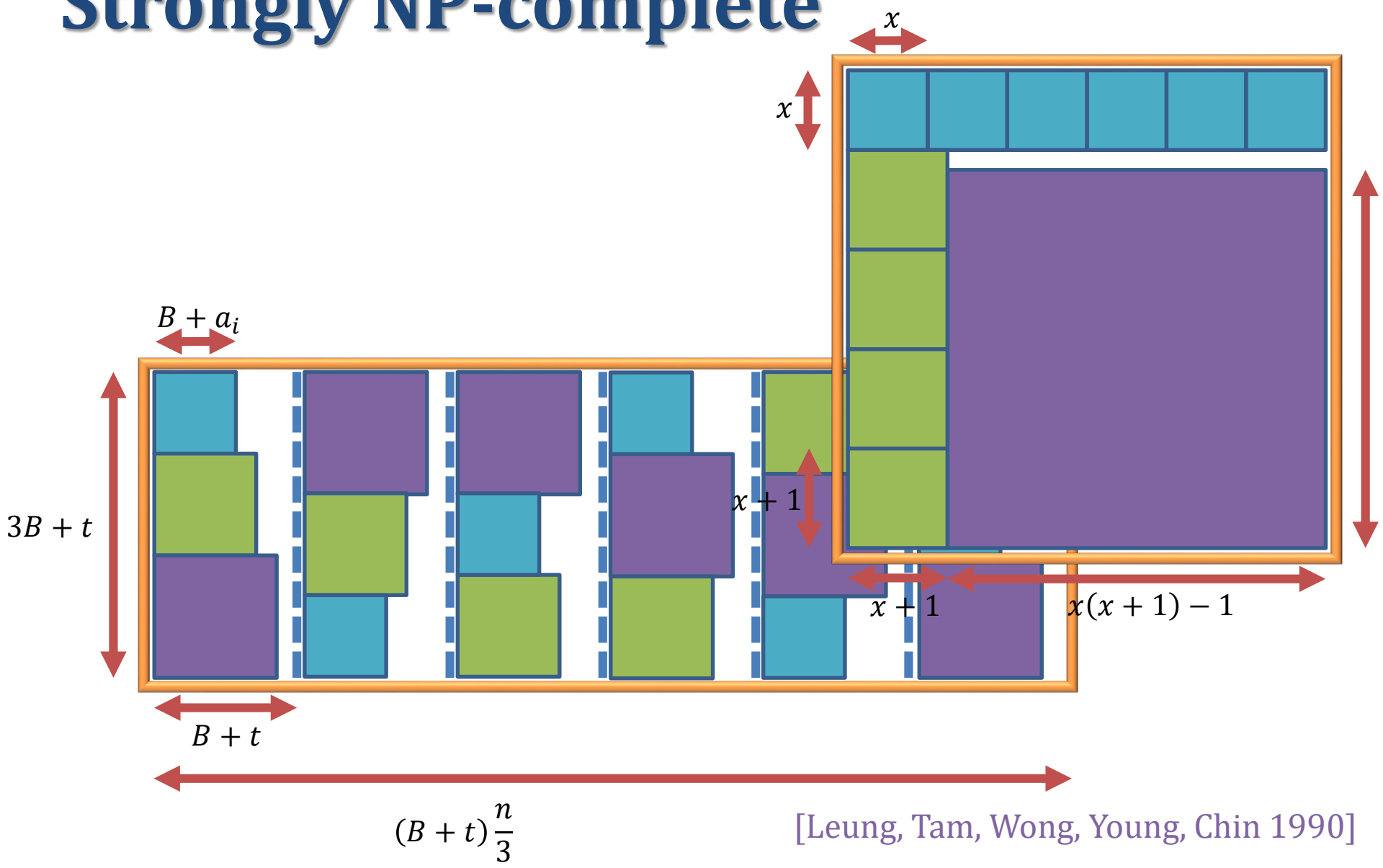
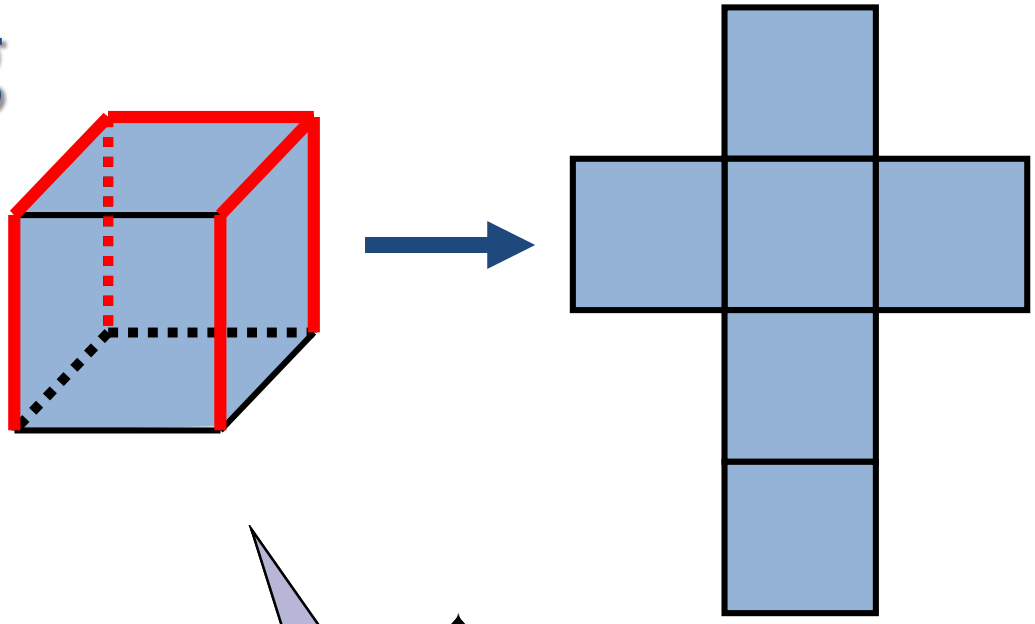
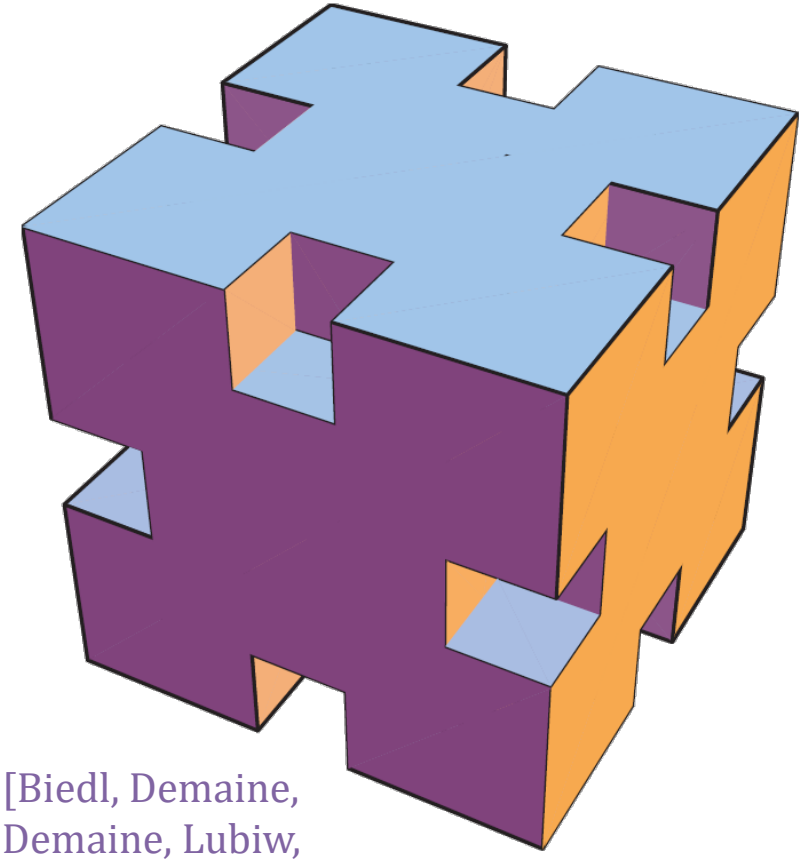
Packing Squares into a Square is Strongly NP-complete



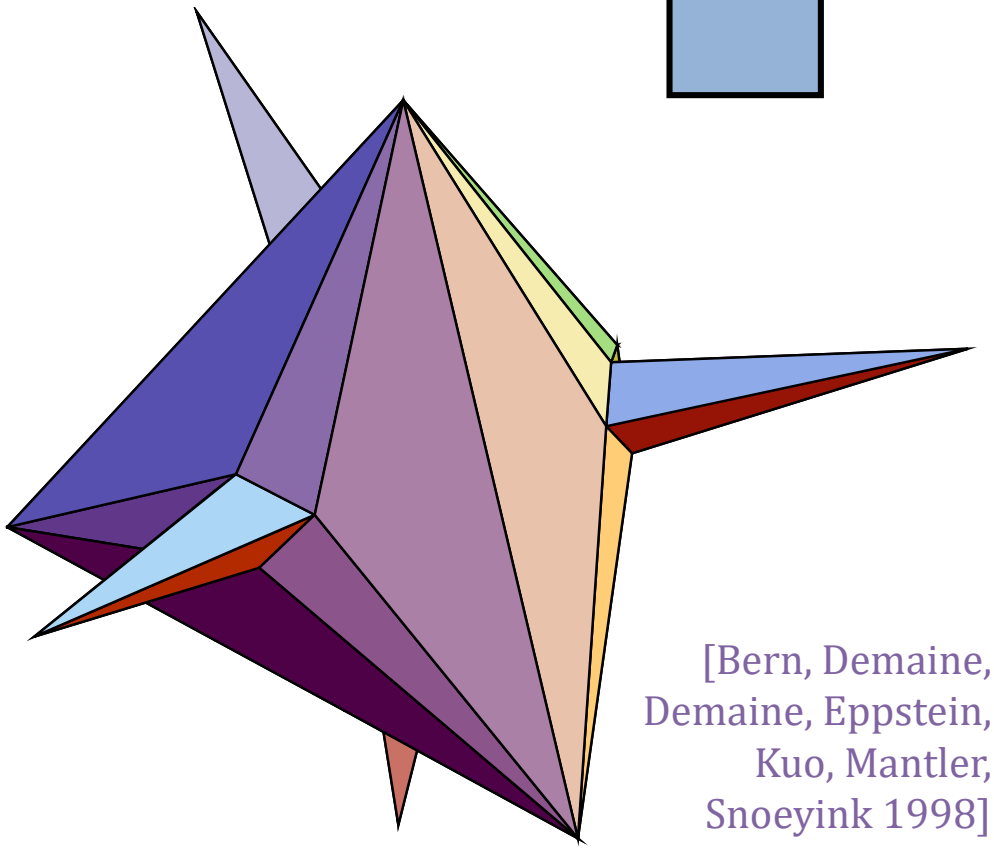
[Leung, Tam, Wong, Young, Chin 1990]

Edge-Unfolding Polyhedra

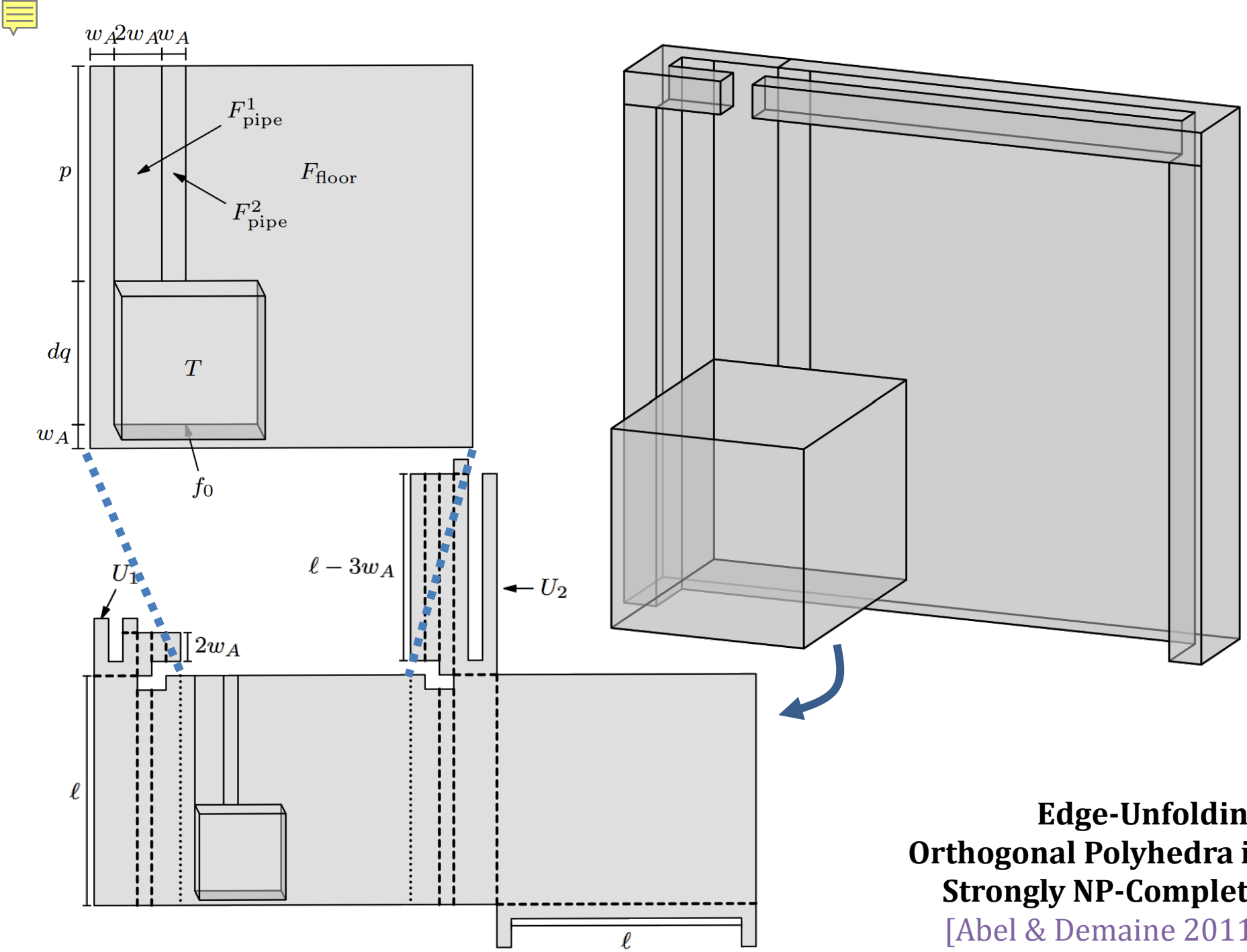




[Biedl, Demaine, Demaine, Lubiw, Overmars, O'Rourke, Robbins, Whitesides 1998]

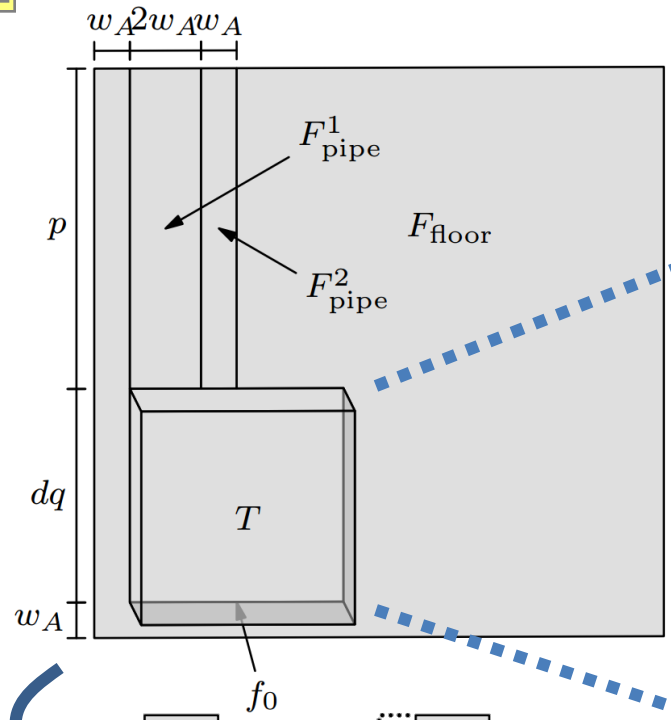


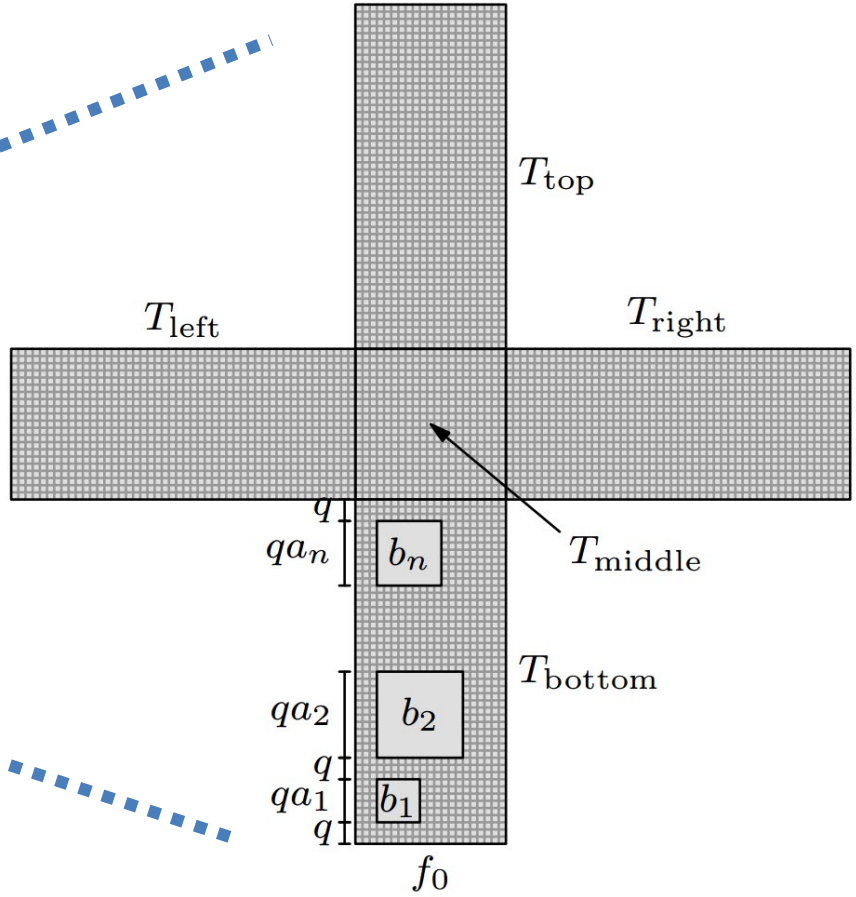
[Bern, Demaine, Demaine, Eppstein, Kuo, Mantler, Snoeyink 1998]

