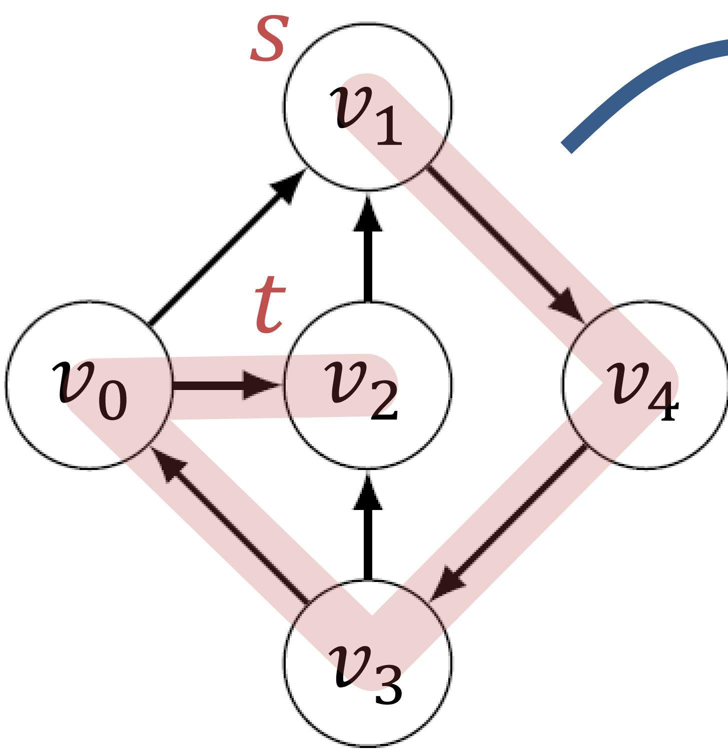
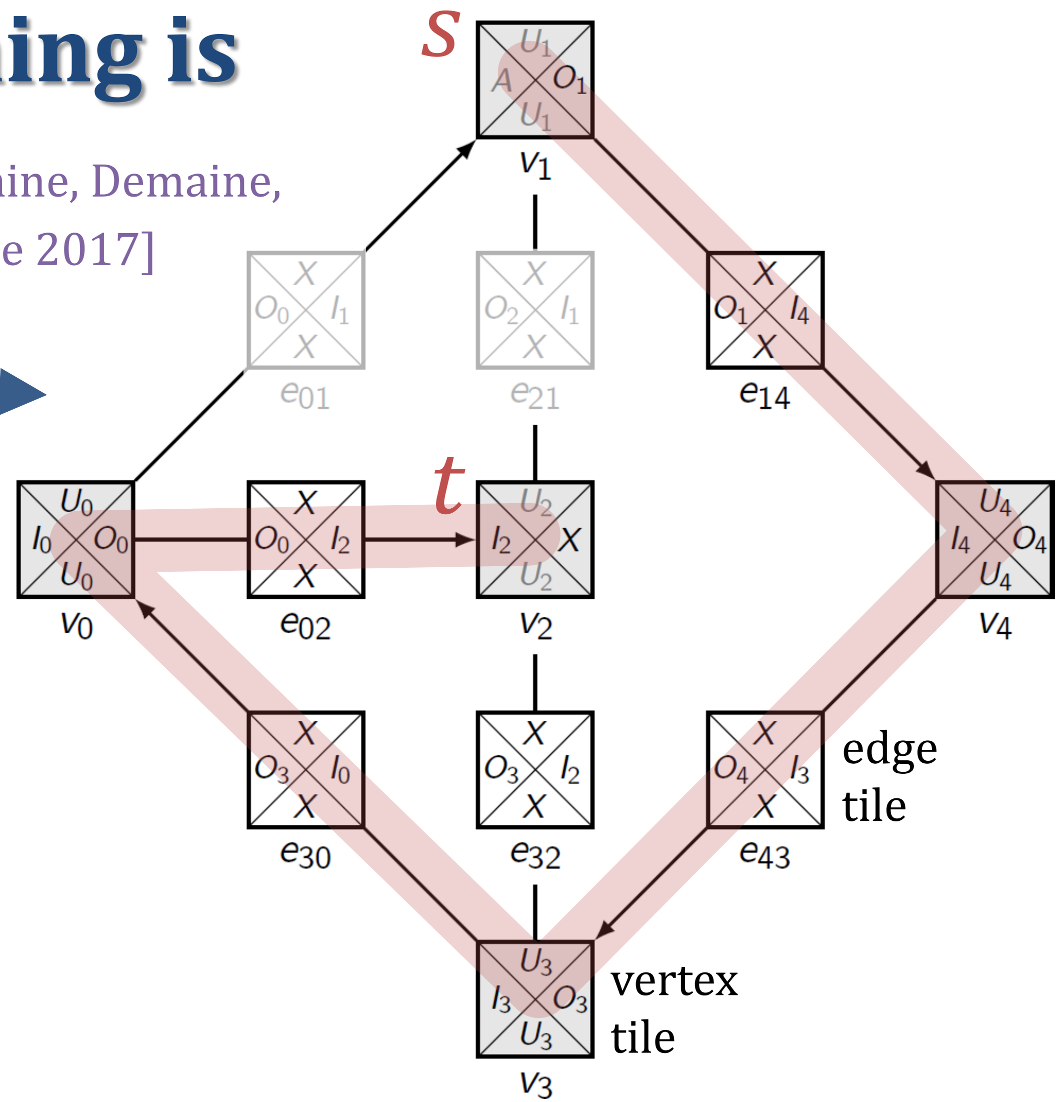
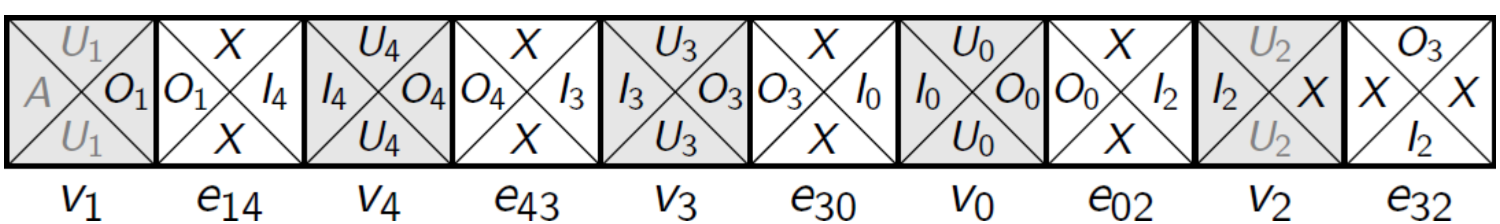
$1 \times n$ Edge Matching is NP-hard [Bosboom, Demaine, Demaine, Hesterberg, Manurangsi, Yodpinyanee 2017]



