2-Colorable Perfect Matching is NP-complete [Schaefer 1978]

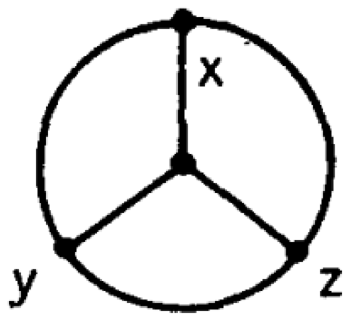


Figure 1(a)

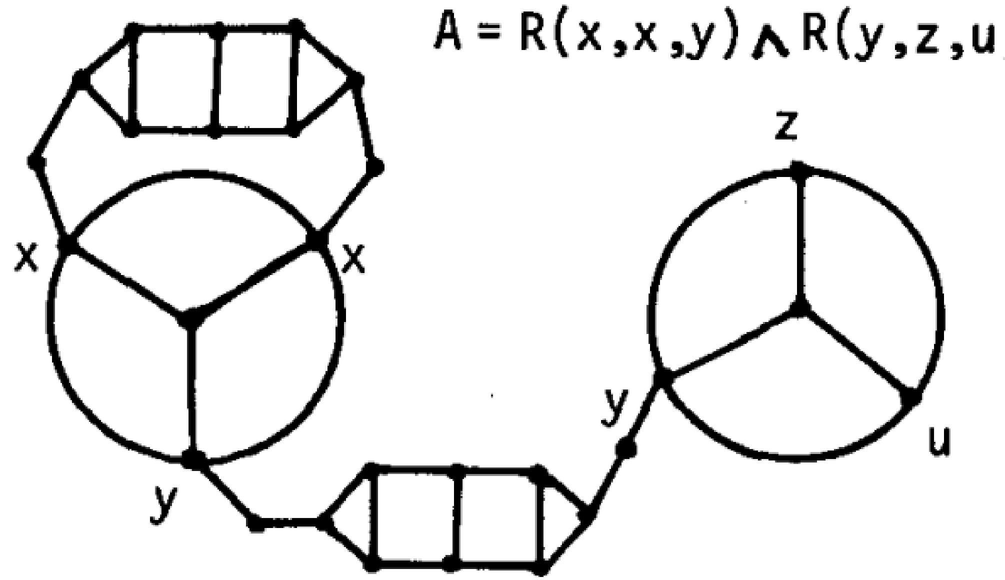


Figure 1(c)