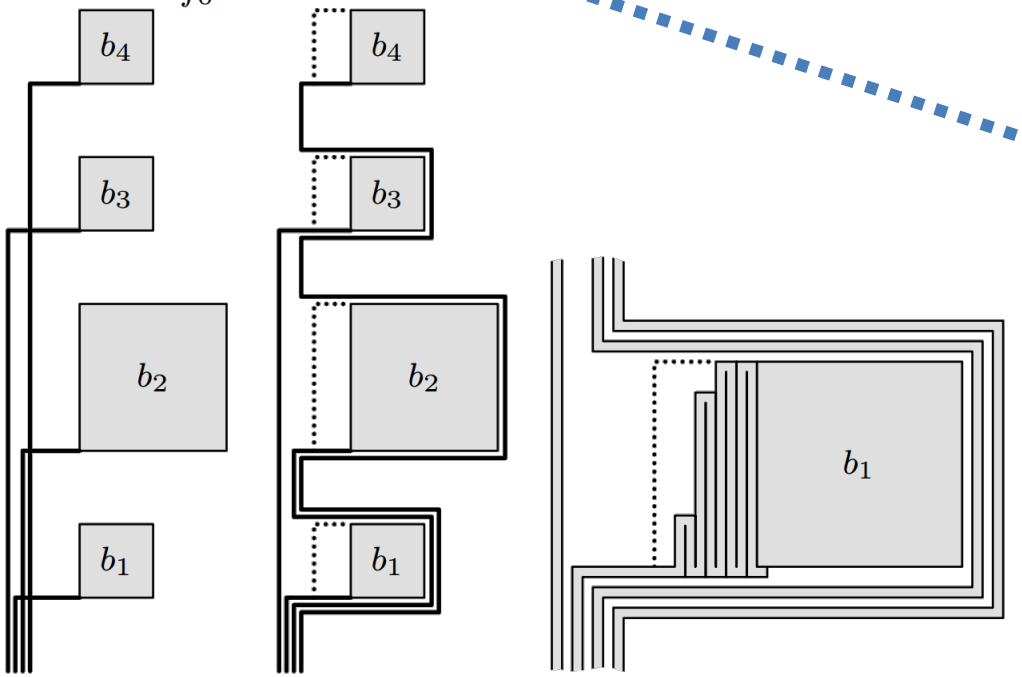


Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

[Abel & Demaine 2011]

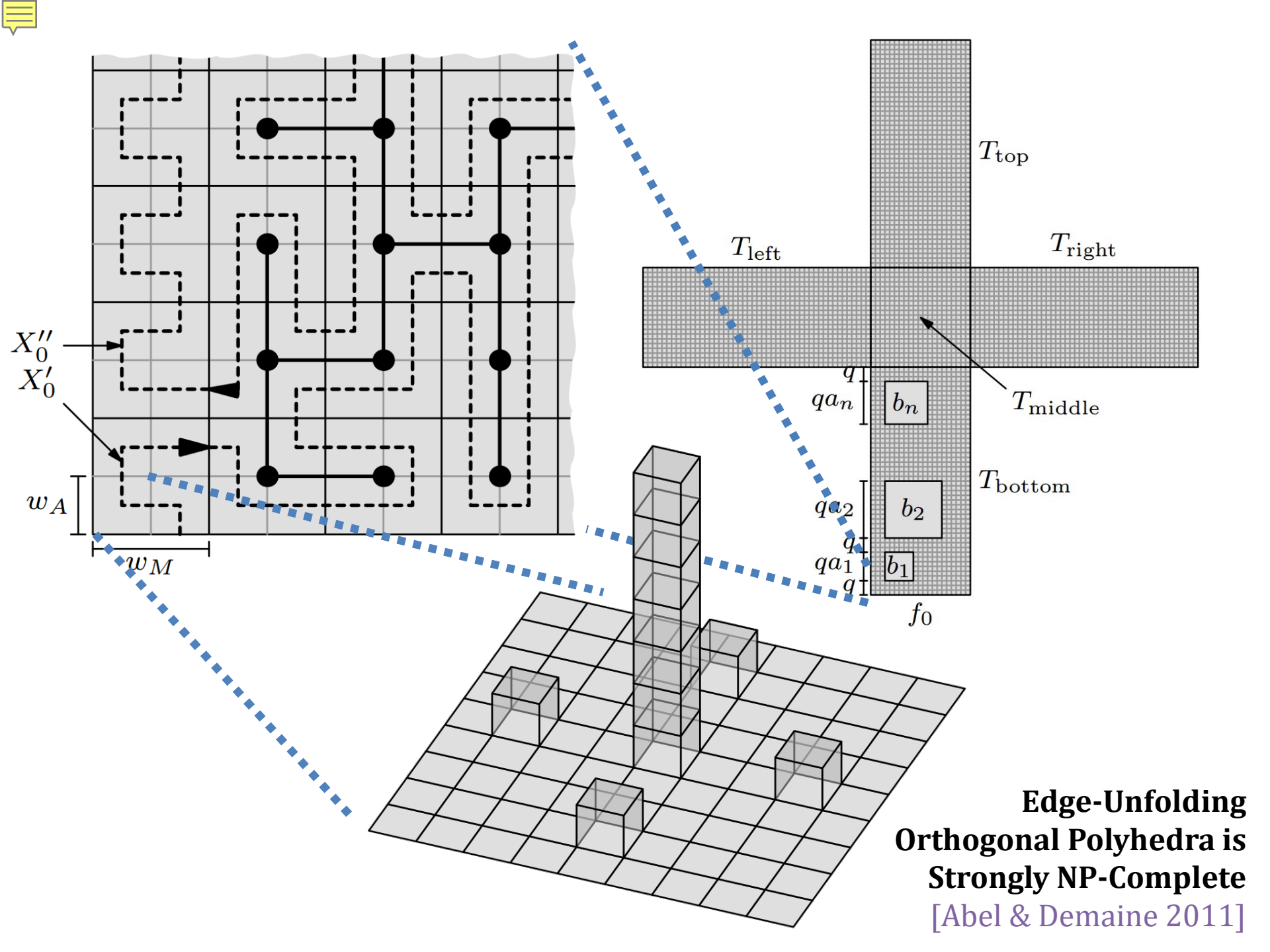






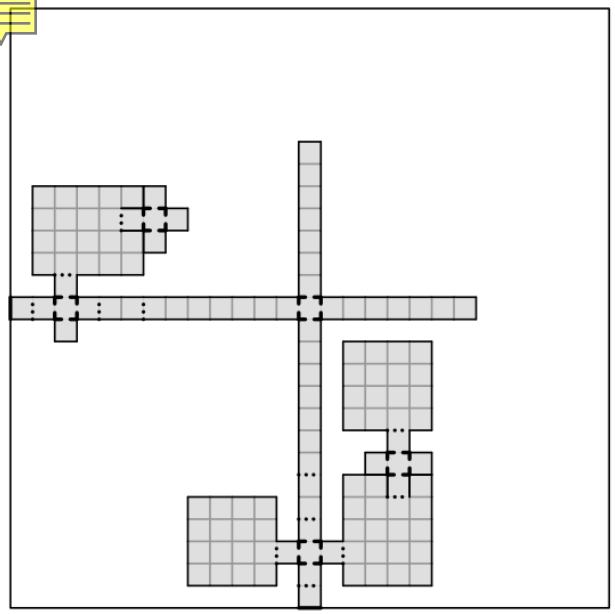
Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

[Abel & Demaine 2011]

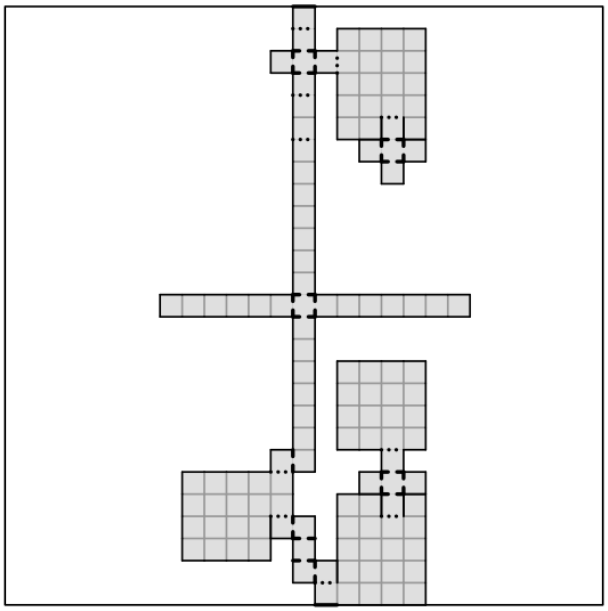


Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

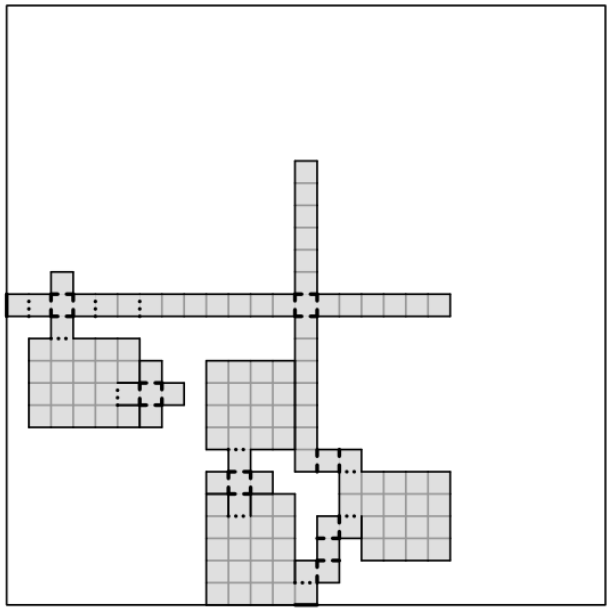
[Abel & Demaine 2011]



(a) [L, L, 13, 13].

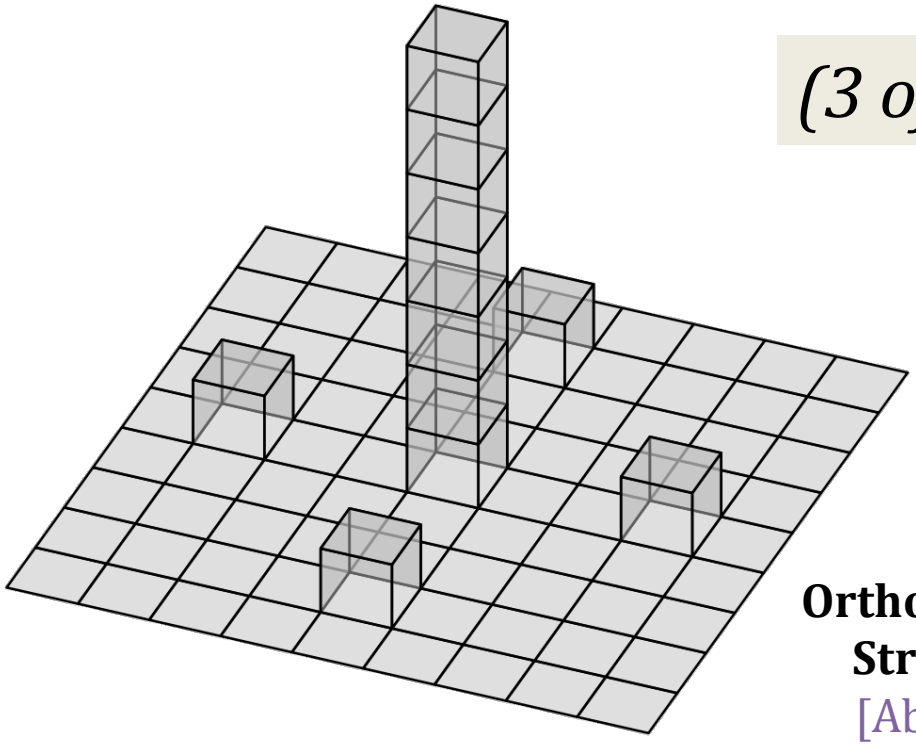


(b) [L, S, 14, 13].



(c) [R, L, 13, 13].

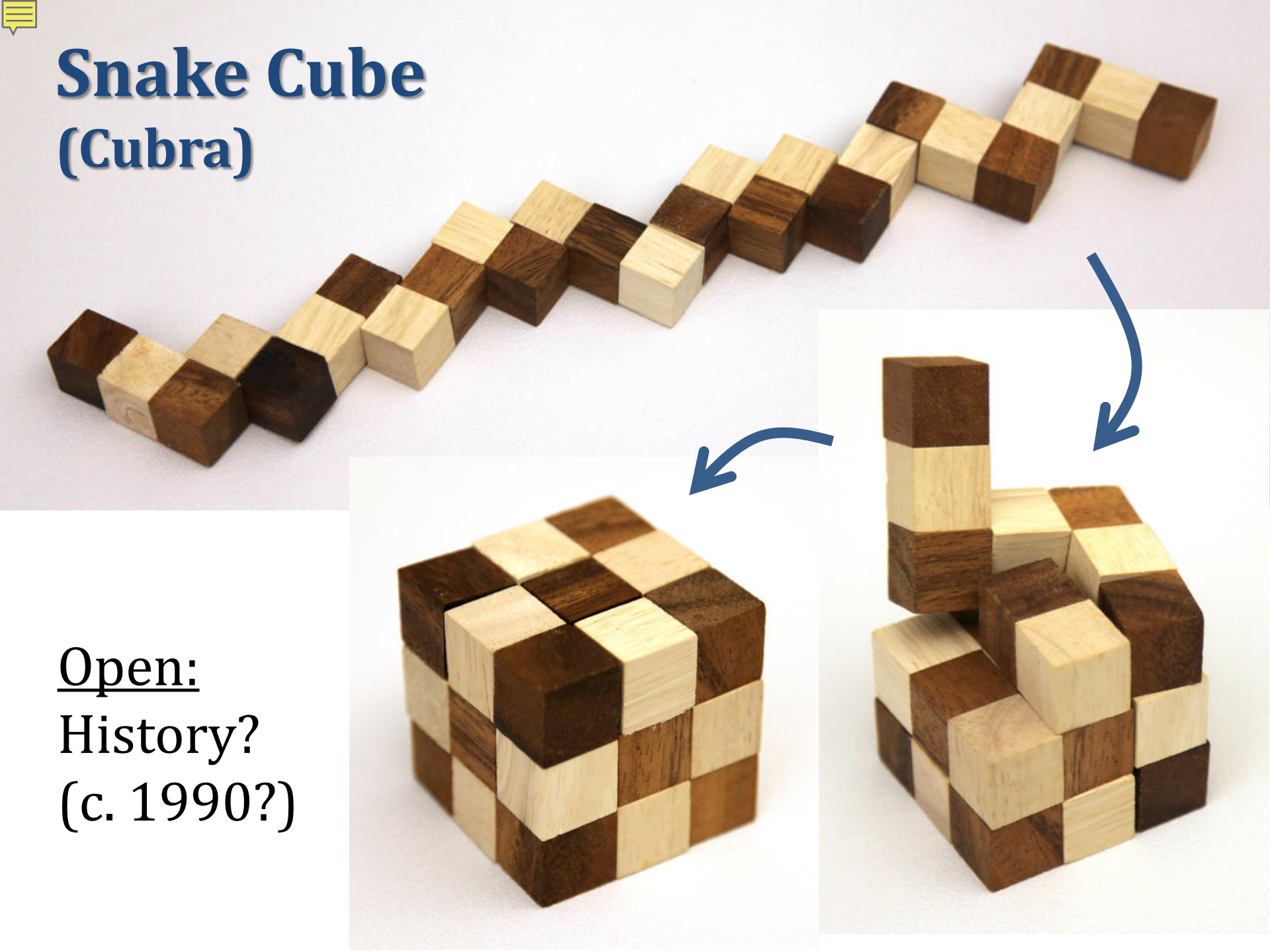
(3 of 10 cases)



Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

[Abel & Demaine 2011]

Snake Cube (Cubra)

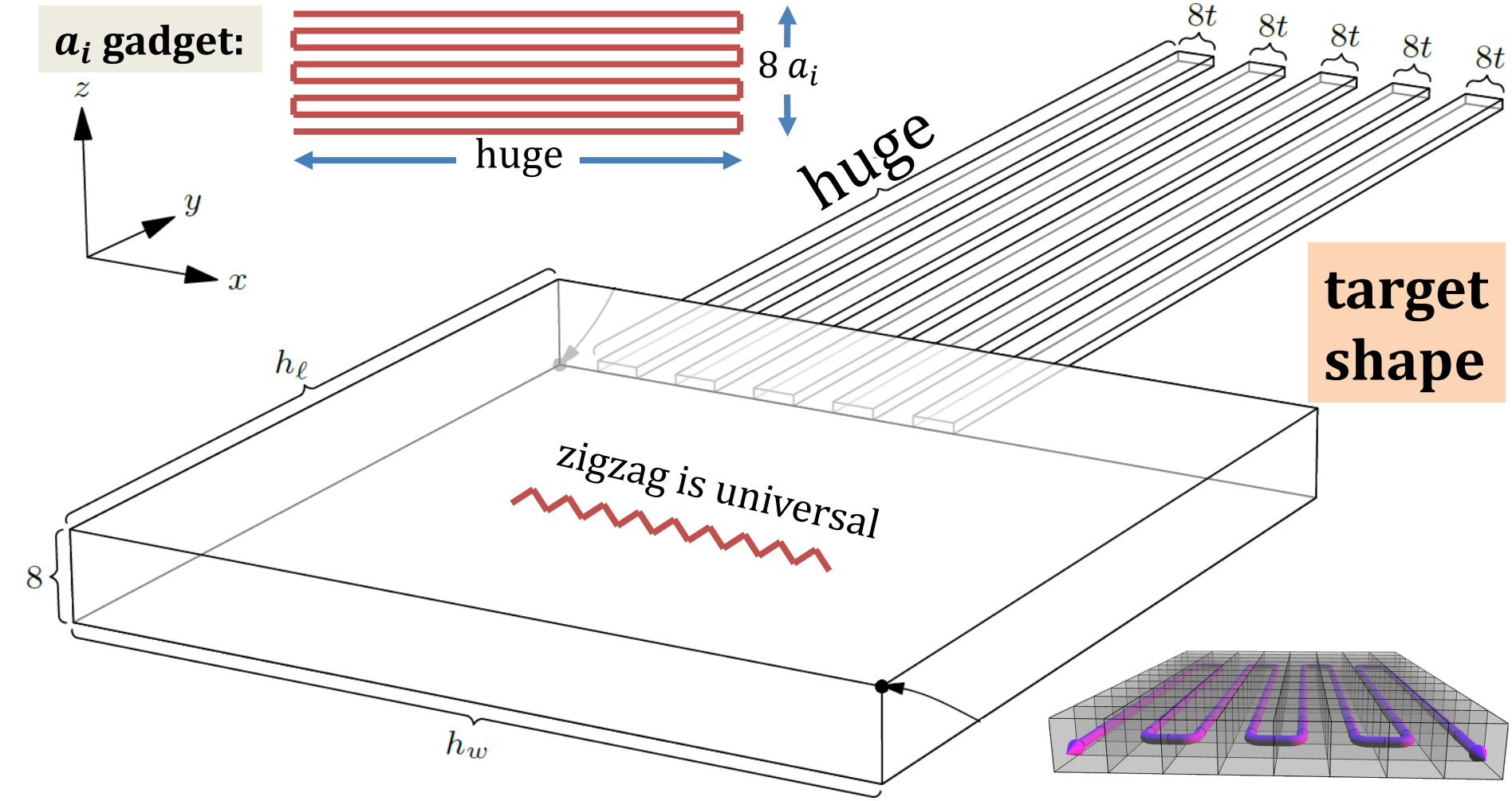


<u>Open:</u>
History?
(c. 1990?)