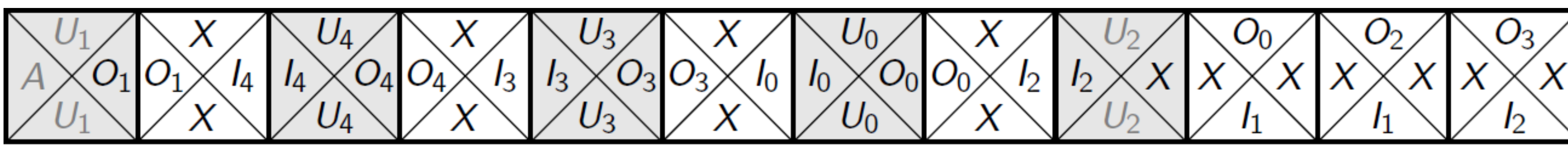




$1 \times n$ Edge Matching is Inapproximable

[Bosboom, Demaine, Demaine, Hesterberg, Manurangsi, Yodpinyanee 2017]

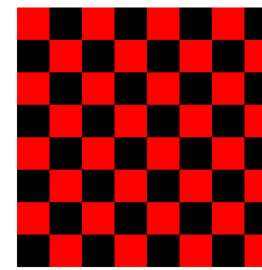
- $1 \times n$ jigsaw & signed/unsigned edge matching puzzles are NP-hard, even to approximate within a factor of 0.9999999762
  - First NP-hardness for $1 \times n$ jigsaw & signed
  - First (correct) inapproximability result