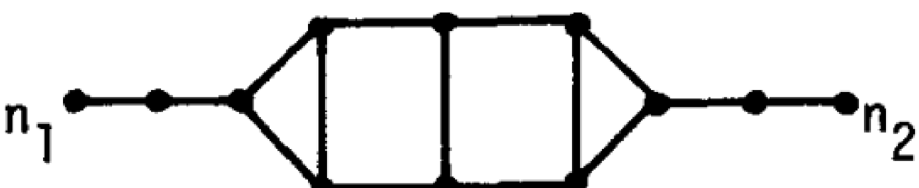
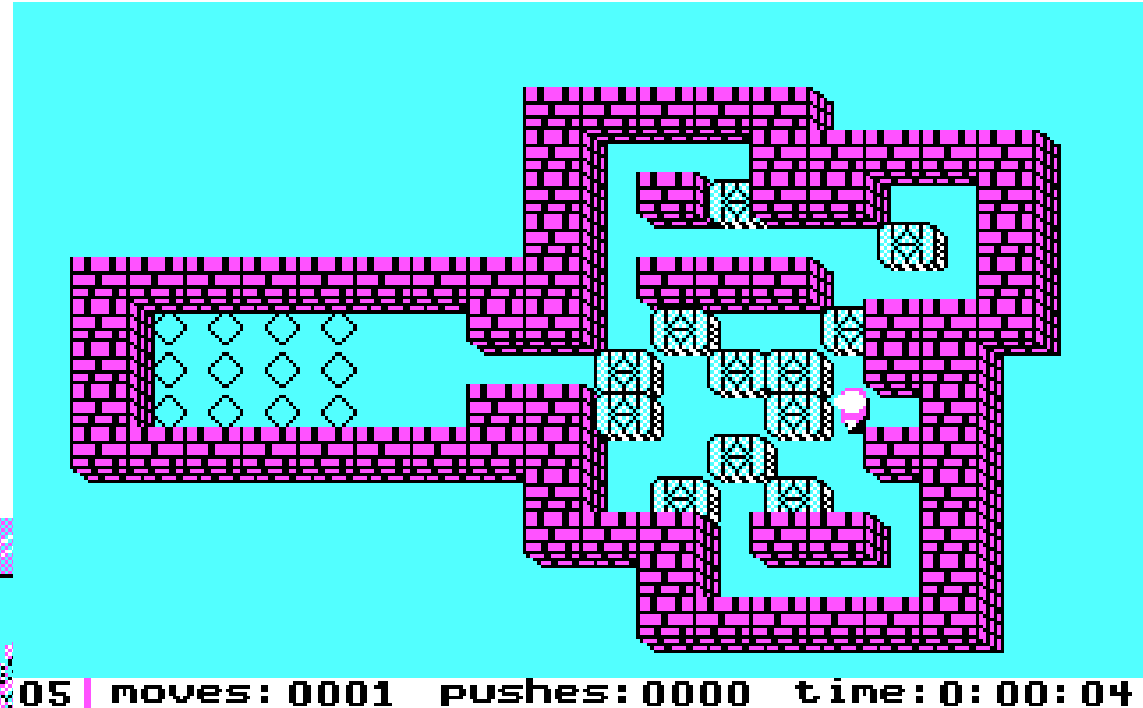
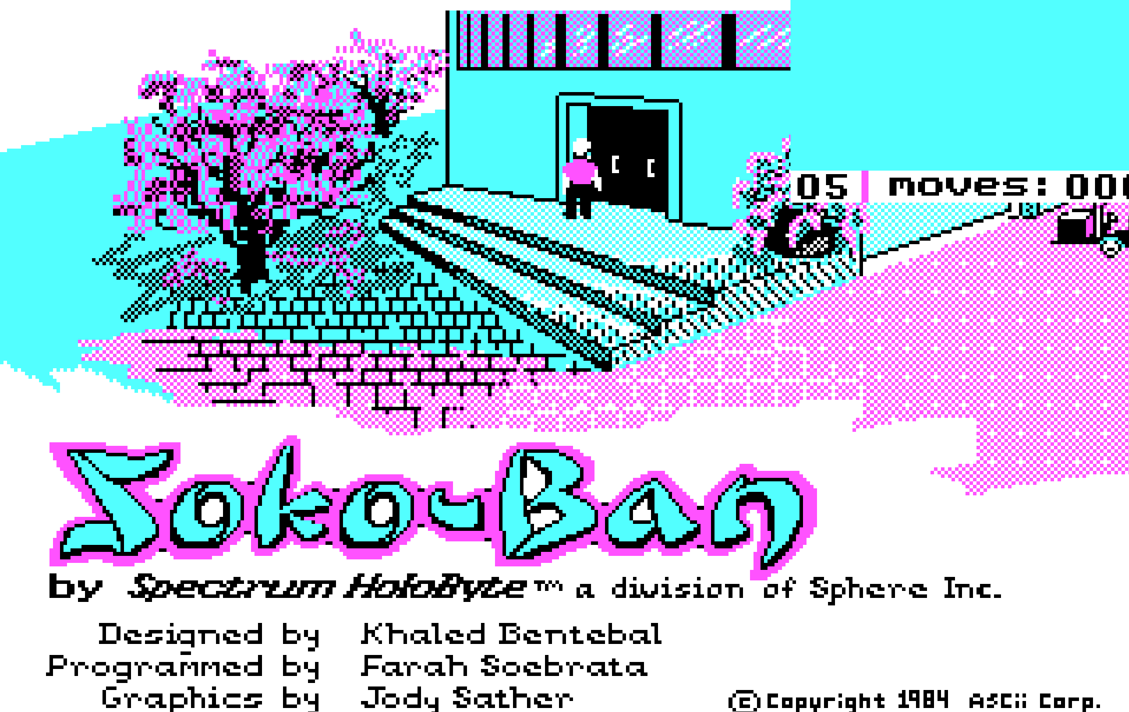


Figure 1(b)