Snake Cube is NP-complete

[Abel, Demaine, Demaine, Eisenstat, Lynch, Schardl 2012]

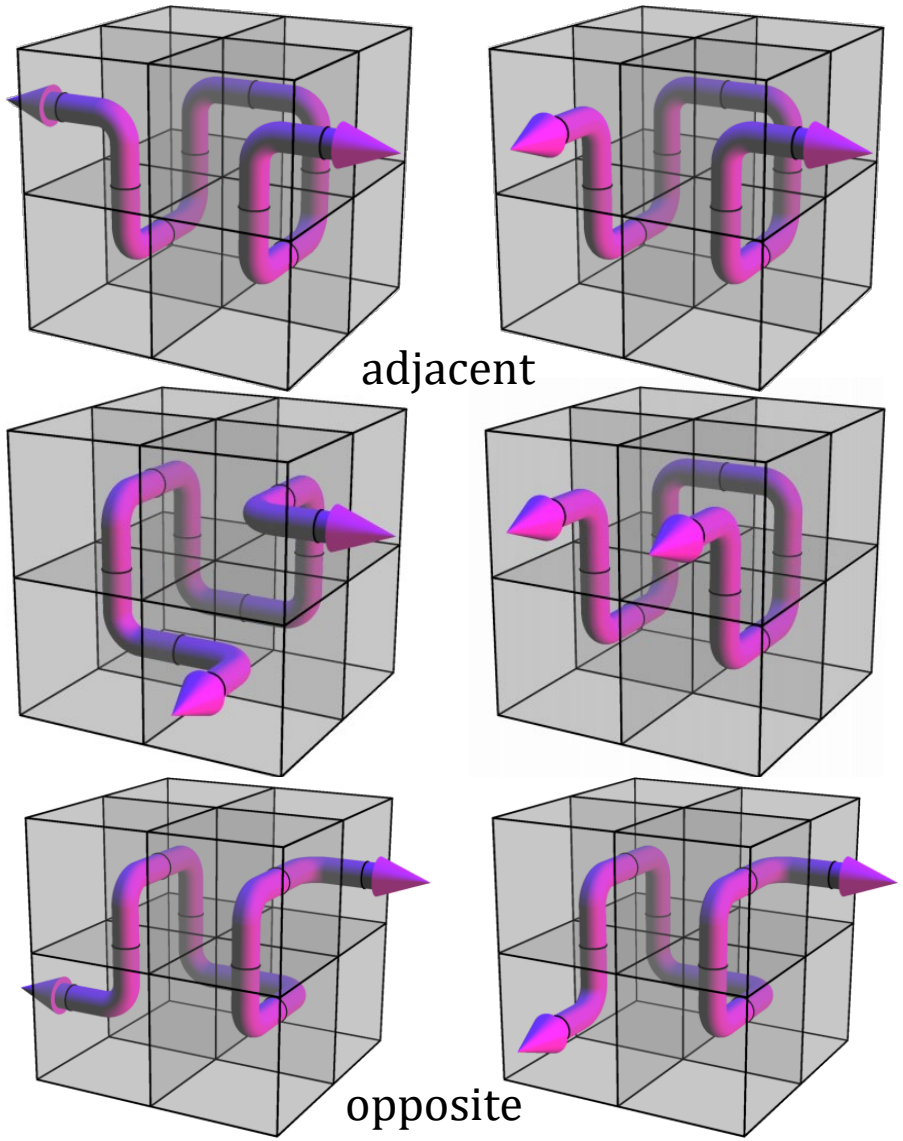
- Reduction from 3-Partition



Snake Cube is NP-complete

[Abel, Demaine, Demaine, Eisenstat, Lynch, Schardl 2012]

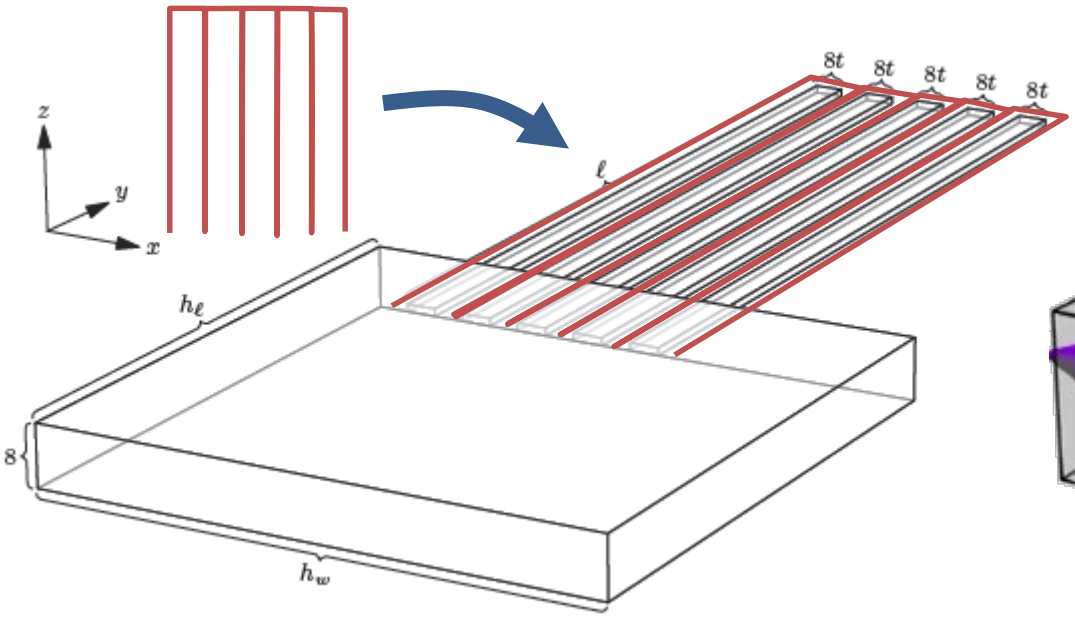
- Zigzag is universal
  - $2 \times 2 \times 2$ refinement makes any Hamiltonian shape
  - $4 \times 4 \times 4$ refinement makes any shape
- **Parity issue:** Path alternates cell parity each step
- <u>Claim:</u> Can start and end at any faces of cells of opposite parity

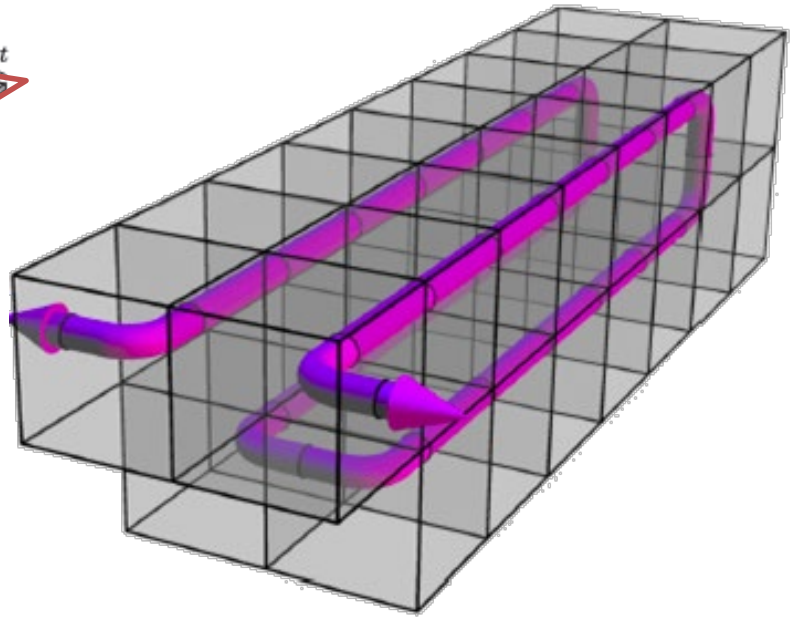


Snake Cube is NP-complete

[Abel, Demaine, Demaine, Eisenstat, Lynch, Schardl 2012]

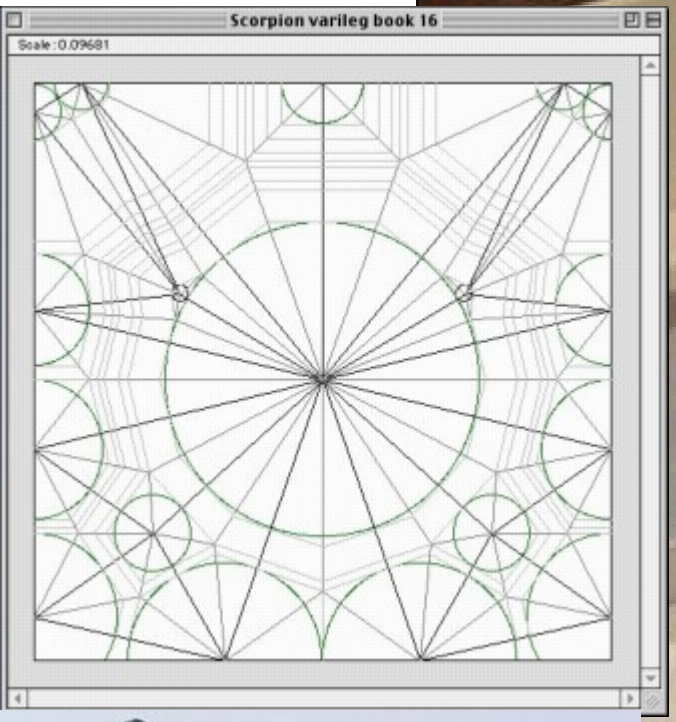
- Reduce target **box** → target **shape**





- Reduce target **cube** → target **box**
- ⇒ NP-hard to fold snake cube into target cube

Disk packing

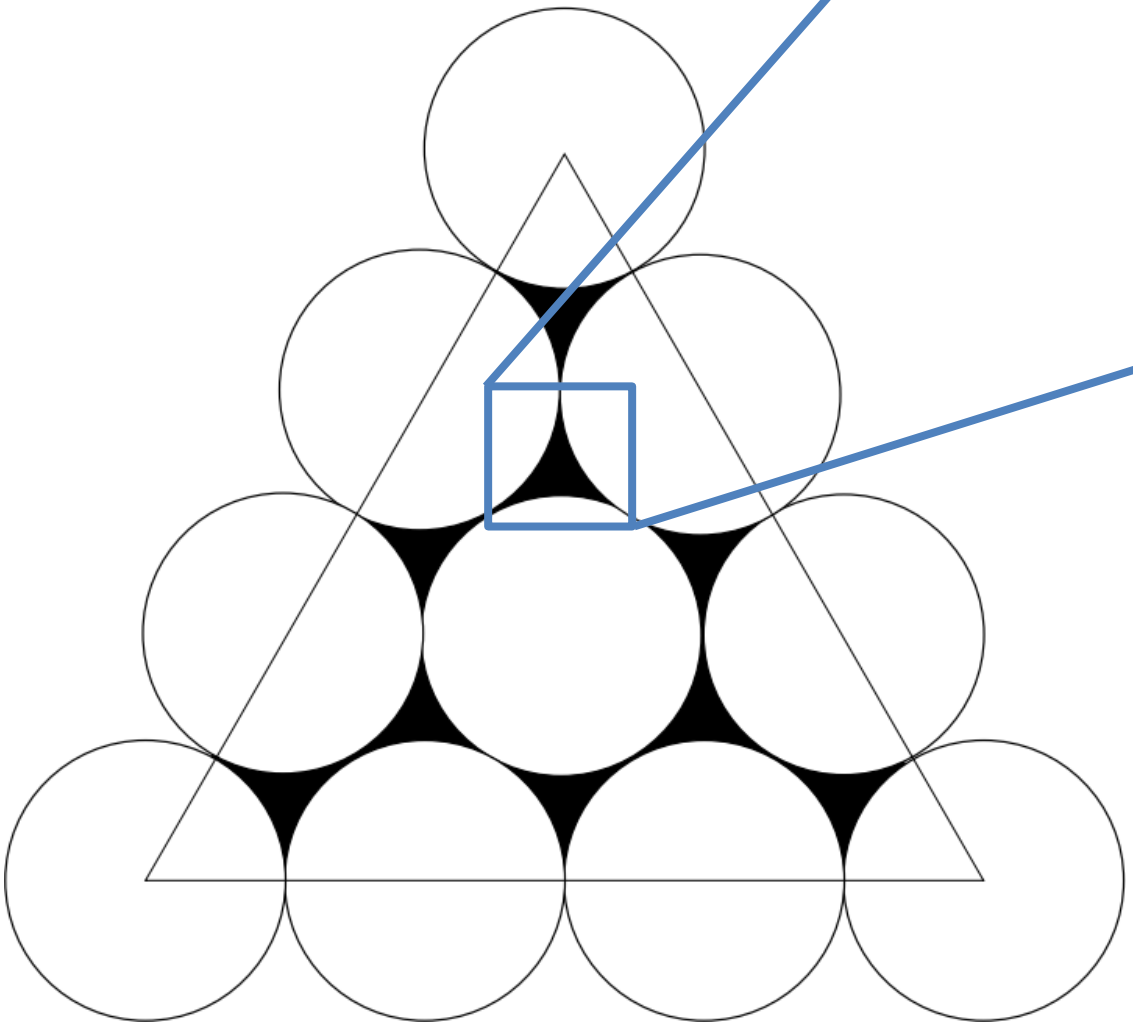


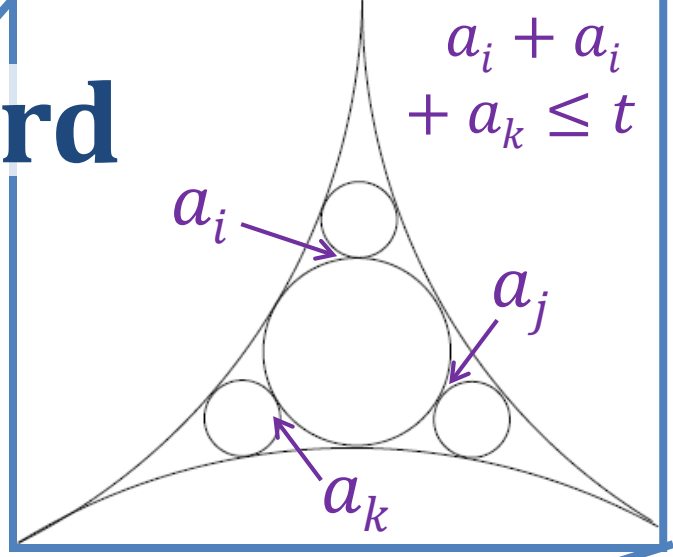
Robert Lang

Viet Elser's
disk packing puzzle

Disk packing is NP-hard

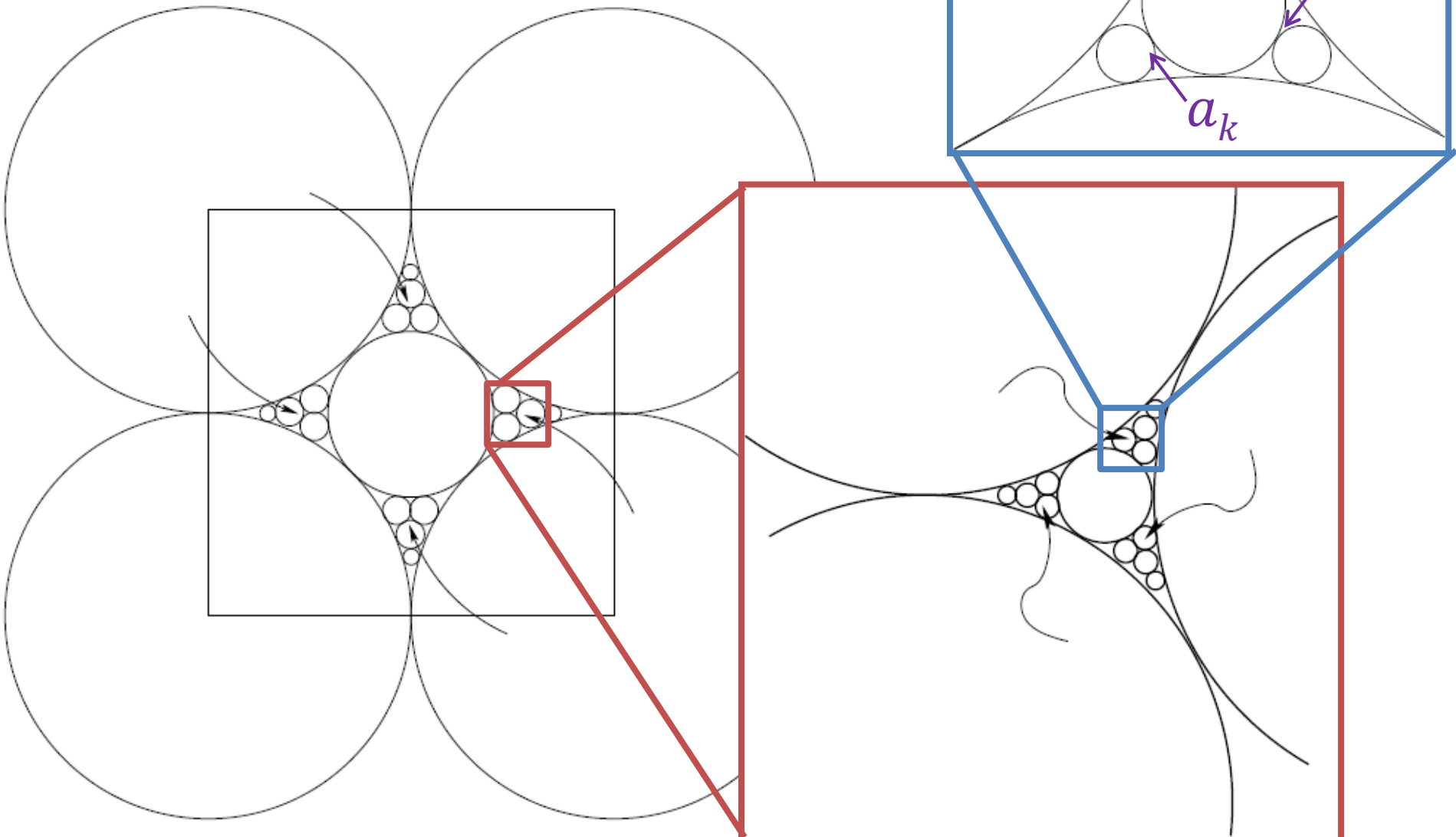
[Demaine, Fekete, Lang 2010]





