Villaamil, Sullivan 2014]

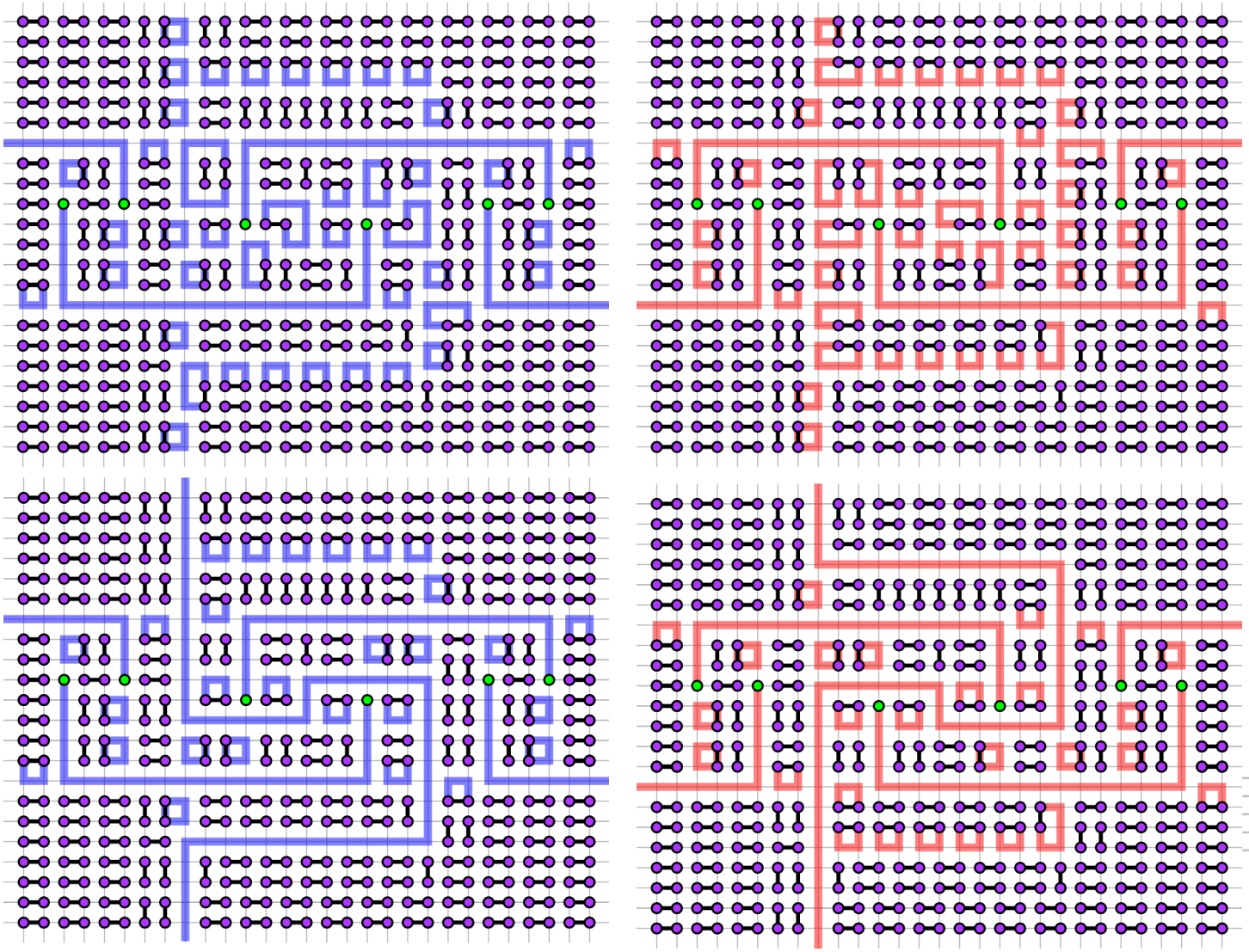
Issue 2: Clause path may be absent



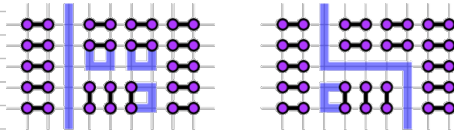


Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O'Brien, Reidl, Sánchez Villaamil, Sullivan 2014]