Pushing 1 × 1 Blocks

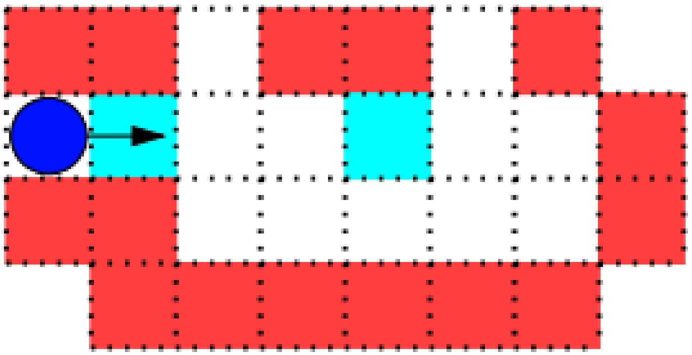


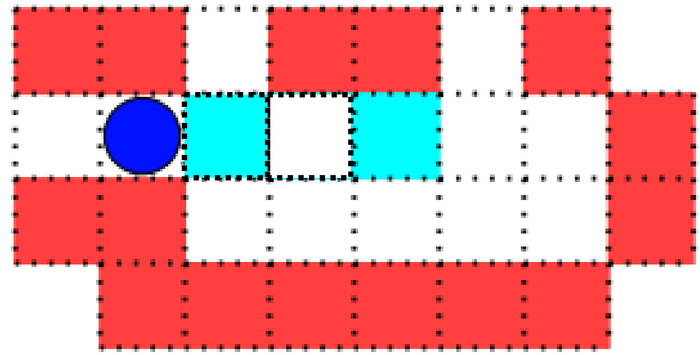


Pushing 1 × 1 Blocks

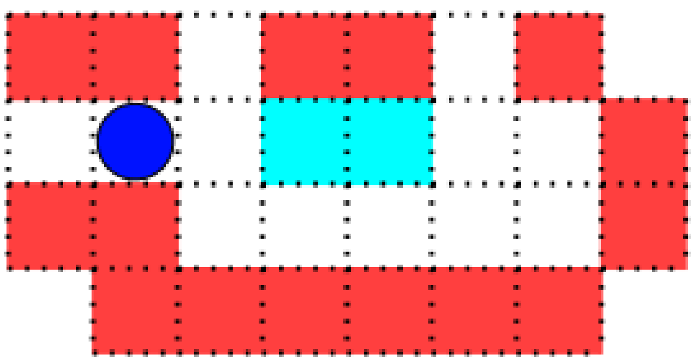
Legend of Zelda: The Minish Cap

Pushing 1 × 1 Blocks

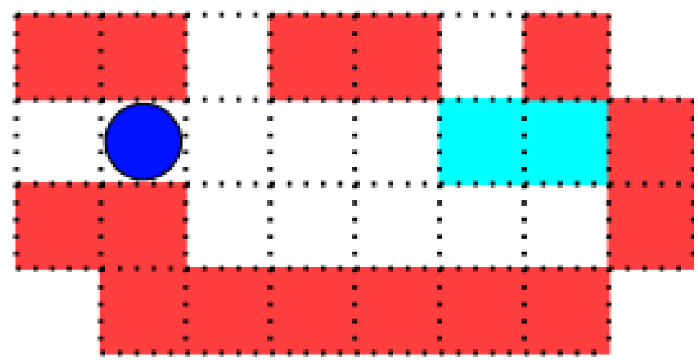




Push-2



PushPush-2

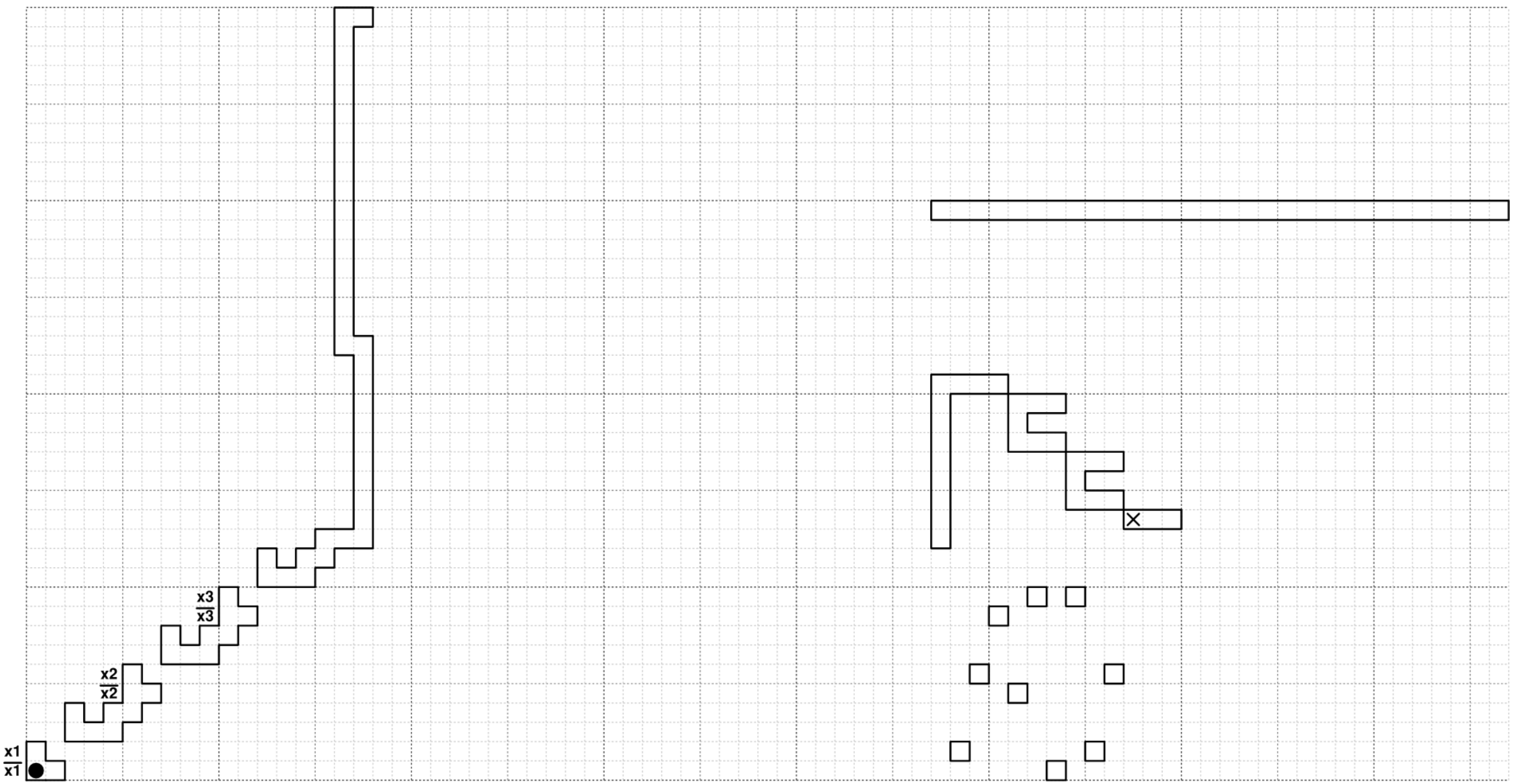


PushPushPush-2

Pushing 1 × 1 Blocks Complexity

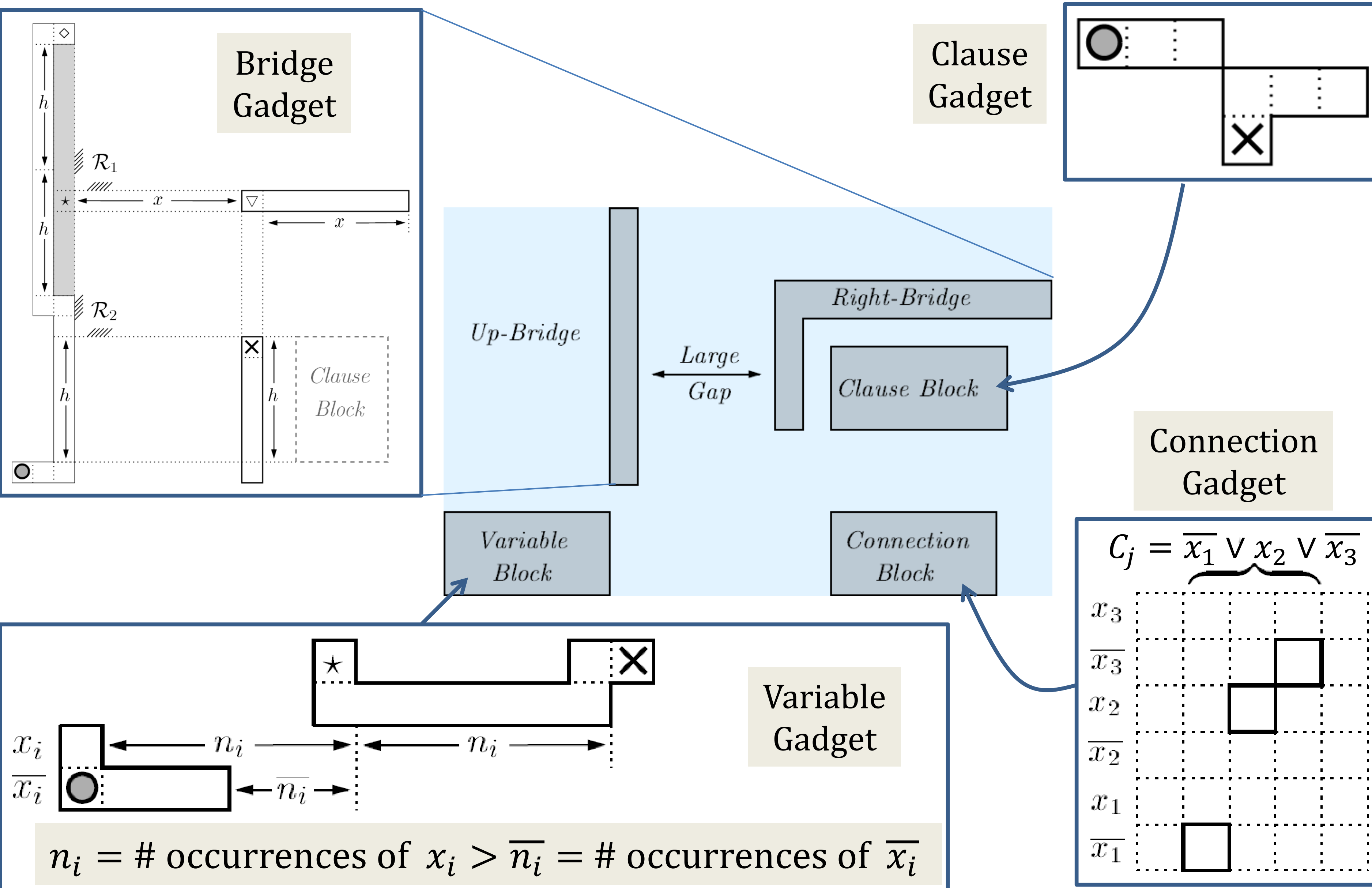
Name	Push	Fixed	Slide	Goal	Complexity	Reference
Push-k	$k \geq 1$	no	min	path	NP-hard	D, D, O'Rourke 2000
Push-$*$	∞	no	min	path	NP-hard	Hoffmann 2000
PushPush-k	$k \geq 1$	no	max	path	PSPACE-complete	D, Hoffmann, Holzer 2004
PushPush-$*$	∞	no	max	path	NP-hard	Hoffmann 2000
Push-1F	1	yes	min	path	NP-hard	DDO 2000
Push-kF	$k \geq 2$	yes	min	path	PSPACE-complete	D, Hearn, Hoffmann 2002
Push-$*$F	∞	yes	min	path	PSPACE-complete	Bremner, O'Rourke, Shermer 1994
Push-kX	$k \geq 1$	no	min	simple path	NP-complete	D, Hoffmann 2001
Push-$*$X	∞	no	min	simple path	NP-complete	Hoffmann 2000
Sokoban	1	yes	min	storage	PSPACE-complete	Culberson 1998

Push-∗ is NP-hard [Hoffmann 2000]