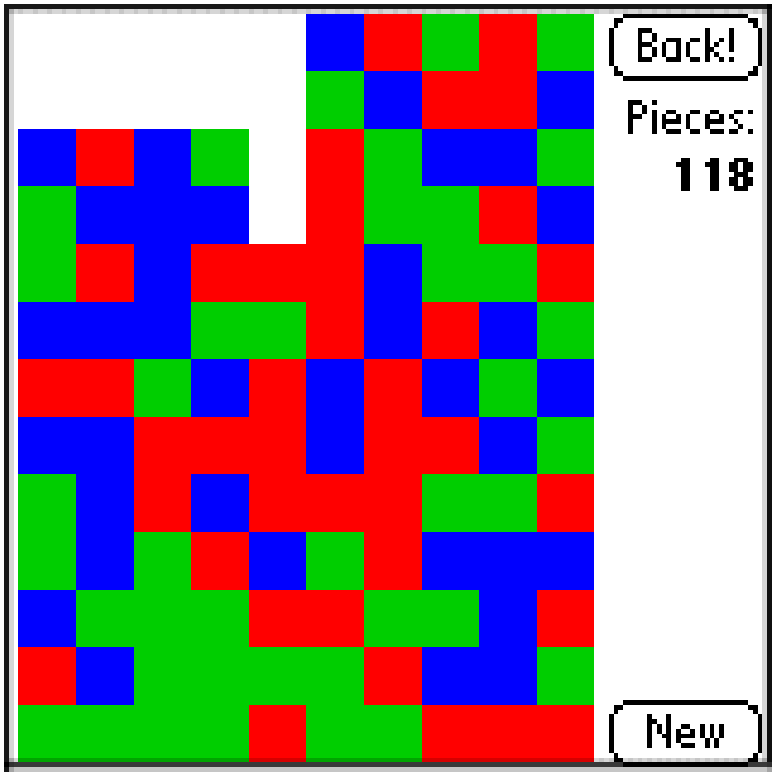
Disk packing is NP-hard

[Demaine, Fekete, Lang 2010]



Clickomania / Same Game
[Schuessler ~2000?]

- **Move** = Remove any connected group of size > 1

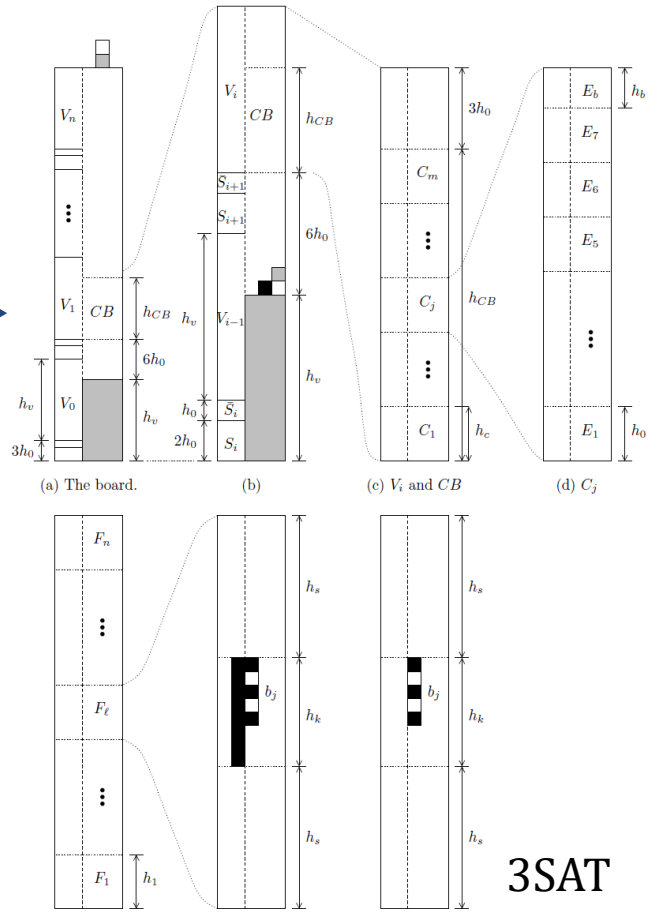


Clickomania Complexity

[Biedl, Demaine, Demaine, Fleischer, Jacobsen, Munro 2000]

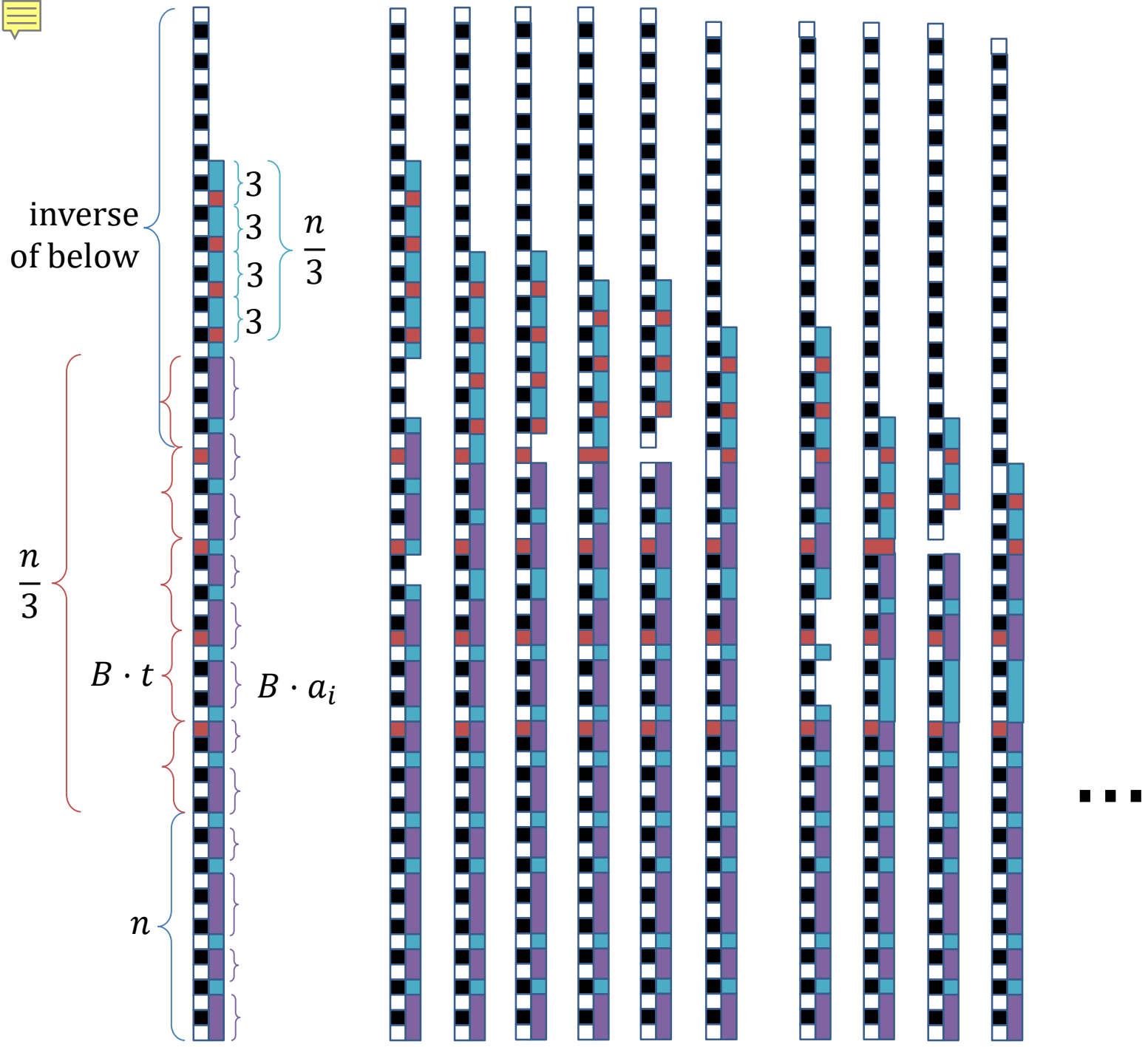
- **Polynomial** for one row/column via CFG
- **NP-hard** for
  - 2 columns & 5 colors
  - 5 columns & 3 colors
- <u>Open:</u>
  - 2 rows
  - 2 colors

$$S \rightarrow \Lambda \mid SS \mid c_i S c_i \mid c_i S c_i S c_i$$



3-partition

3SAT



[Biedl, Demaine, Demaine, Fleischer, Jacobsen, Munro 2000]

$$B = \frac{4}{3}n$$

SCORE

155

LEVEL
0

LINES
1

MIT Green Bldg.
April 2012

In Honor of your Intellectual Contribution to the Art of Tetris,

FOR PROVING NP-COMPLETENESS IN MAXIMIZATION OF LINES, TETRISES, PIECES PLAYED, OR MINIMIZATION OF SQUARE HEIGHT,

we masters of the Harvard Tetris Society hereby confer the title of

TETRIS MASTER

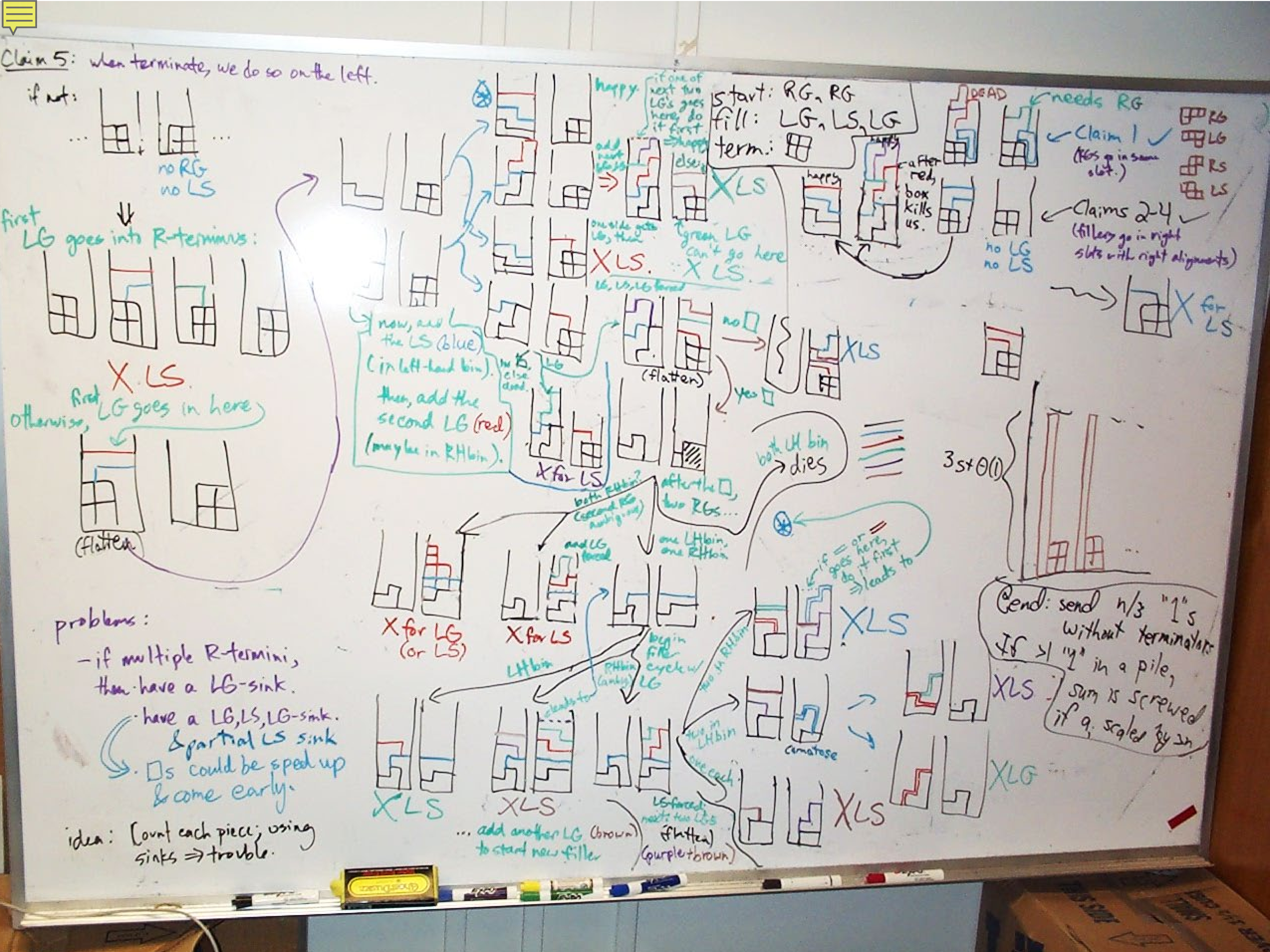
upon

Erik D. Demaine

on the sixteenth day of the twelth month in the year 17 Anno Tetri (2002)

David Renard
HTS President

Gregory M. Angelo
HTS Treasurer.



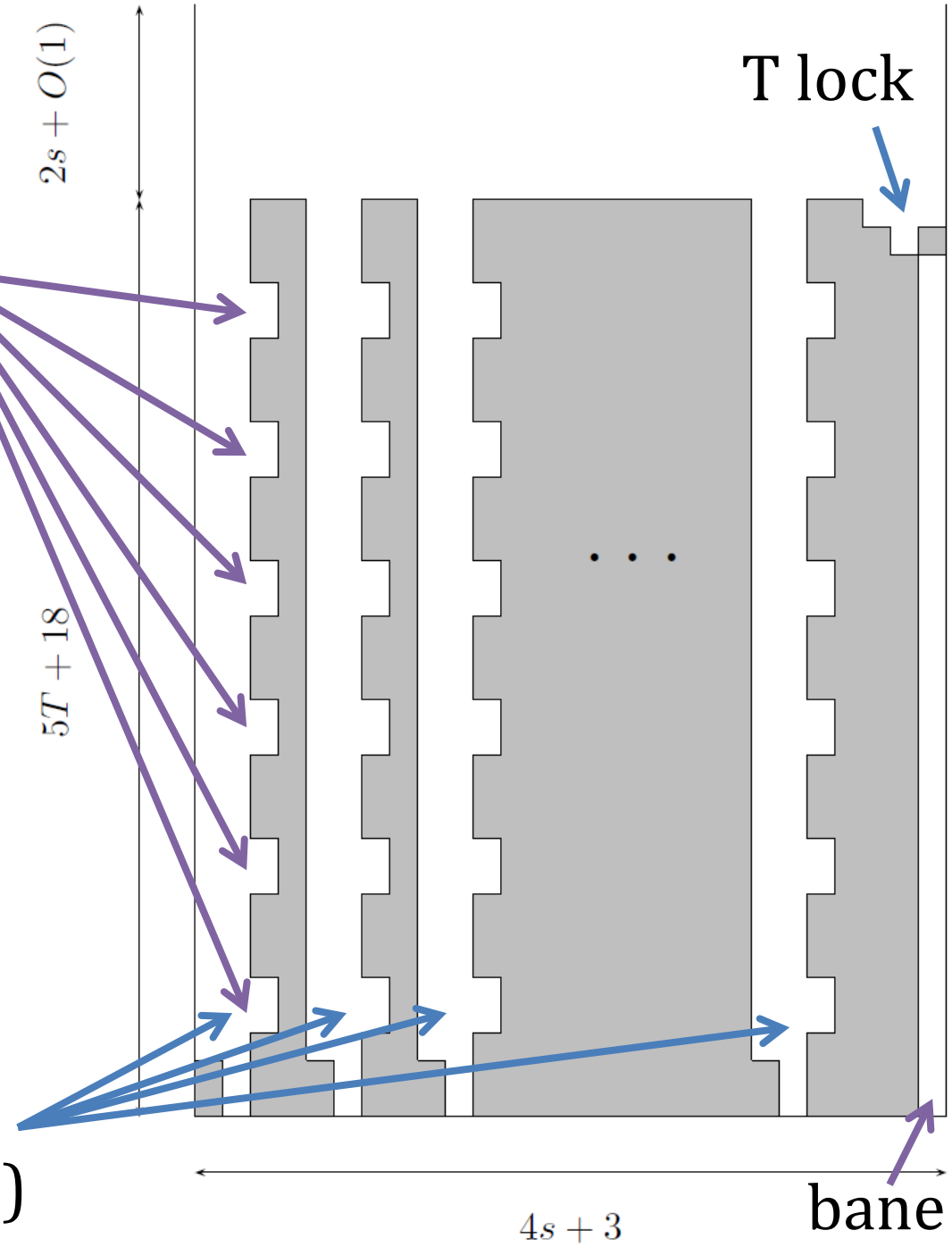
Claim 5: when terminate, we do so on the left.
if not:
no RG
no LS
first LG goes into R-terminus:
X LS
otherwise, first LG goes in here
(flatten)
problems:
- if multiple R-termini, then have a LG-sink.
· have a LG,LS,LG-sink. & partial LS sink
· □s could be sped up & come early.
idea: Count each piece; using sinks ⇒ trouble.
now, add the LS (blue) (in left-hand bin).
then, add the second LG (red) (maybe in RH bin).
X for LS
happy.
if one of next two LG's goes here, do it first ⇒ happy
add next blocks
else
X LS
one side gets LG, then
X LS.
green LG can't go here
X LS
LG, LS, LG forced
no □
XLS
(flatten)
yes □
both LH bin
dies
after the □, two RGs...
both RH bin (second RG ambiguous)
one LH bin, one RH bin
and LG forced
X for LG (or LS)
X for LS
LH bin
RH bin (ambig) LG
begin filler cycle w/ LG
leads to
X LS
X LS
... add another LG (brown) to start new filler
LS forced; next two LGs
(flatten)
(purple+brown)
two in RH bin
two in LH bin
one each
if = or = goes here, do it first ⇒ leads to
XLS
comatose
XLS
XLG
XLS
start: RG, RG
fill: LG, LS, LG
term:
happy
after red, box kills us.
DEAD
needs RG
Claim 1 ✓ (RGs go in same slot.)
RG
LG
RS
LS
Claims 2-4 ✓ (fillers go in right slots with right alignments)
no LG
no LS
X for LS
3s+Θ(1)
Gend: send n/3 "1"s without terminators.
If >1 "1" in a pile, sum is screwed if a_i scaled by 3n