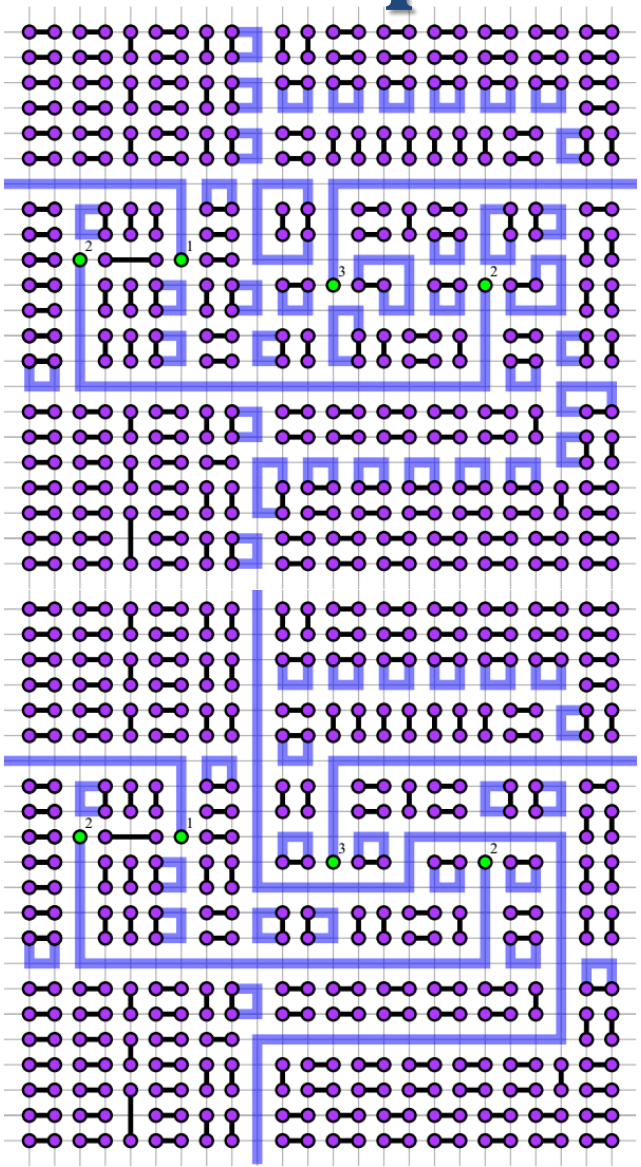


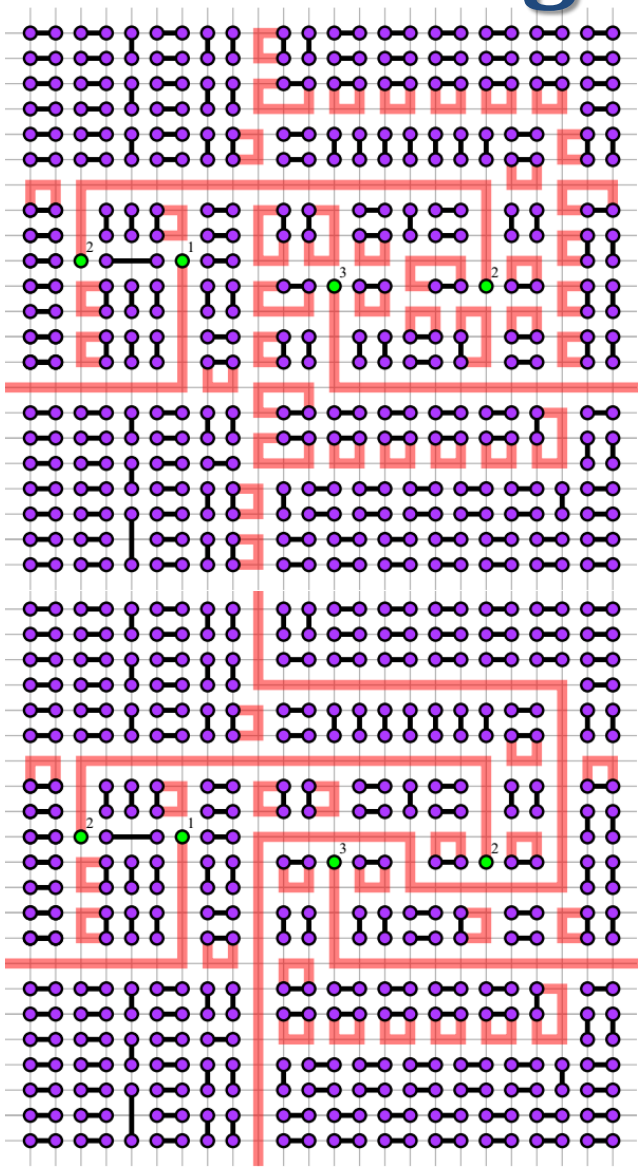
Issue 3: Gadget size parity

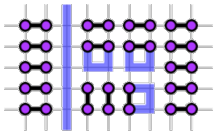


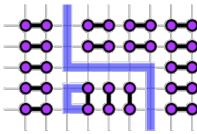
Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O'Brien, Reidl, Sánchez Villaamil, Sullivan 2014]