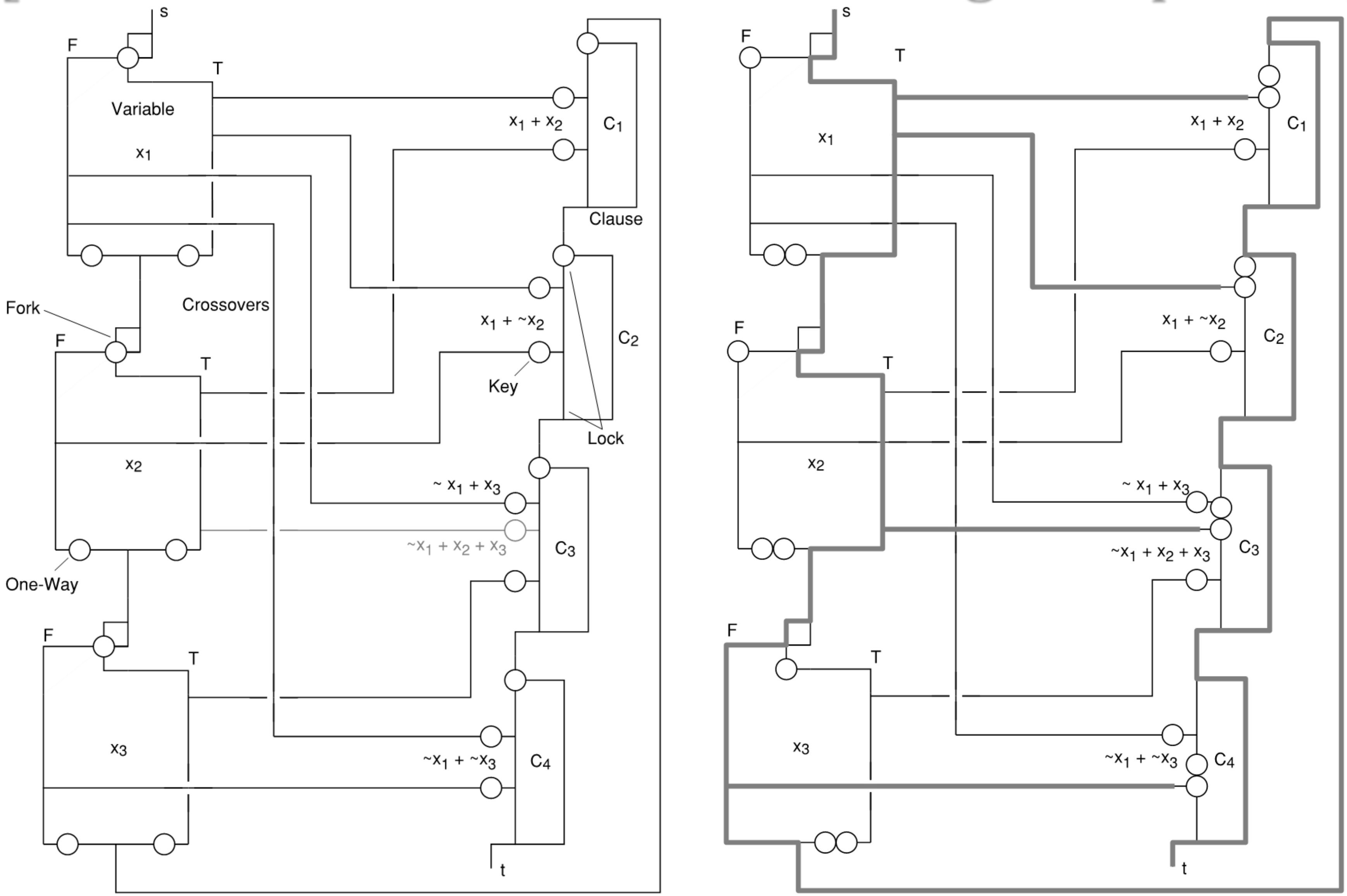
$(x1 \vee x2 \vee \overline{x3}) \wedge (\overline{x2} \vee x3 \vee \overline{x1}) \wedge (x3 \vee x1 \vee x2)$

Push-∗ is NP-hard [Hoffmann 2000]