Initial Board

$\approx t$ notches
(target sum)

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]

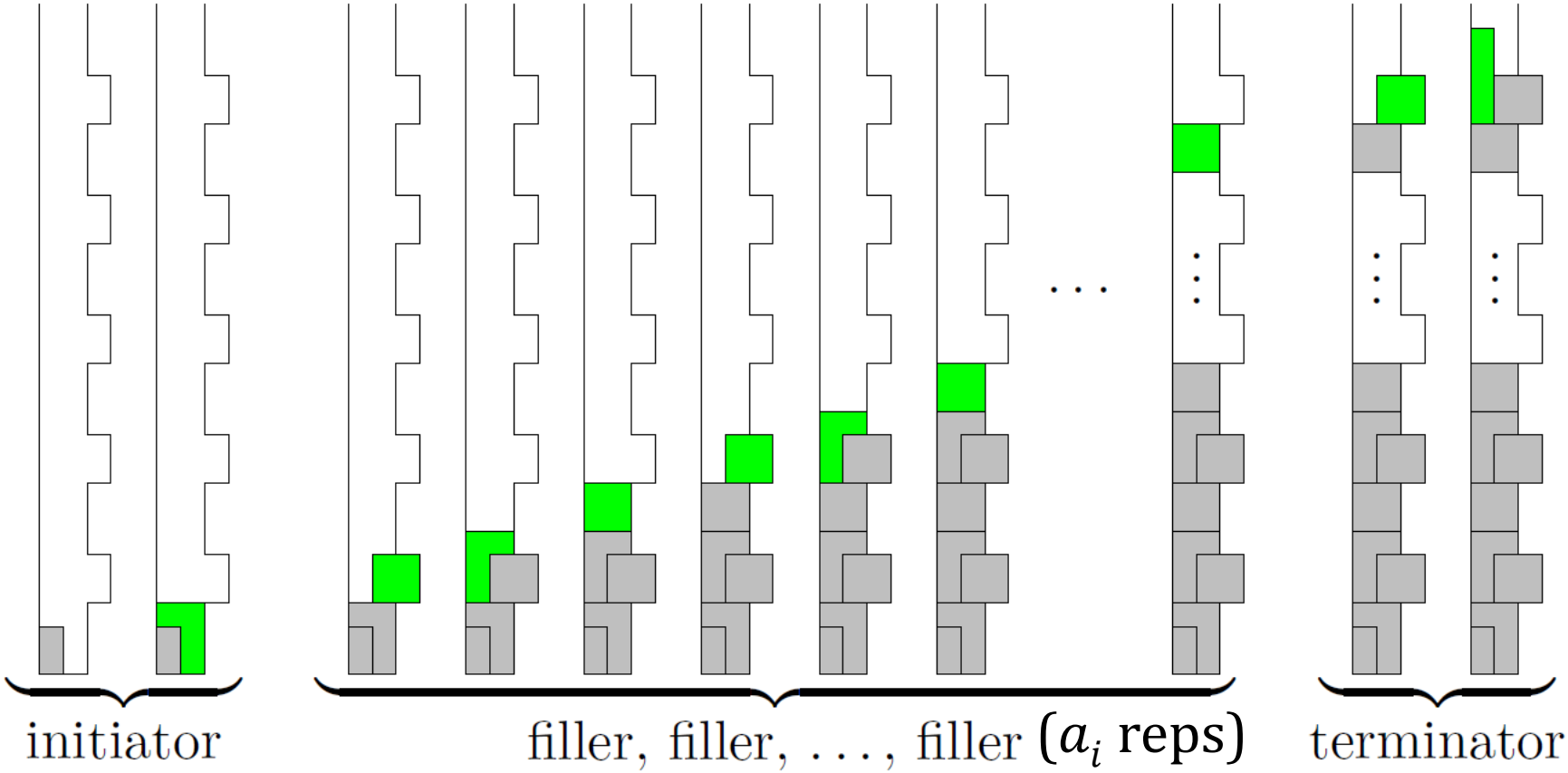
(it is possible to actually get here)



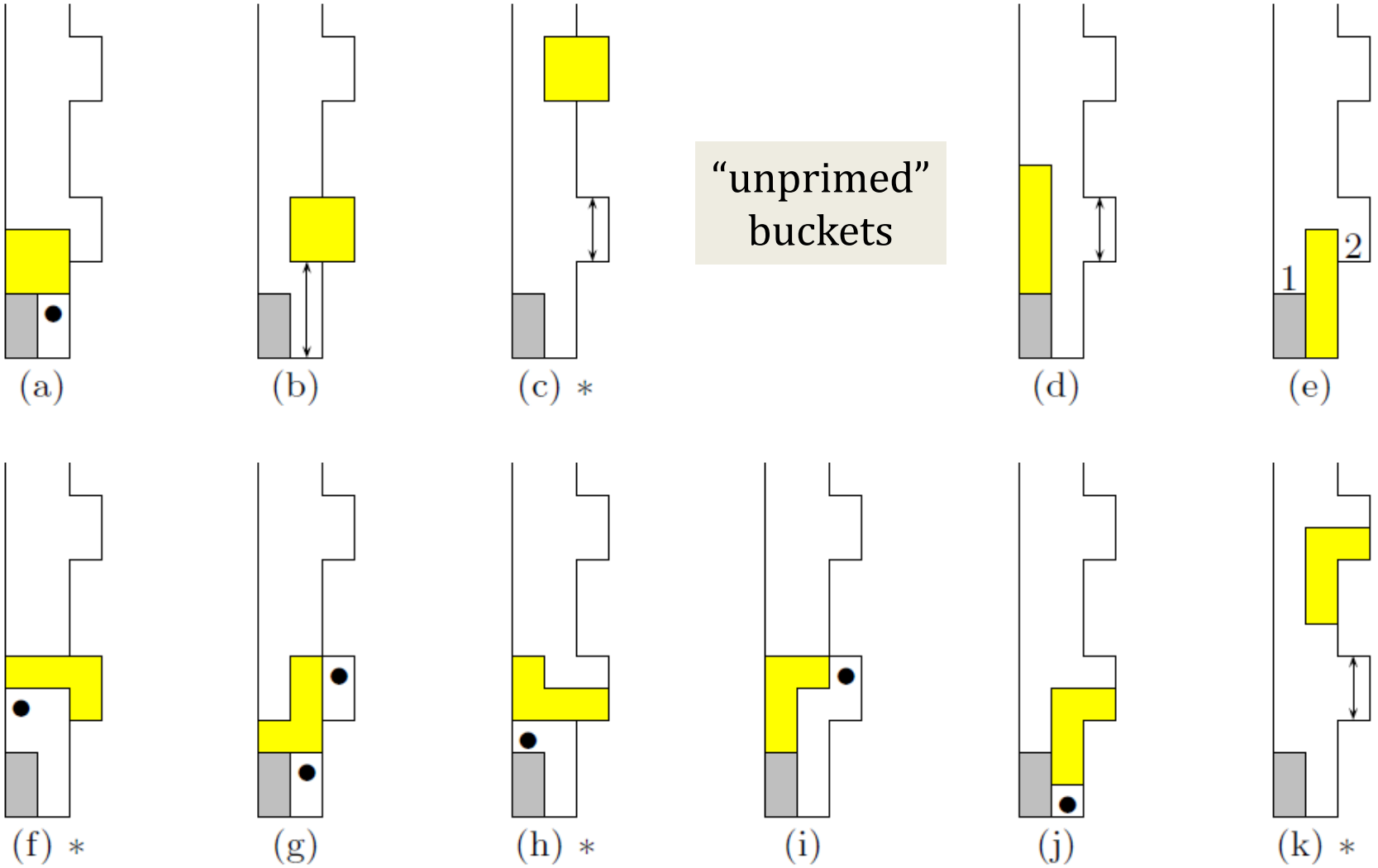
$n/3$ buckets
(one per sum)

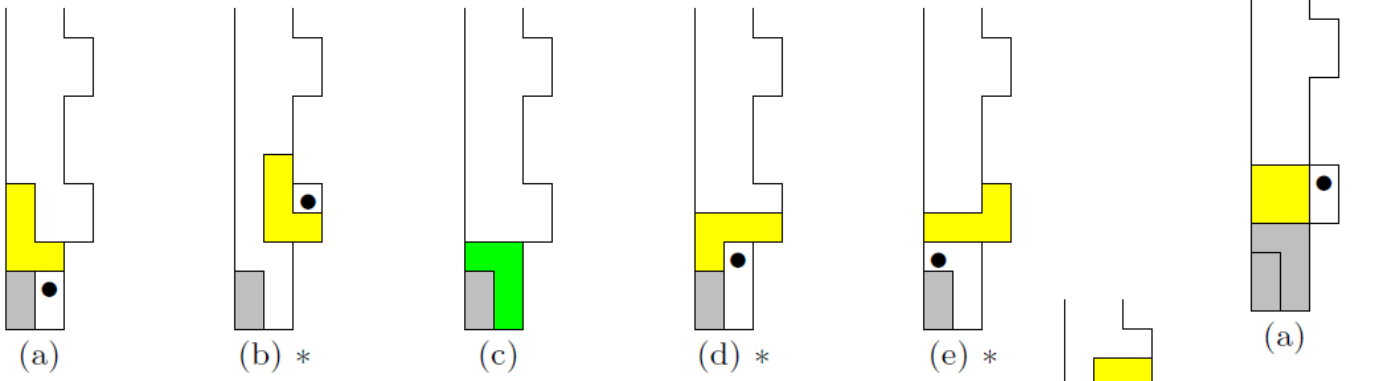
Piece Sequence [Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]

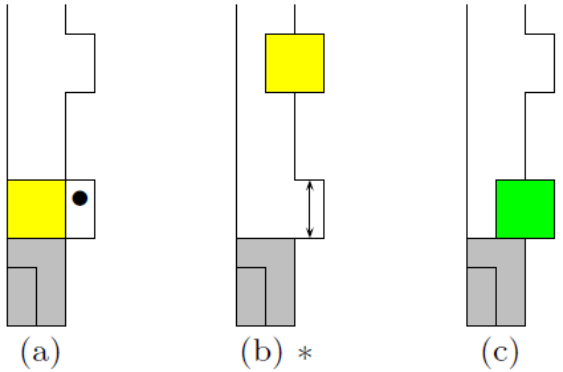
- For each input a_i:

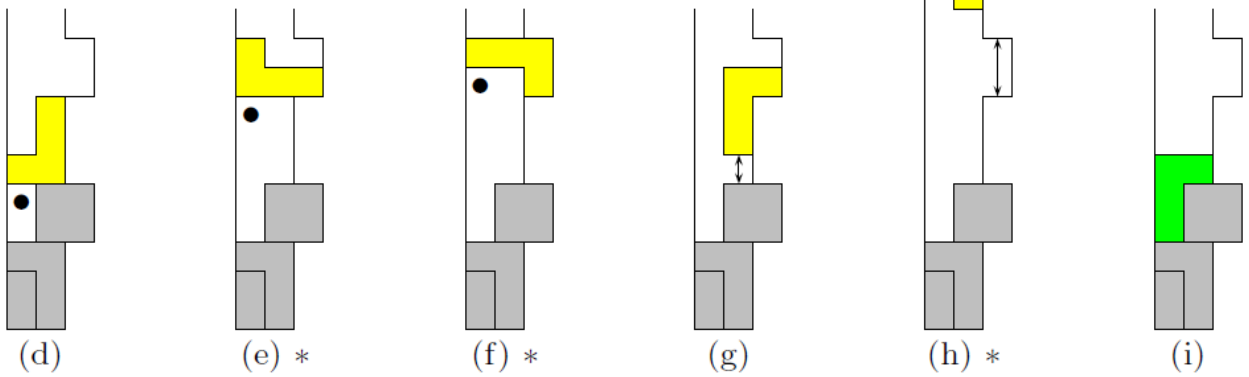


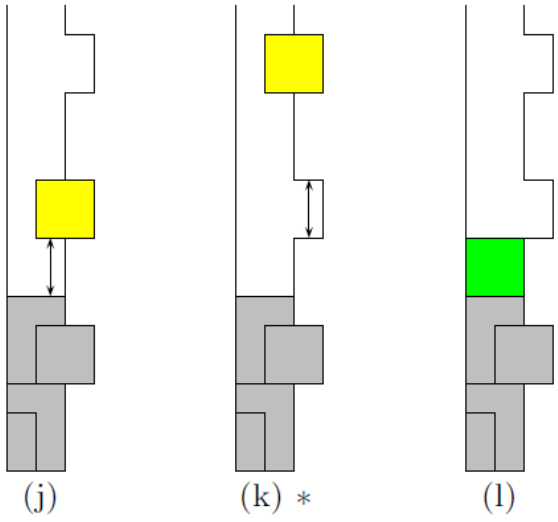
Failure to Launch [Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]









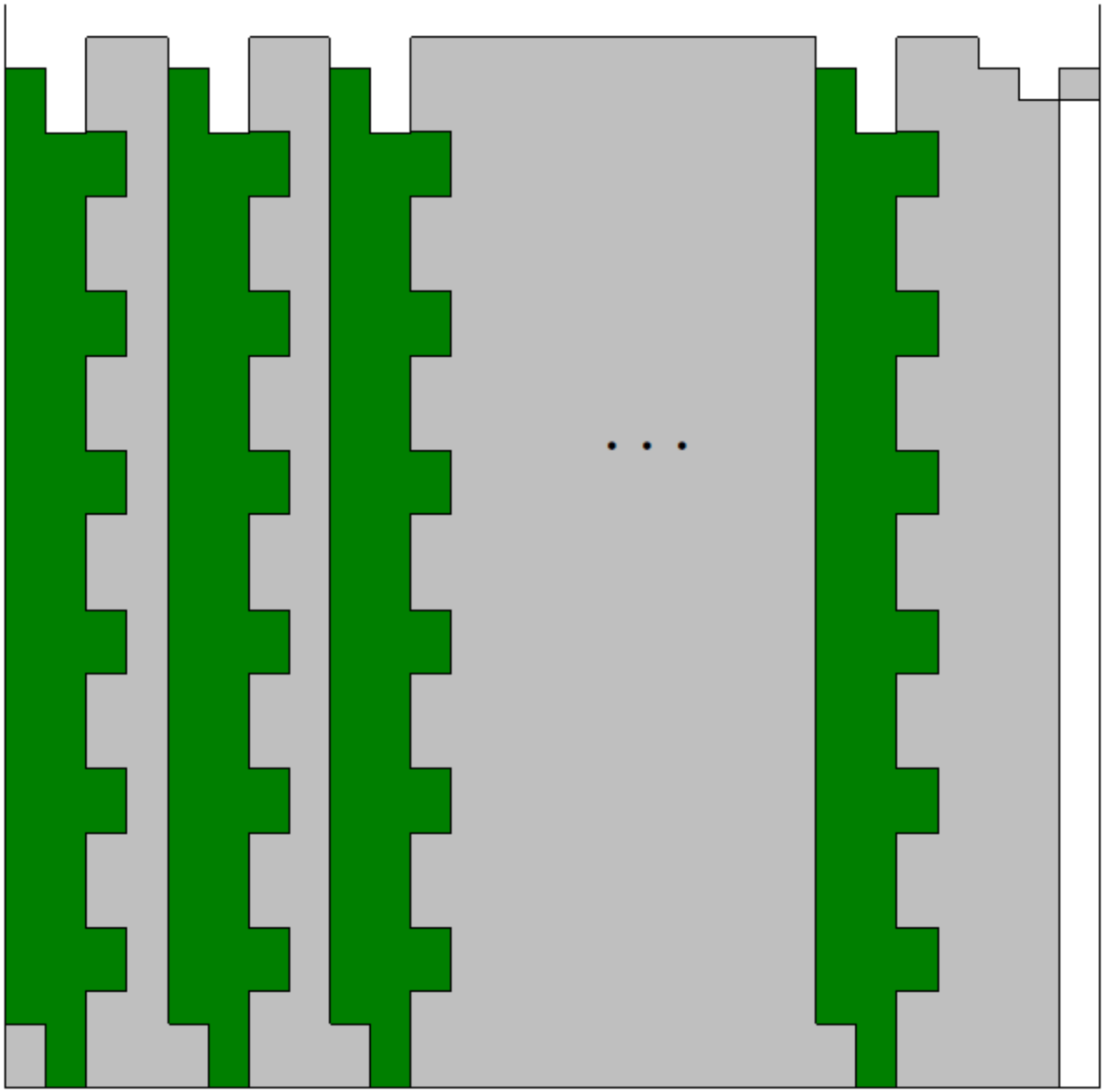


Forced Moves

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]