- In fact, show stronger **gap hardness**: NP-hard to distinguish between perfectly solvable vs. $<$ 99.99999762% solvable instances

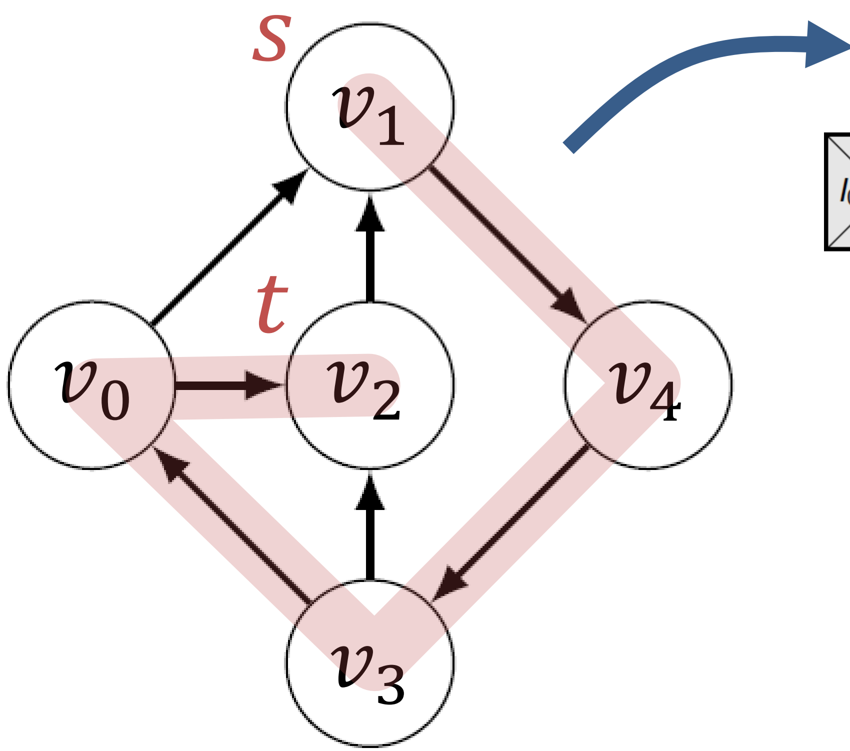
What Does Approximation Mean?