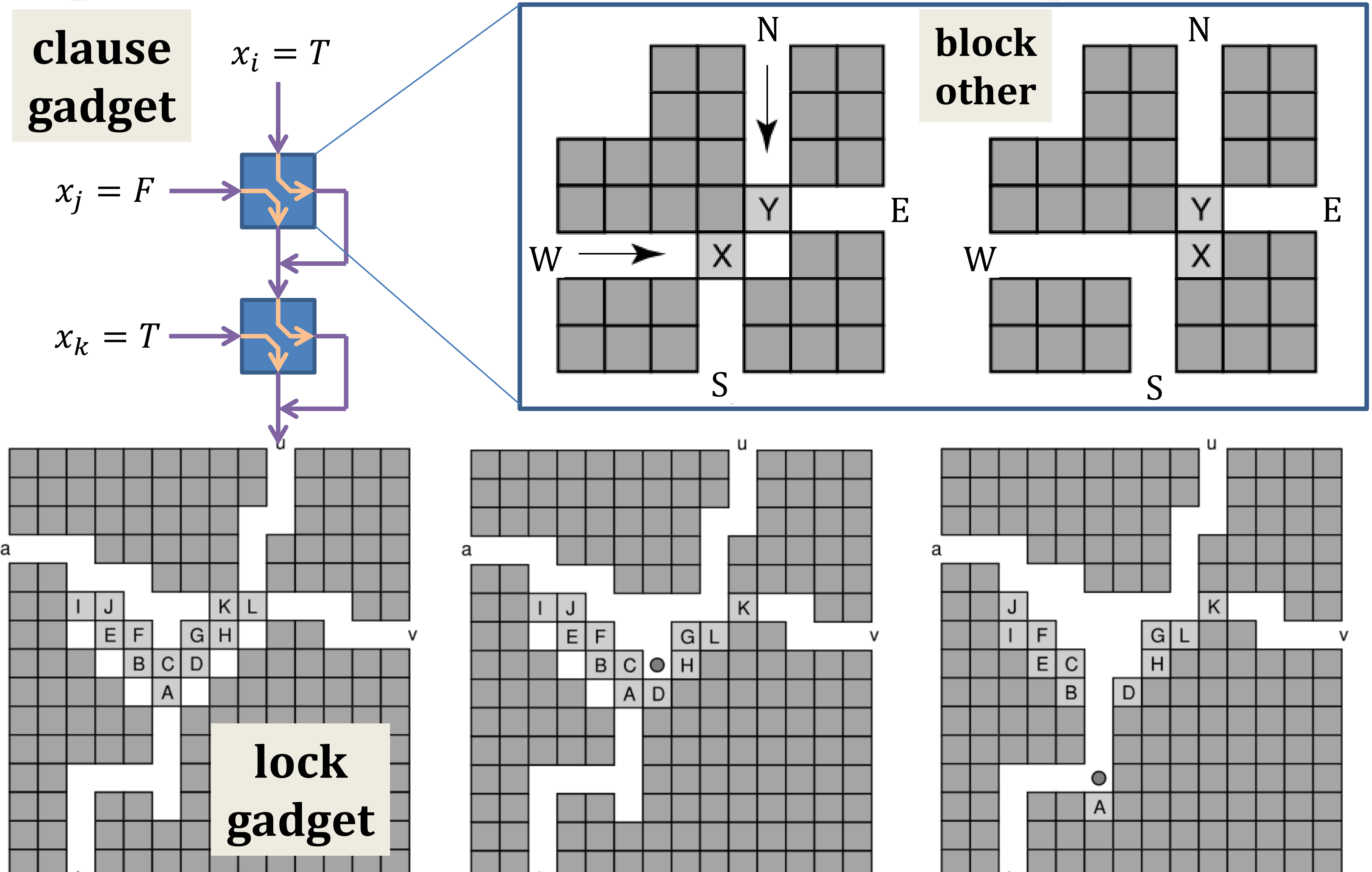
PushPush-1 is NP-hard in 3D

[O'Rourke & Smith Problem Solving Group 1999]