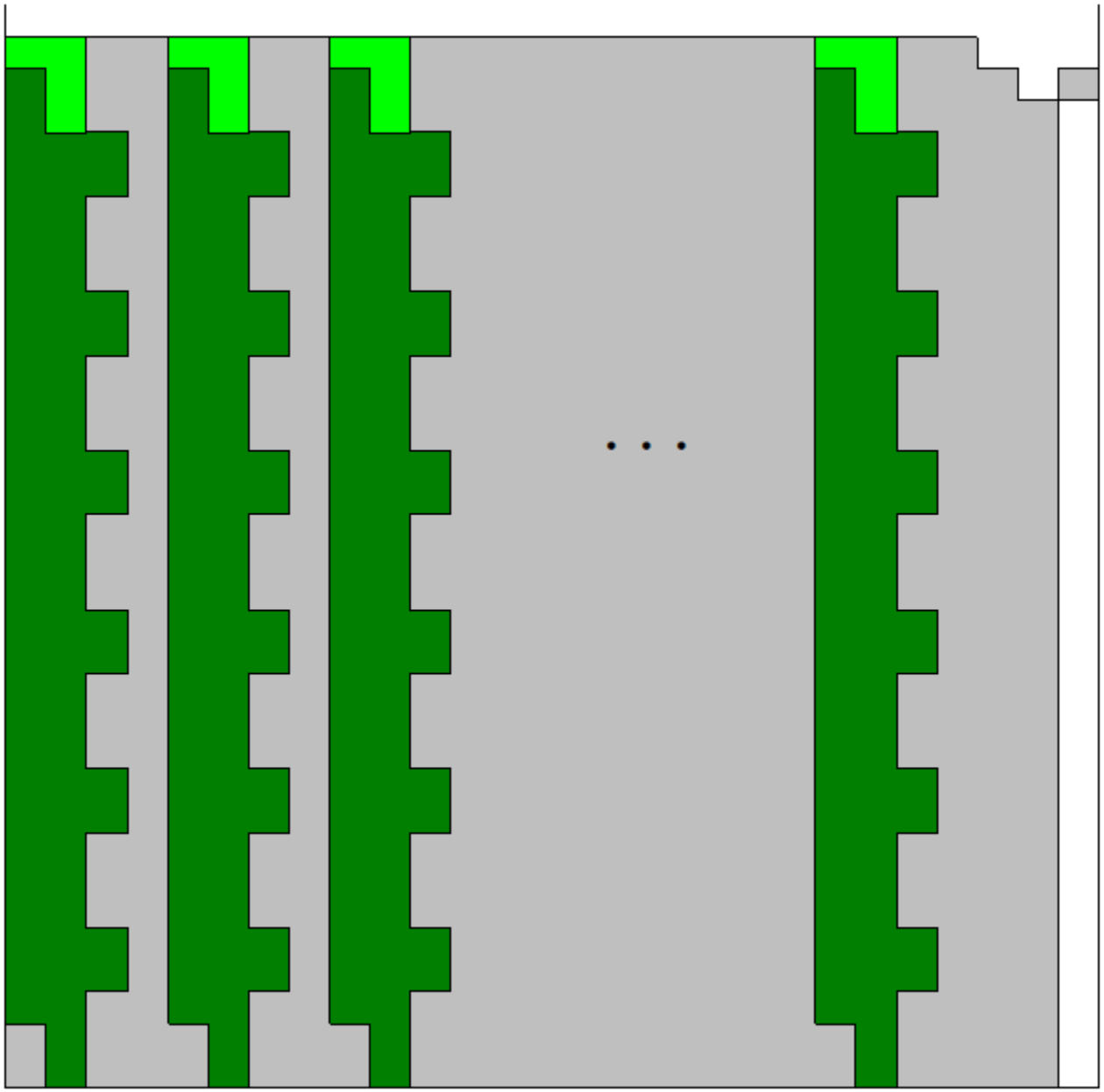
Finale Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]



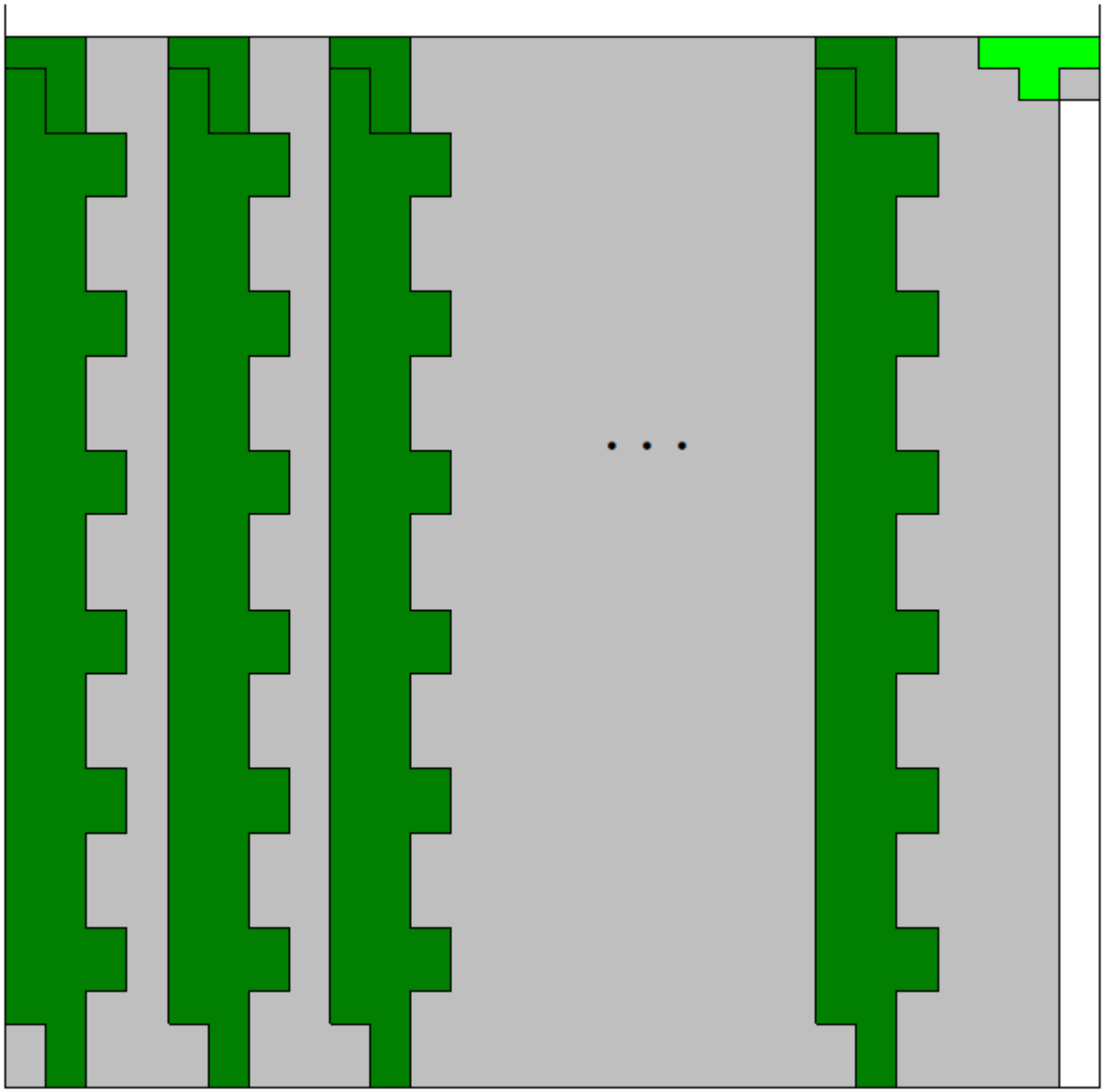
Finale Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]



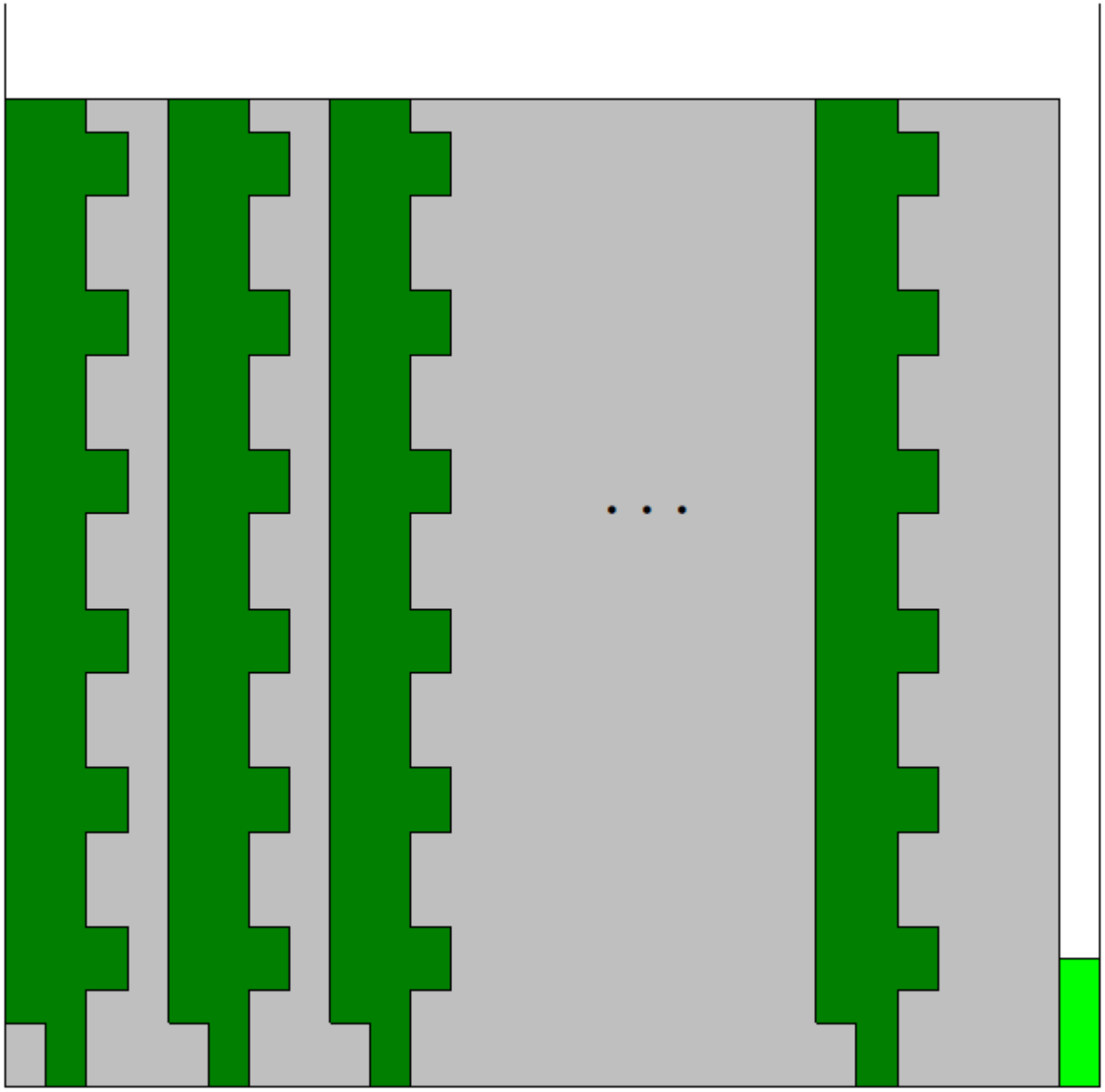
Finale Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]



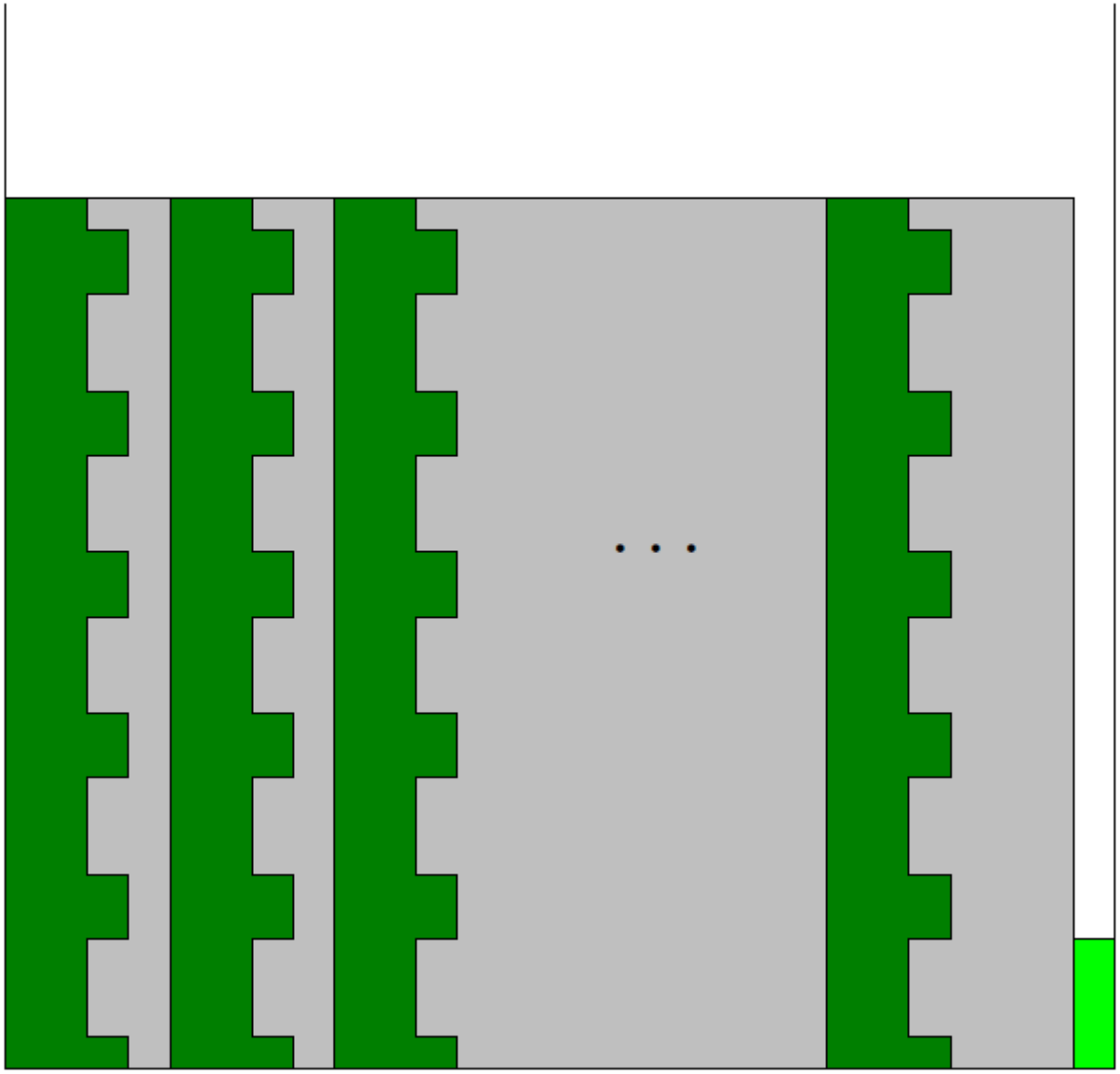
Finale Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]



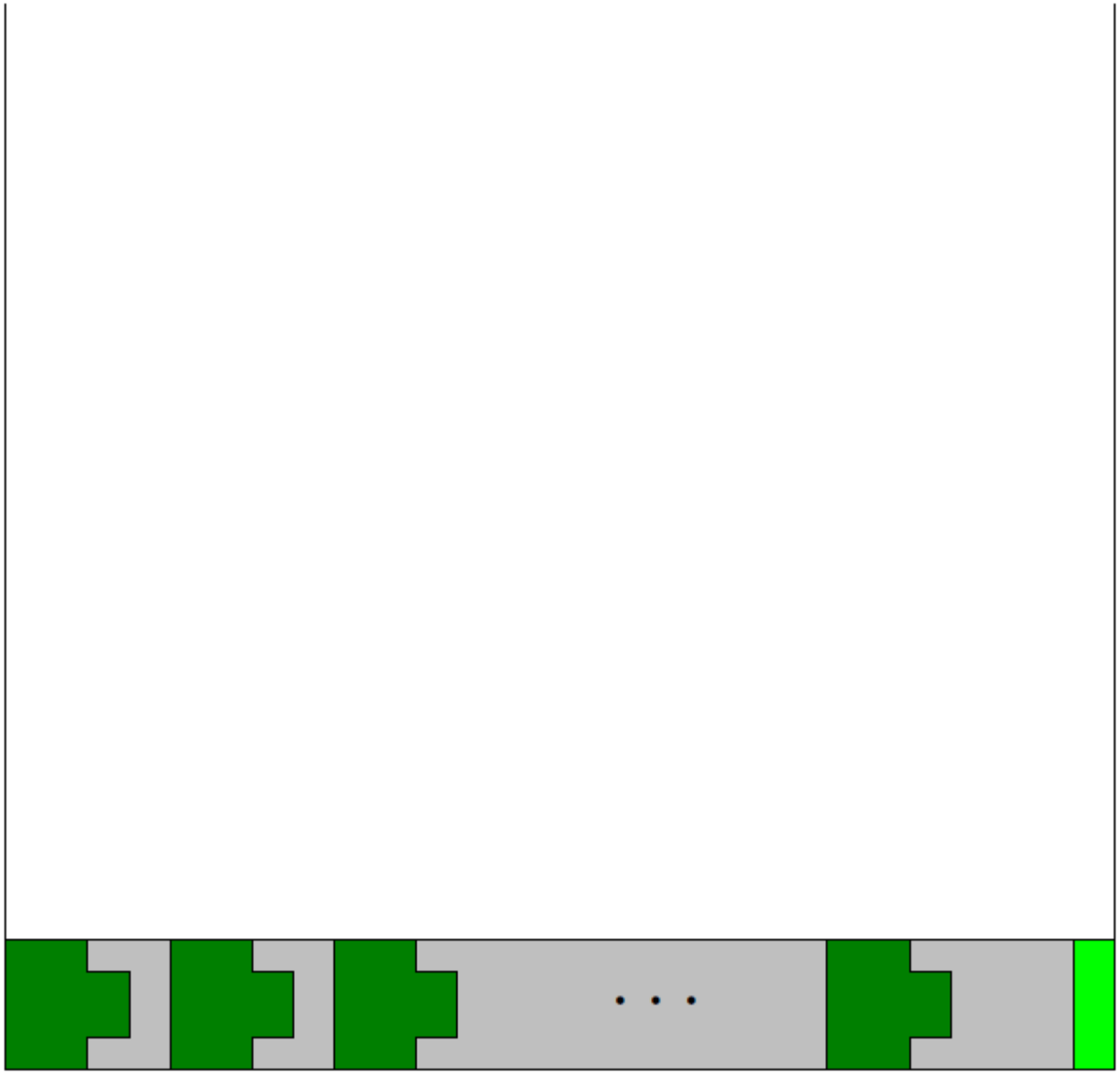
Finale Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]