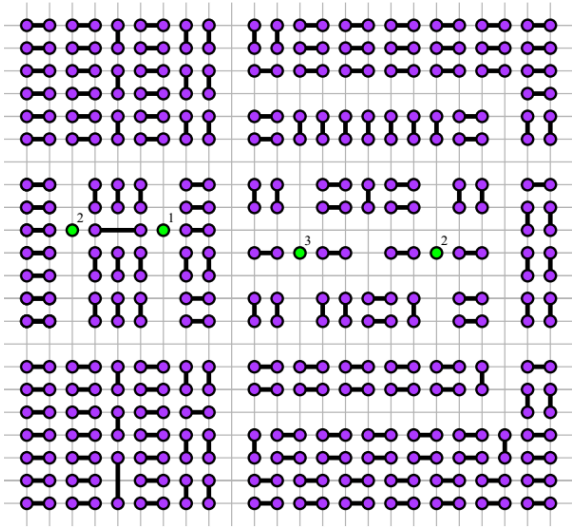




July 28, 2014

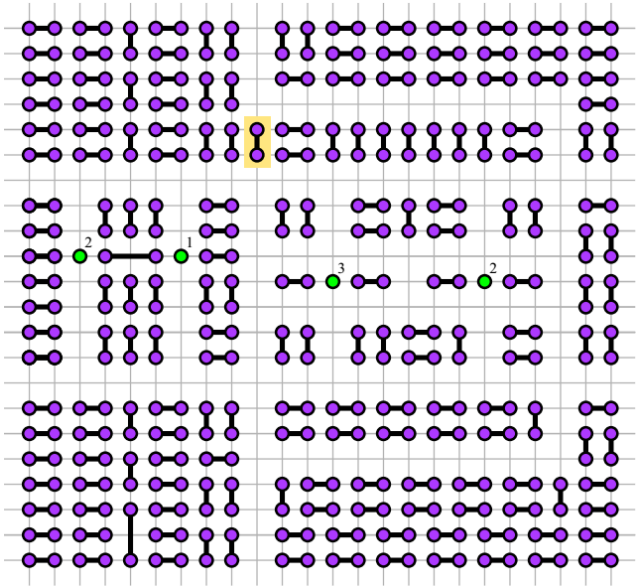
Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

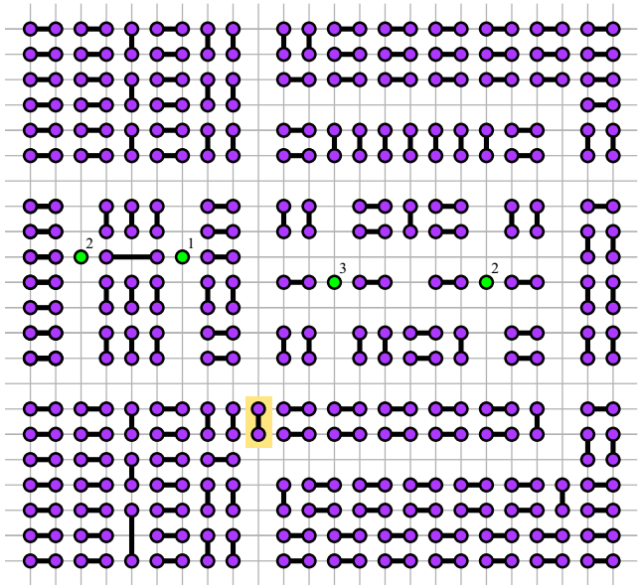
[Adcock, Demaine, Demaine, O'Brien, Reidl, Sánchez Villaamil, Sullivan 2014]