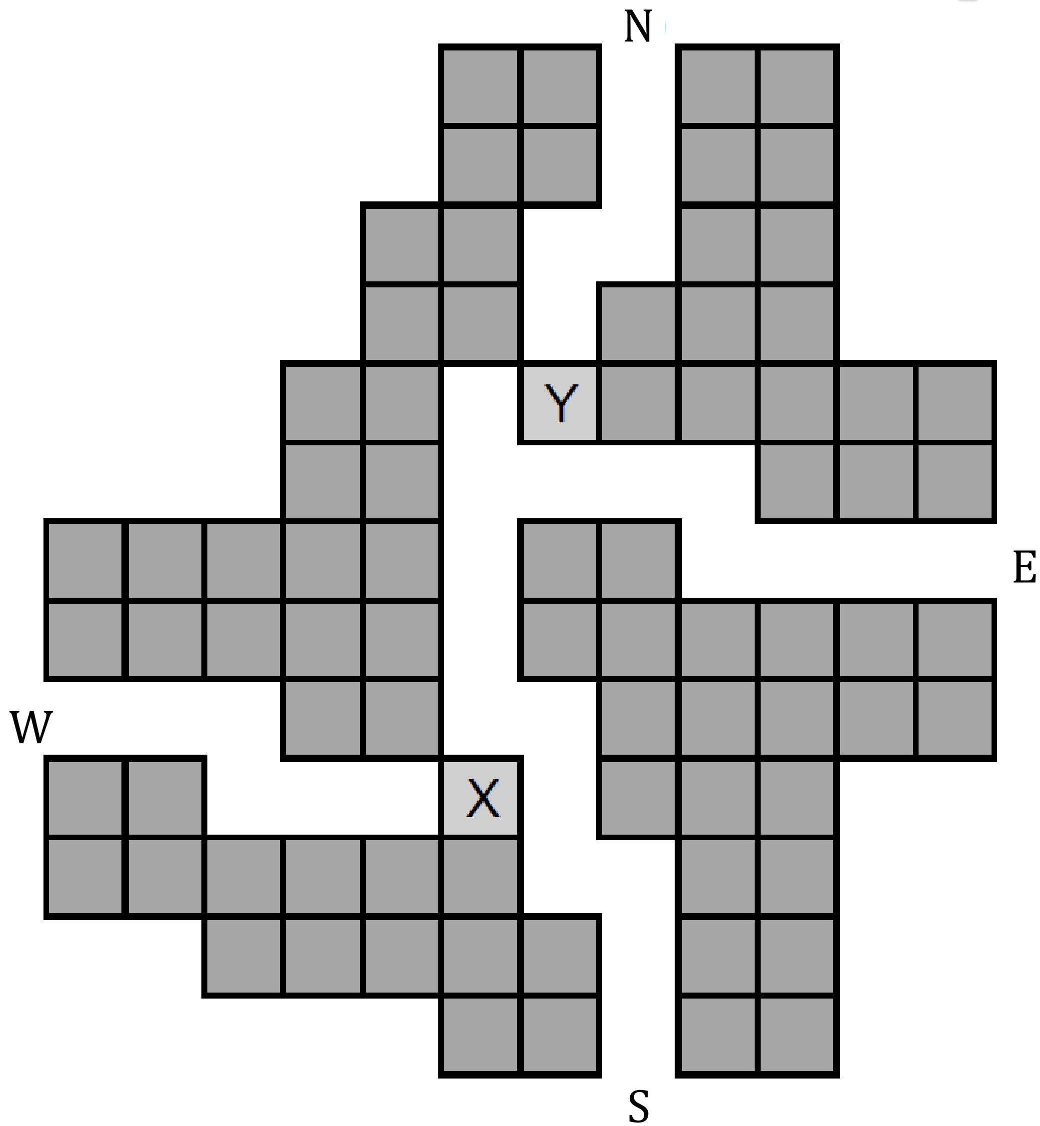
(Push)Push-1 is NP-hard in 2D

[Demaine, Demaine, O'Rourke 2000]



(Push)Push-1 is NP-hard in 2D

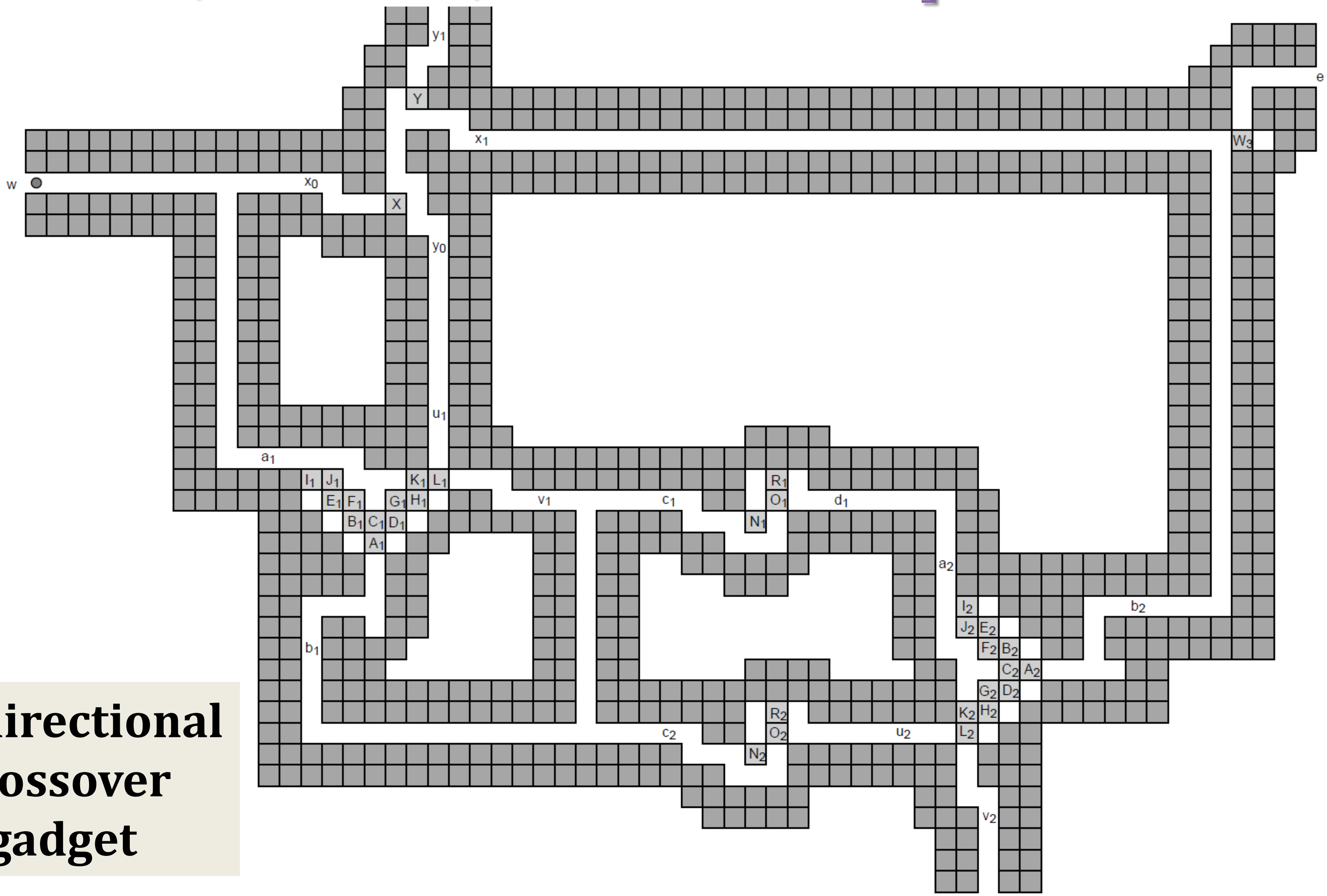
[Demaine, Demaine, O'Rourke 2000]