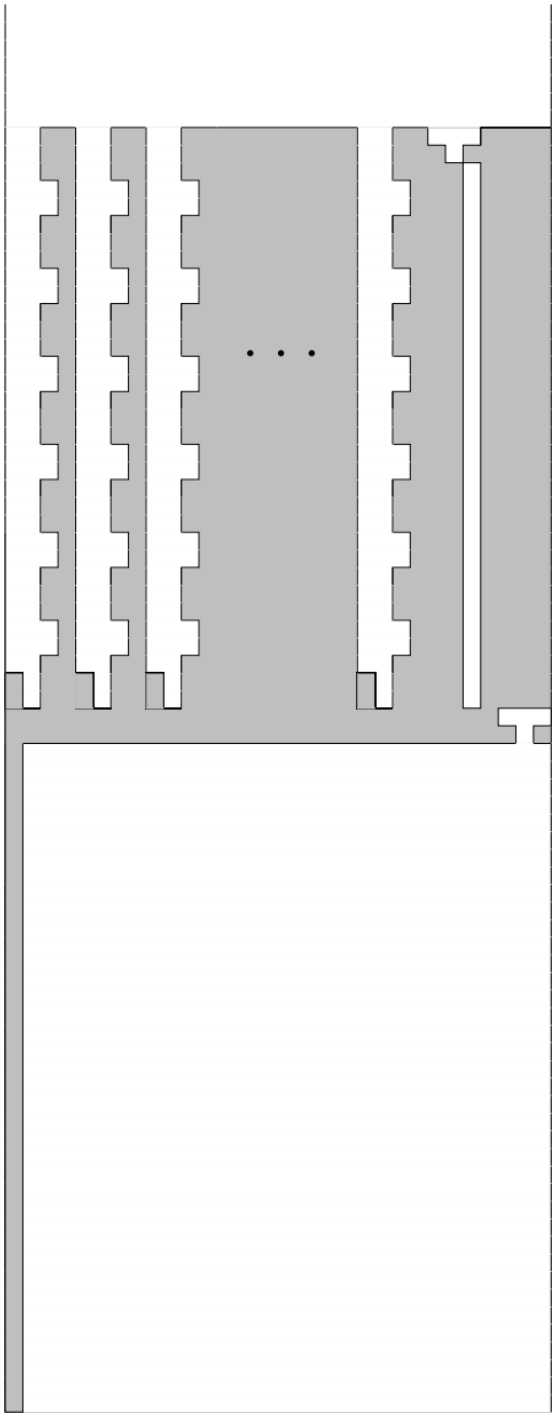
Finale Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]



Hardness of Approximation

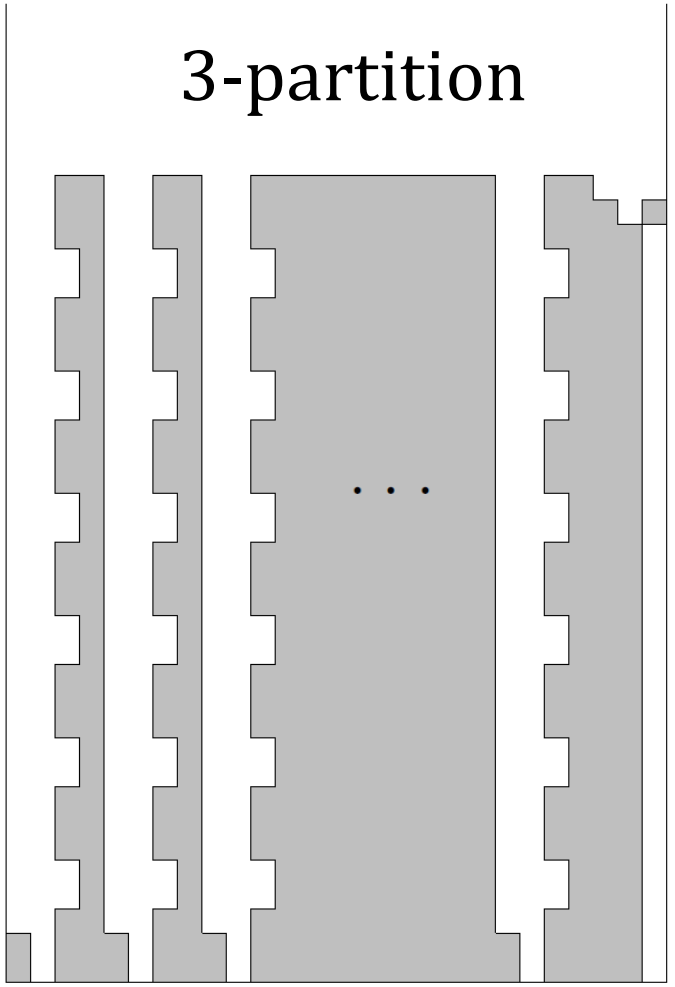
[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]



Tetris Open Problems

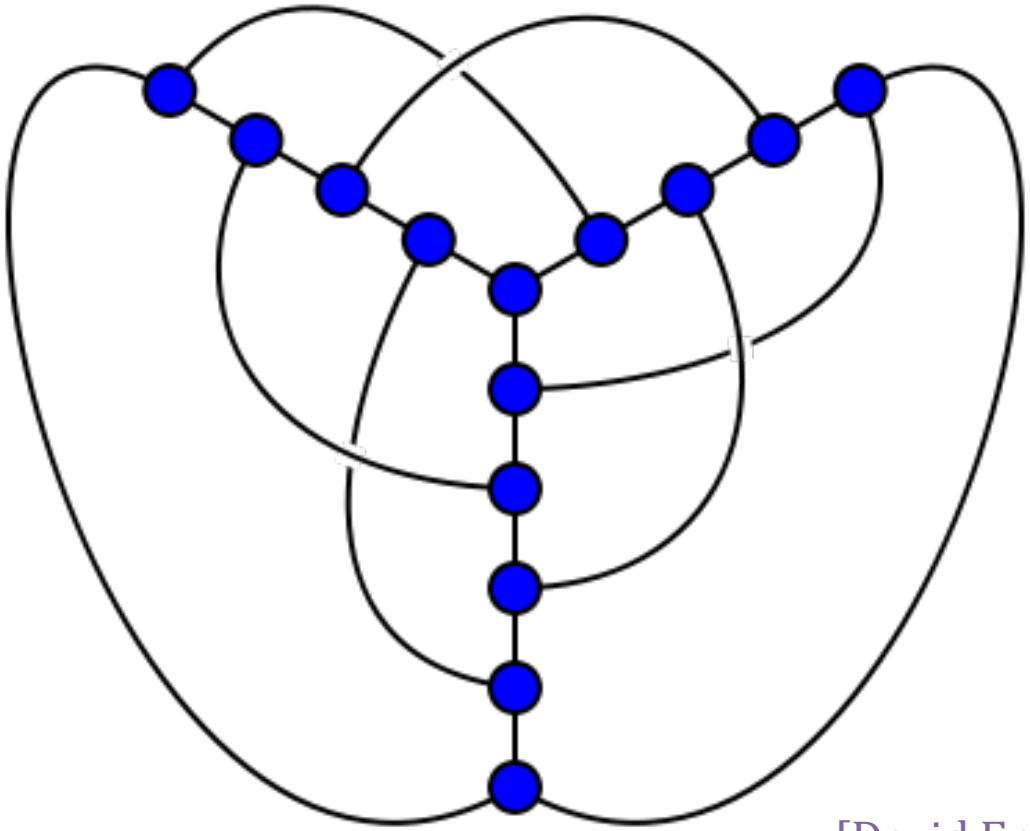
[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]

- Complexity of Tetris with
  - Initially empty board?
  - $O(1)$ columns?
  - $O(1)$ rows?
  - Restricted piece sets (e.g. )?
  - No last-minute slides?
- Is two-player Tetris PSPACE-complete?
- What can we say about online (regular) Tetris?



1-planar Graph [Ringel 1985]

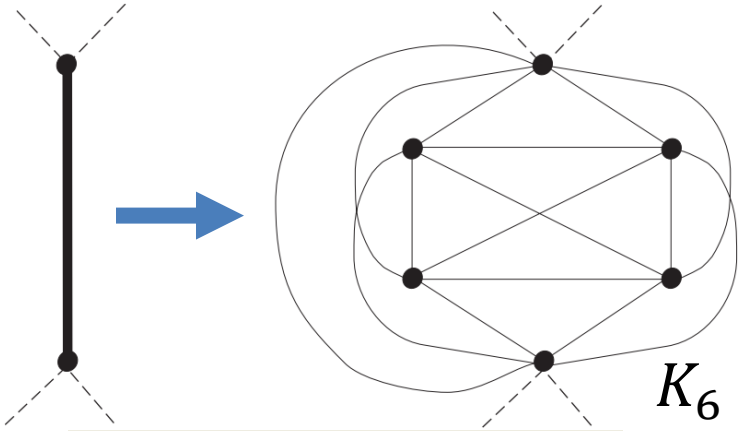
- Each edge has at most one crossing



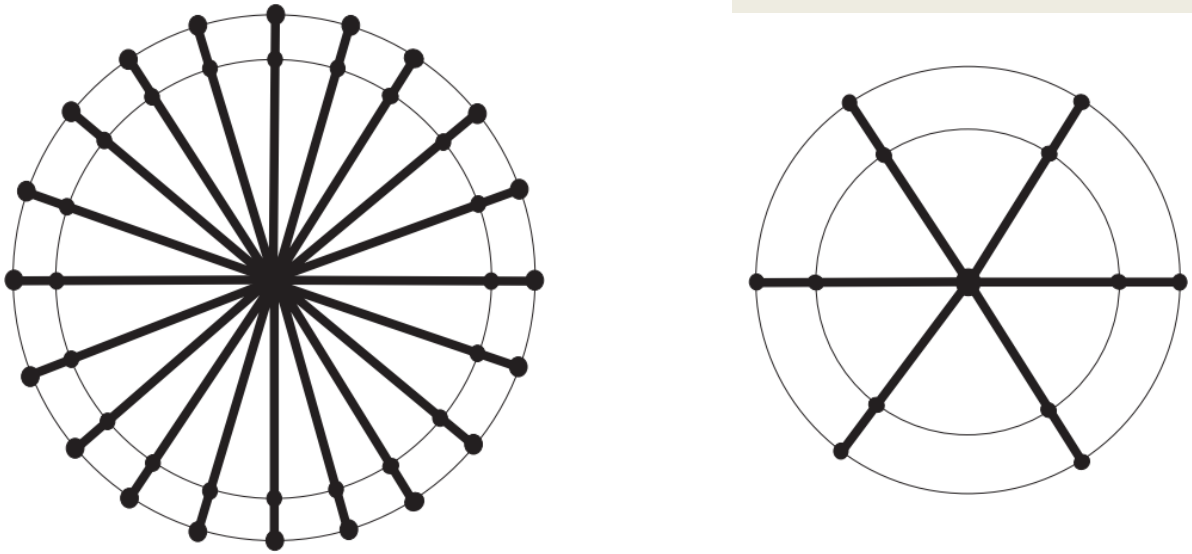
[David Eppstein]

1-planarity is NP-complete

[Grigoriev & Bodlaender 2007]



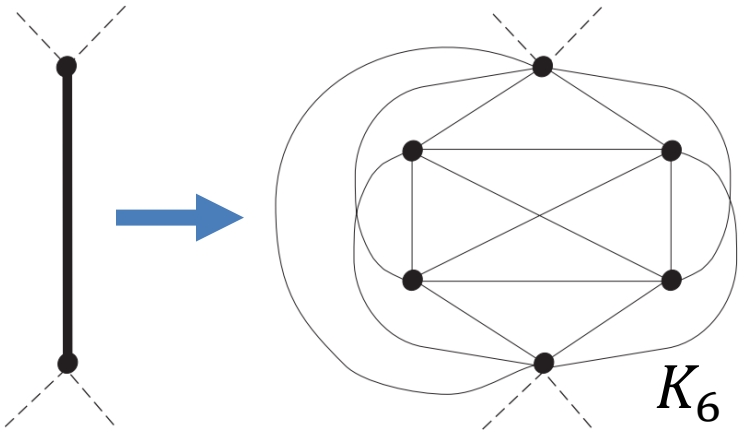
uncrossable edge

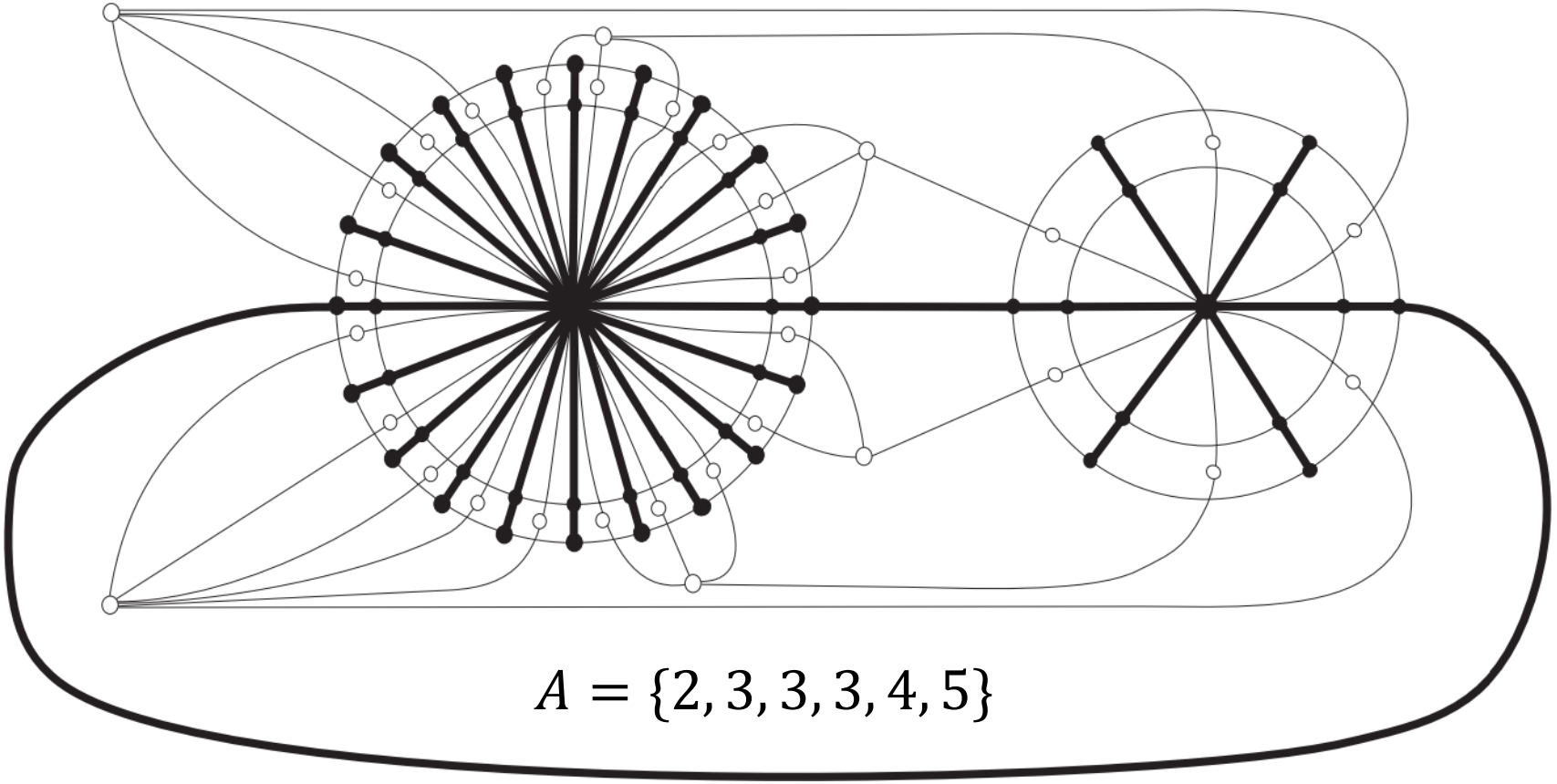


double wheel

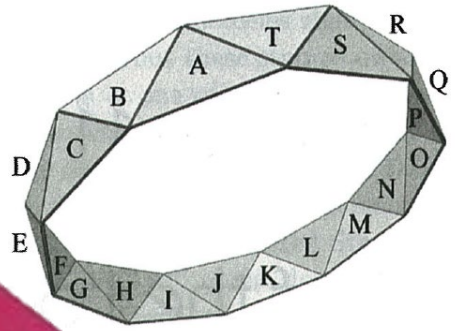
1-planarity is NP-complete

[Grigoriev & Bodlaender 2007]





GeoLoop & Ivan's Hinge



[Kenneth Stevens 1993]

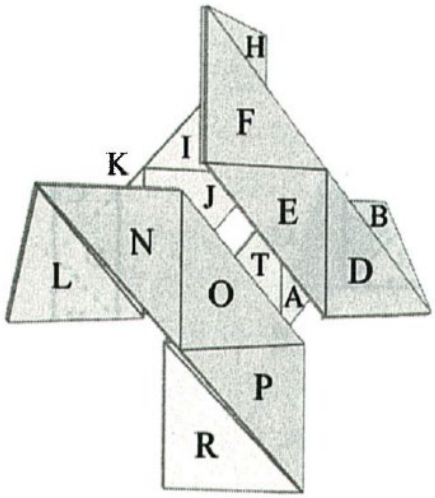
[Jan Essebaggers & Ivan Moscovich 1993]