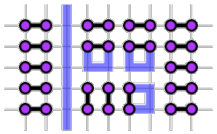
cross/ prevent blue

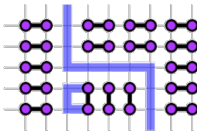
cross/ prevent red





Issue 4: Crossing versions





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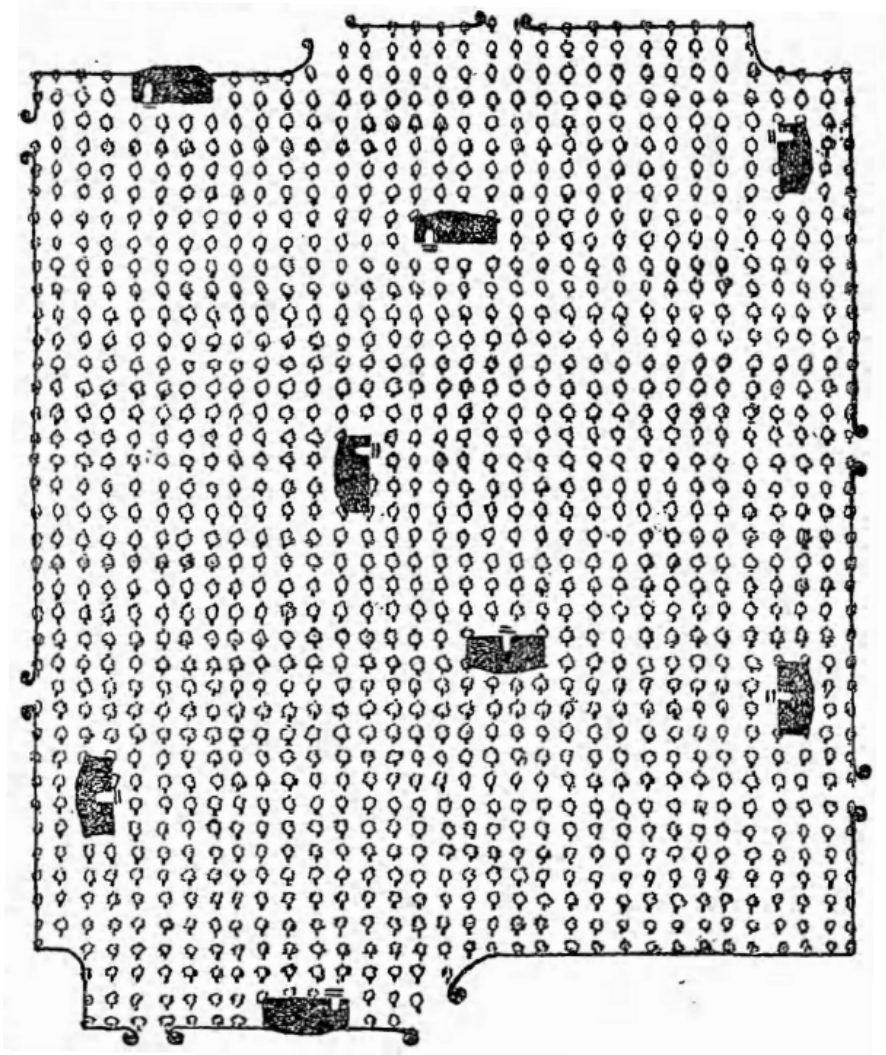
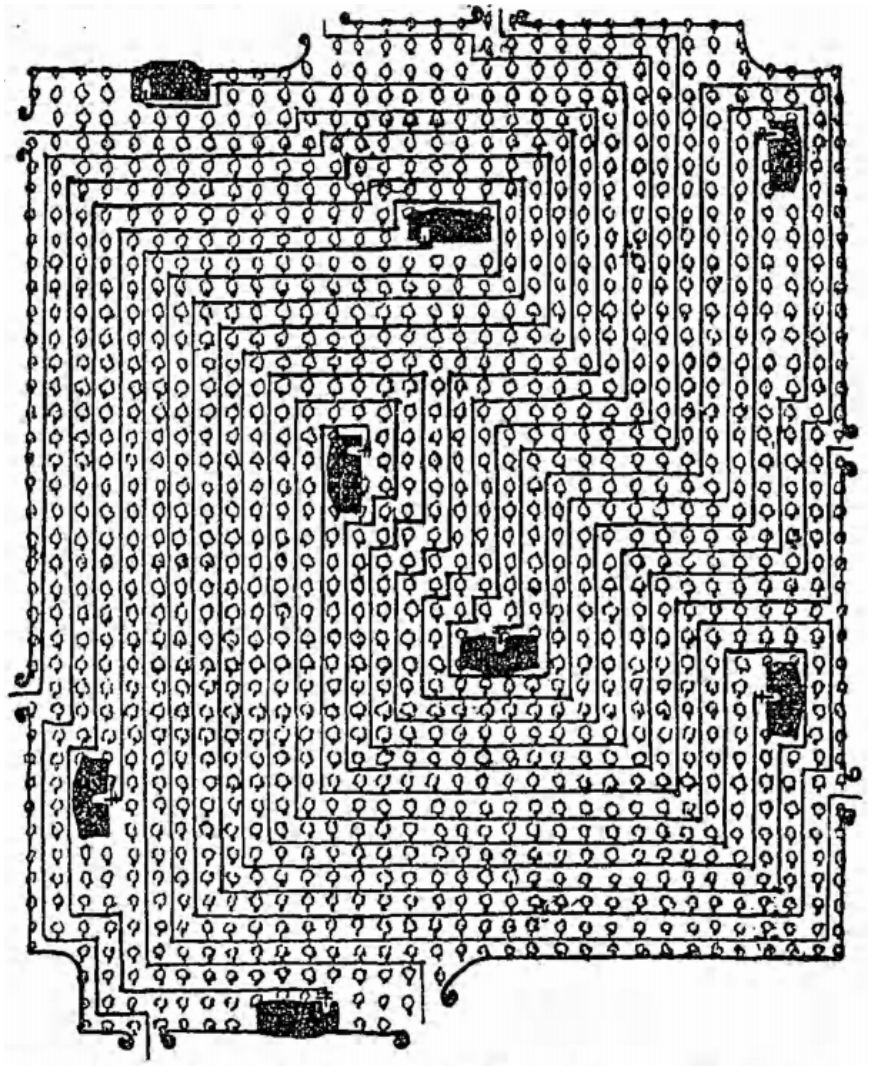
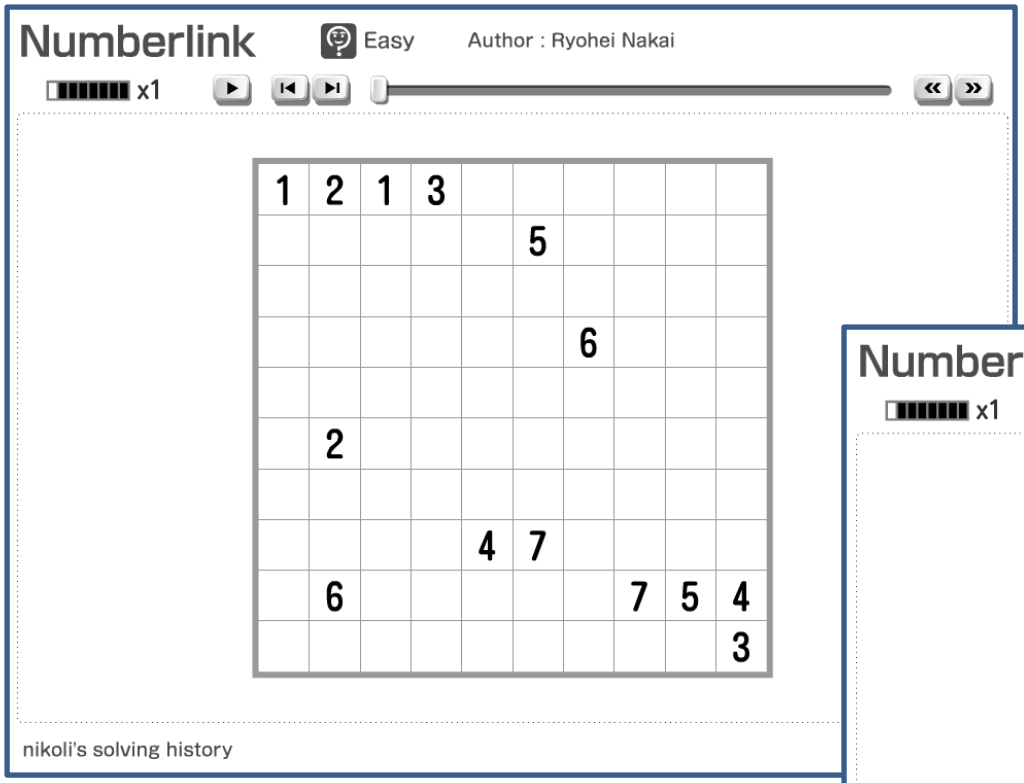
Sam Loyd's "The Puzzled Neighbors"