NAND crossover gadget

(Push)Push-1 is NP-hard in 2D

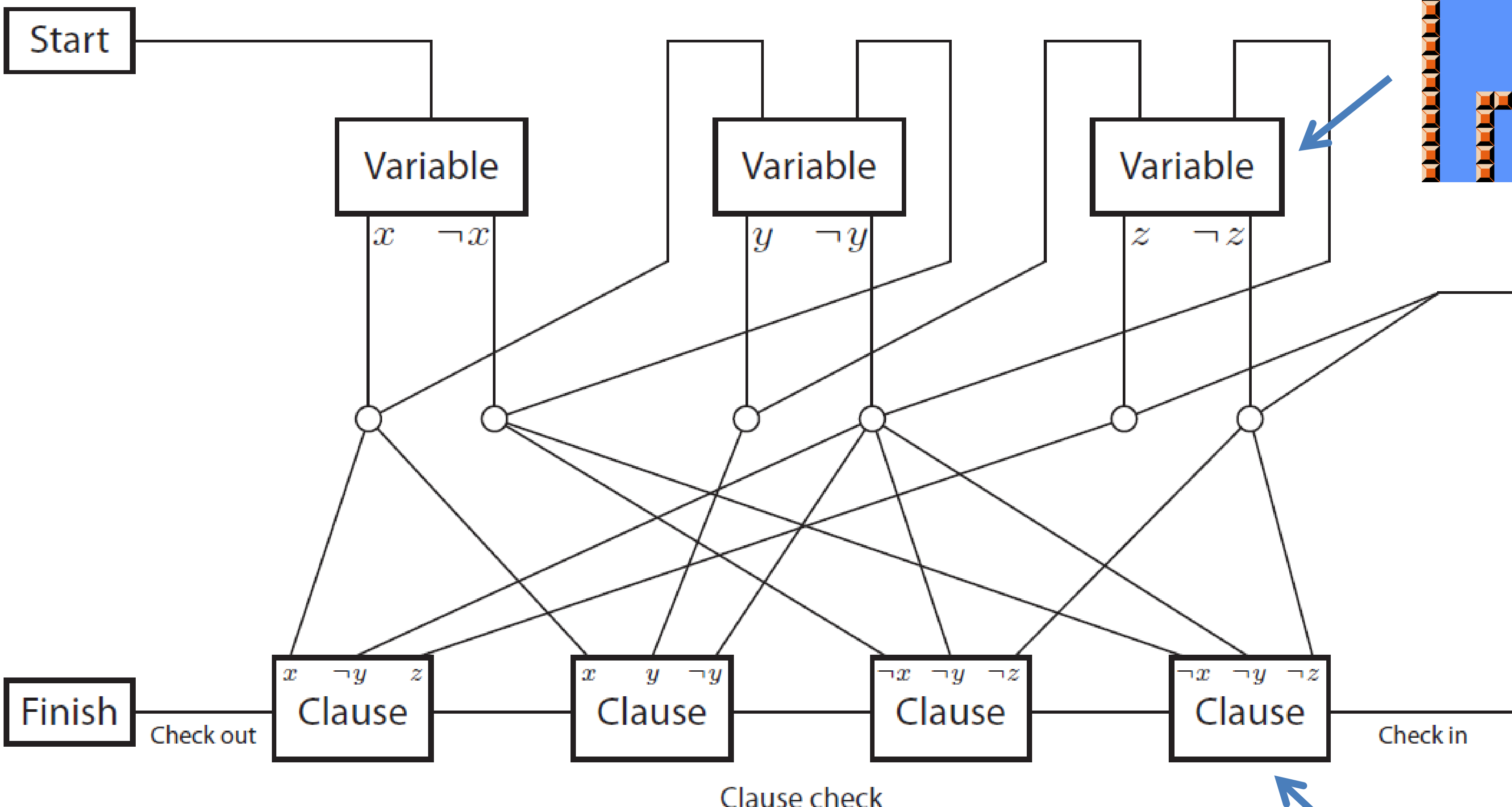
[Demaine, Demaine, O'Rourke 2000]

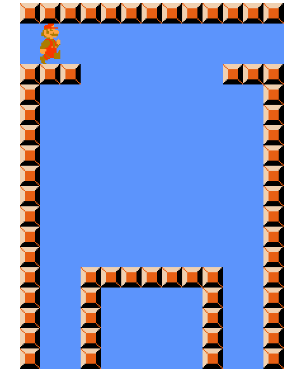


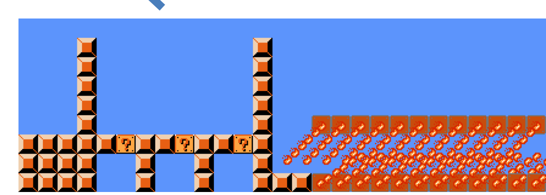
unidirectional crossover gadget

Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]







$(x \text{ OR } \neg y \text{ OR } z)$ & $(x \text{ OR } y \text{ OR } \neg y)$ &
$(\neg x \text{ OR } \neg y \text{ OR } \neg z)$ & $(\neg x \text{ OR } \neg y \text{ OR } \neg z)$