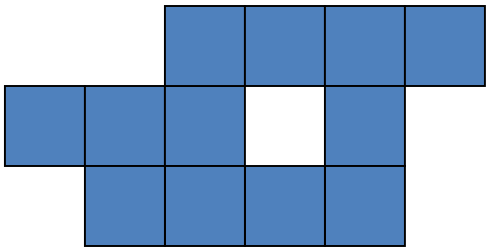
GeoLoop & Ivan's Hinge

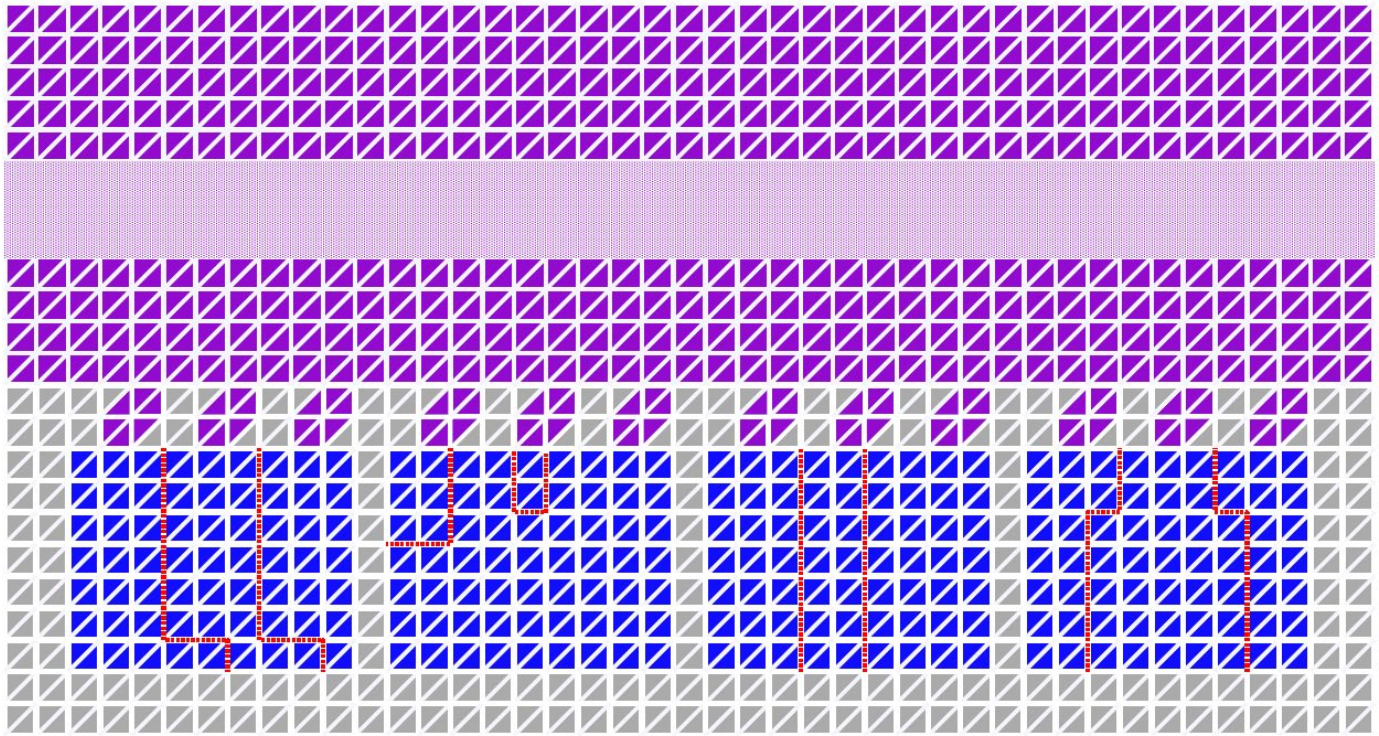
[Abel, Demaine, Demaine, Horiyama, Uehara 2014]

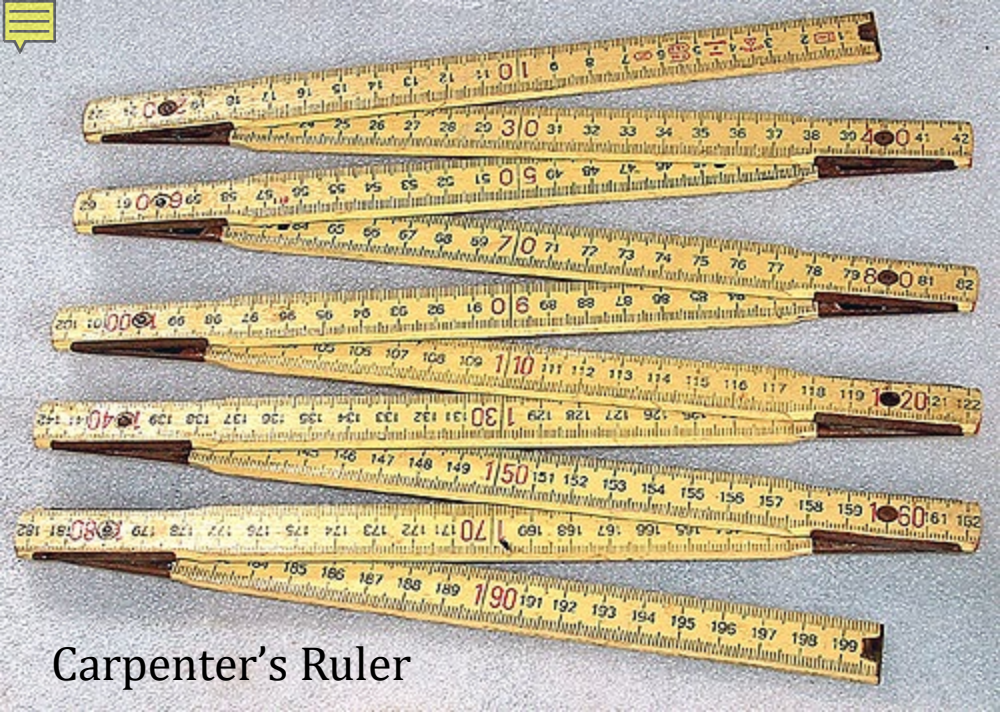
NP-complete

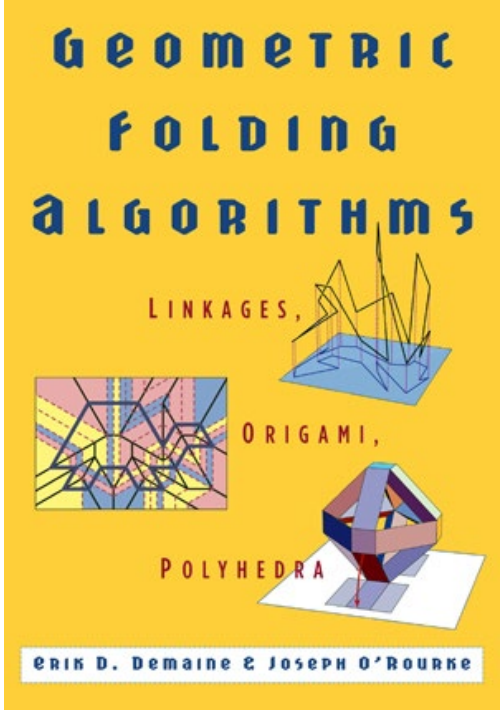
universal & polynomial








Carpenter's Ruler



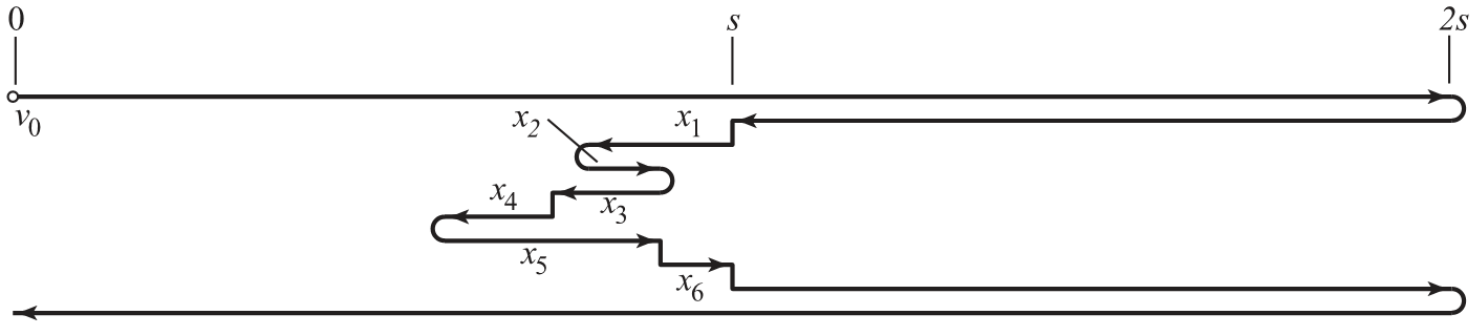


Figure 2.5. Ruler folding reduction. Here $x_1 + x_3 + x_4 = x_2 + x_5 + x_6$.

[Hopcroft, Joseph, Whitesides 1985]

(Simple) Map Folding

[Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000]

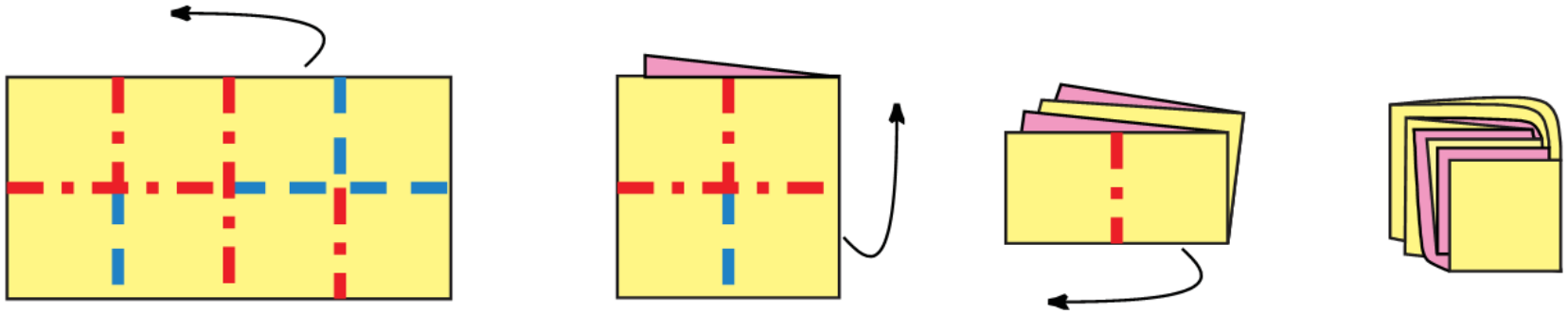
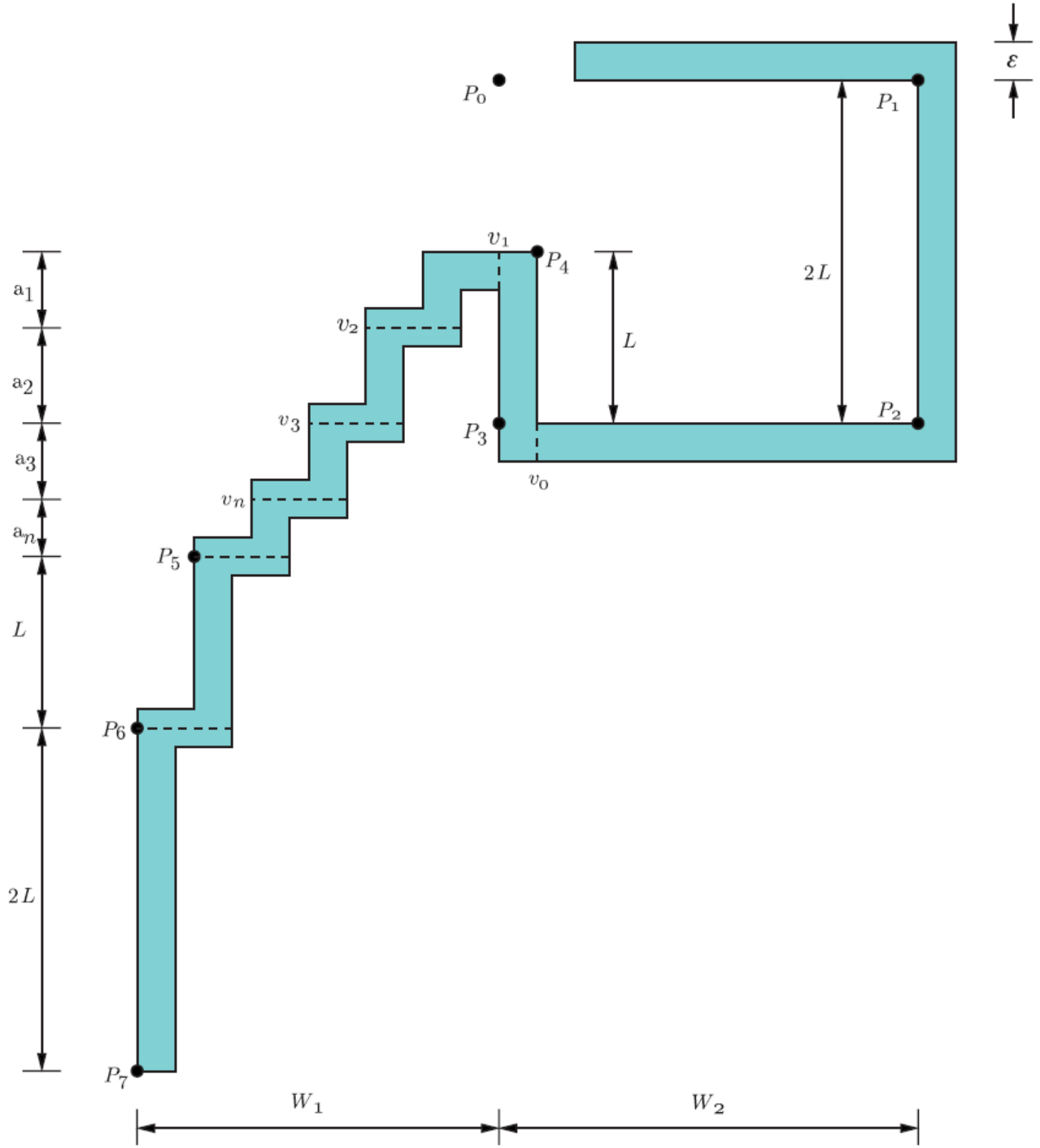
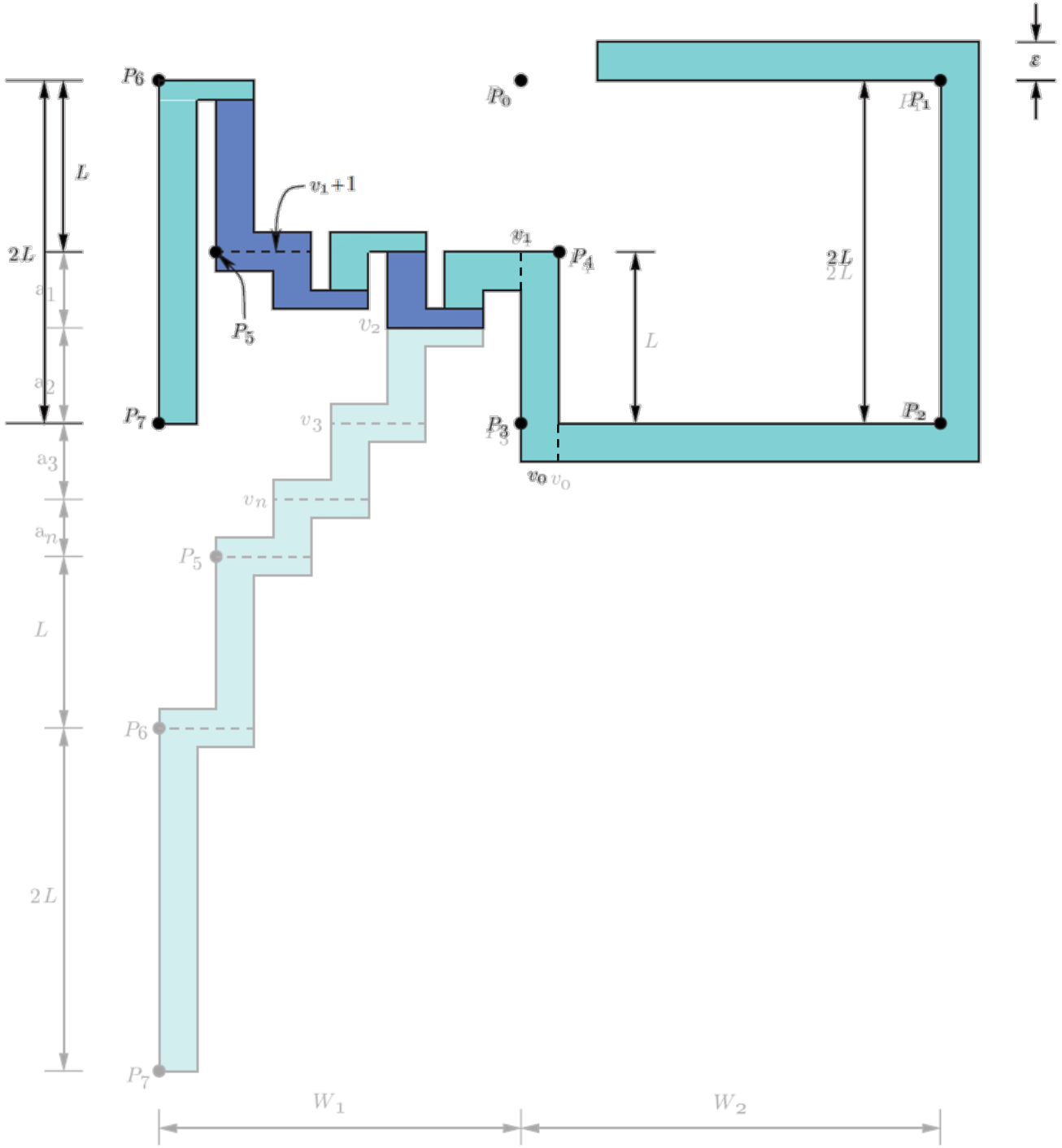


Figure 14.4. Folding a 2×4 map via a sequence of three all-layers simple folds.



[Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000]



[Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000]