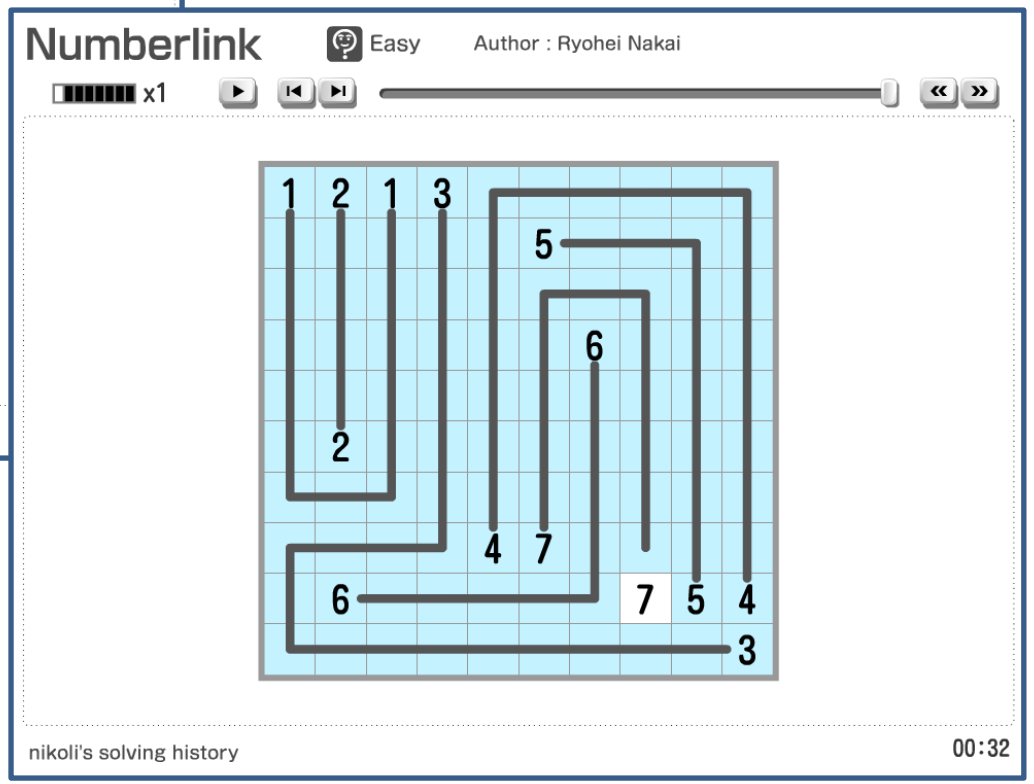
[Brooklyn Daily Eagle, 1897]



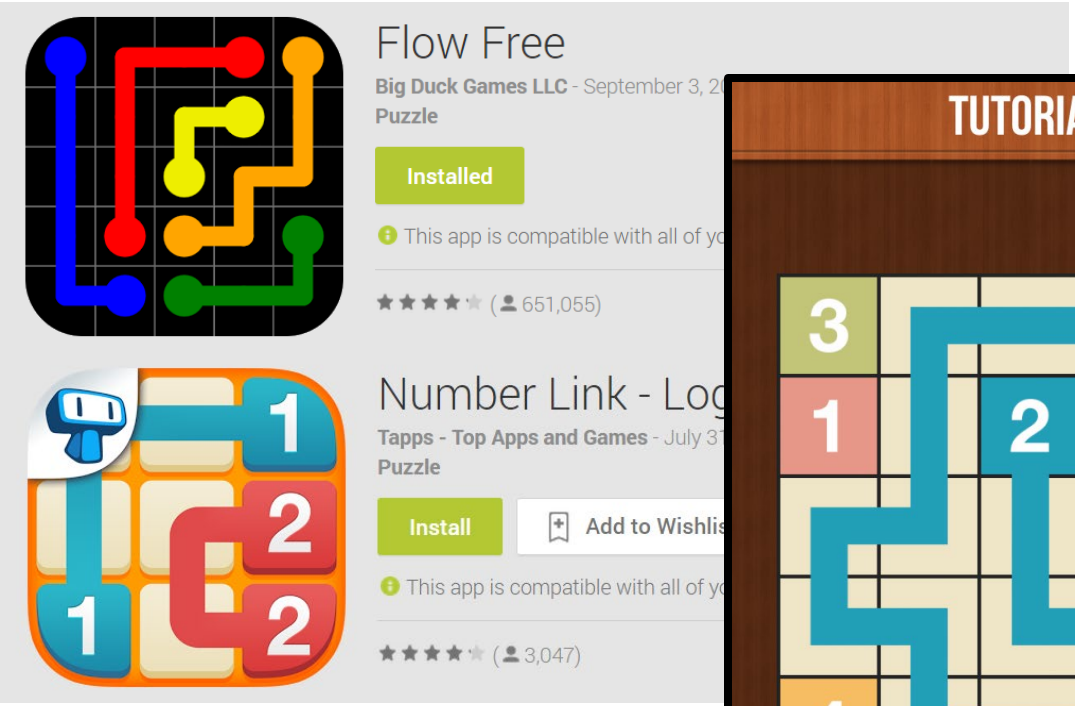


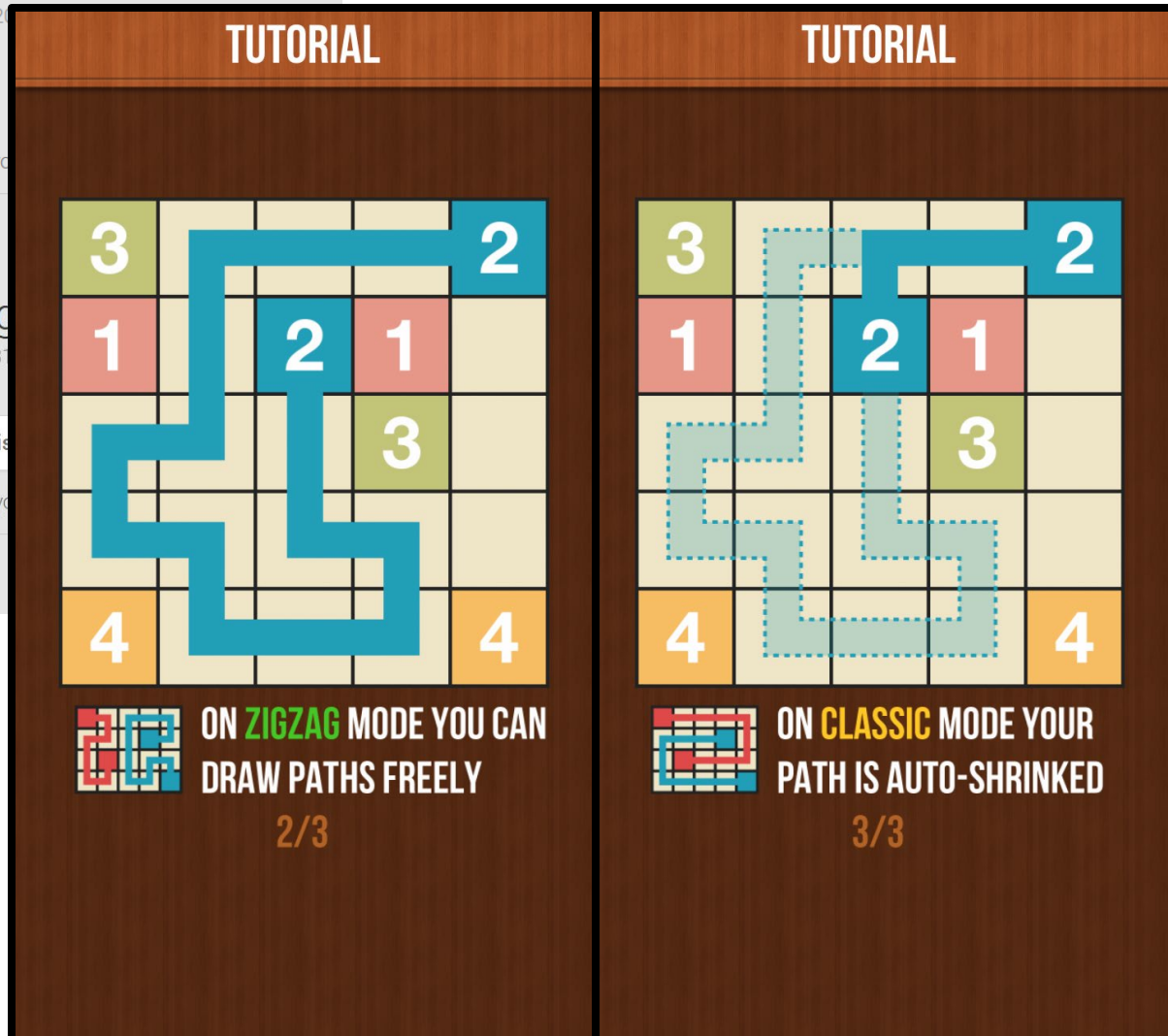
Nikoli Numberlink Puzzle





Nikoli Numberlink Puzzle





Nikoli Numberlink Puzzle is NP-complete

[Kotsuma & Takenaga 2010]

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ナンバーリンクのNP完全性と問題の列挙

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あらまし ナンバーリンクとは、格子状の盤面に同じ数字が 2 個づつ与えられた問題に対し、同じ数字どうしを交差しない線でつなぎ合わせるパズルである。本稿では、盤面に空白のマスを許さない場合においてもナンバーリンクが NP 完全となることを証明する。また、与えられた盤面のサイズに対し、ナンバーリンクの数字と正当な線からなる盤面を逆探索を用いて列挙するアルゴリズムを提案する。

キーワード パズル、NP 完全性、列挙アルゴリズム

NP-Completeness and Enumeration of Number Link Puzzle

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Abstract Number Link is a puzzle on a square grid. Some numbers are given in cells so that each number appears in two cells, and the object of the puzzle is to connect the cells with the same number by non-crossing lines drawn on the cells. In this paper, we first prove that Number Link is NP-complete even if an answer that has a cell on which no line is drawn is not admitted. Second, we propose an algorithm to enumerate the problems of Number Link with a given size using reverse-search algorithm.

Key words puzzle, NP-completeness, enumeration algorithm

1. はじめに

計算機科学の分野において、様々な問題の計算複雑さを調べることがは重要である。パズルに対しても計算複雑さについての研究は数多く行われており、その多くに対して計算量が明らかにされている [1]〜[4]。本稿ではナンバーリンク [5] と呼ばれるパズルを扱う。ナンバーリンクとは、格子状の盤面に同じ数字が 2 個づつ与えられた問題に対し、数字の与えられていない各マスに上下左右のうち 2 個のマスを結ぶ線を引くことにより、同じ数字どうしを交差しない線でつなぎ合わせるパズルである。

本稿では、まずナンバーリンクの計算量を明らかにする。ナンバーリンクにおいて盤面のマスの空白を許す場合は、グリッドグラフ上の頂点間パス問題と考えられ、すでに NP 完全であることが知られている [6]。しかし、パズルとしてのナンバーリンクは、通常その解答において冗長な迂回をした連結がなく、また空白のマスが生じないことが求められる。そこで、本研究ではこれらの条件を満たす場合においても NP 完全となることを証明する。

次に逆探索に基づいたナンバーリンクの問題の列挙について、与えられた盤面のサイズに対し、ナンバーリンクの数字と冗長な迂回を含まない連結線からなる盤面を列挙するアルゴリズムを提案する。ある指定された条件を満たす解を全て探し出す問題を列挙問題、または数え上げ問題と呼び、組み合わせ理論や幾何学、計算機科学において重要な問題の 1 つである。列挙アルゴリズムの手法の 1 つとして、逆探索 [8] と呼ばれる手法が注目されている。本稿で提案するアルゴリズムは逆探索を用い、任意の出力から次の出力までの計算時間および必要な領