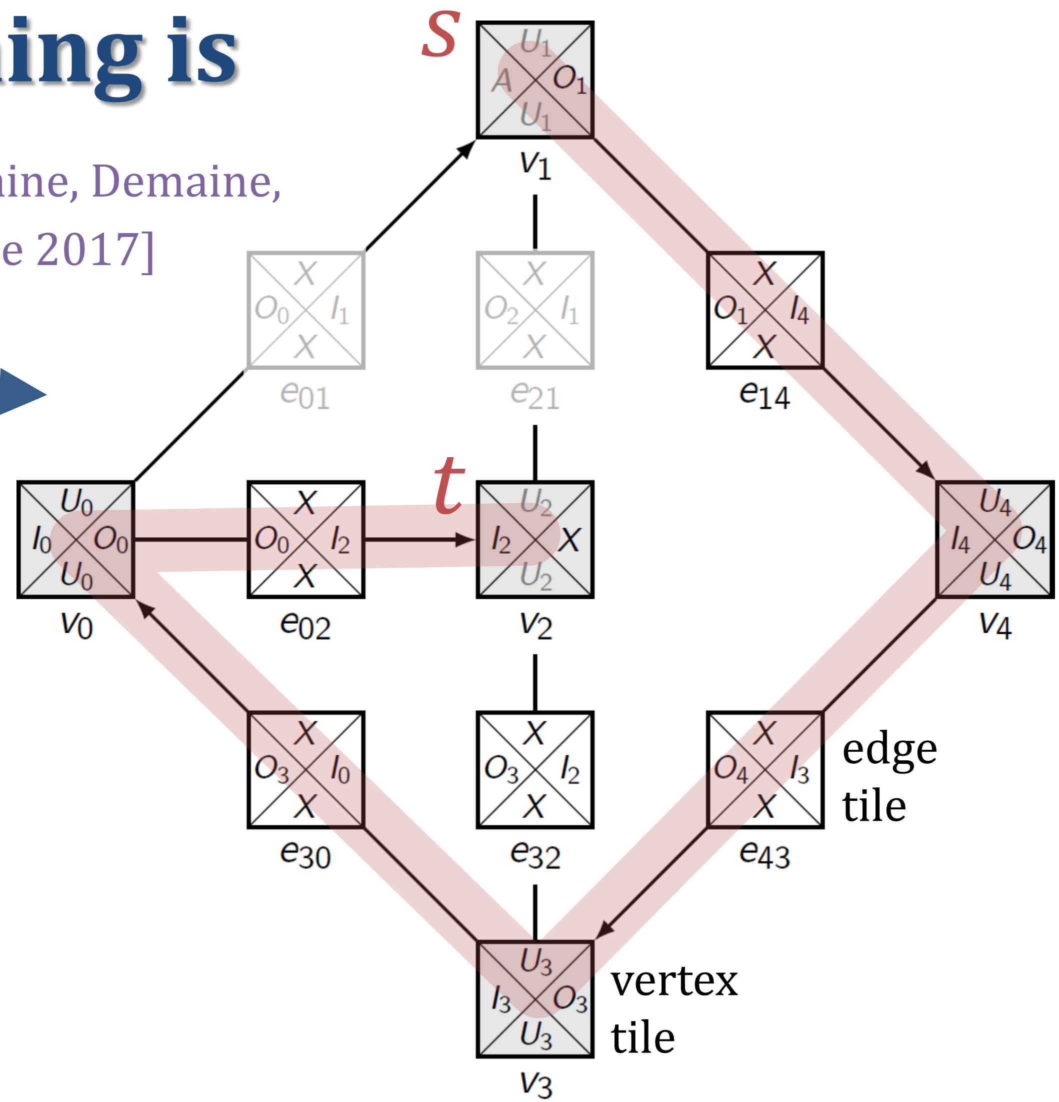
1. Place the maximum number of tiles **without mismatches**
   - Most meaningful for jigsaw puzzles (no overlaps)
   - $\frac{1}{2}$-approximation by checkerboard
   - $\frac{2}{3}$-approximation for $1 \times n$ via matching

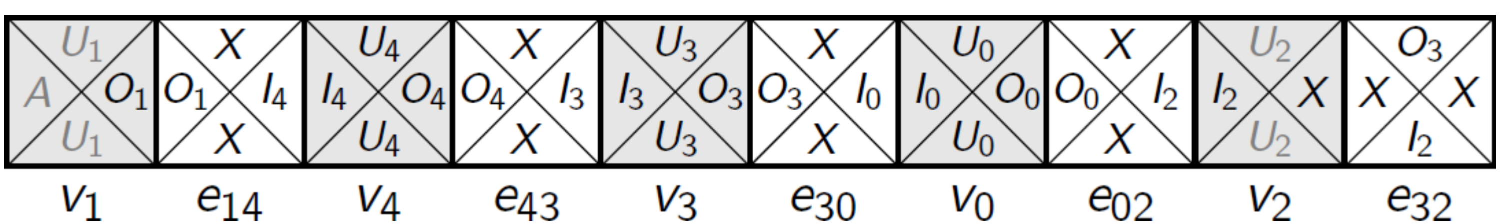


2. Place **all tiles** to maximize the number of edge matches [Antoniadis & Lingas 2010]
   - Dual to minimizing number of mismatches, which is NP-hard to distinguish between 0 and 1
   - $\frac{1}{2}$-approximation for $1 \times n$ via matching

$1 \times n$ Edge Matching is NP-hard [Bosboom, Demaine, Demaine, Hesterberg, Manurangsi, Yodpinyanee 2017]

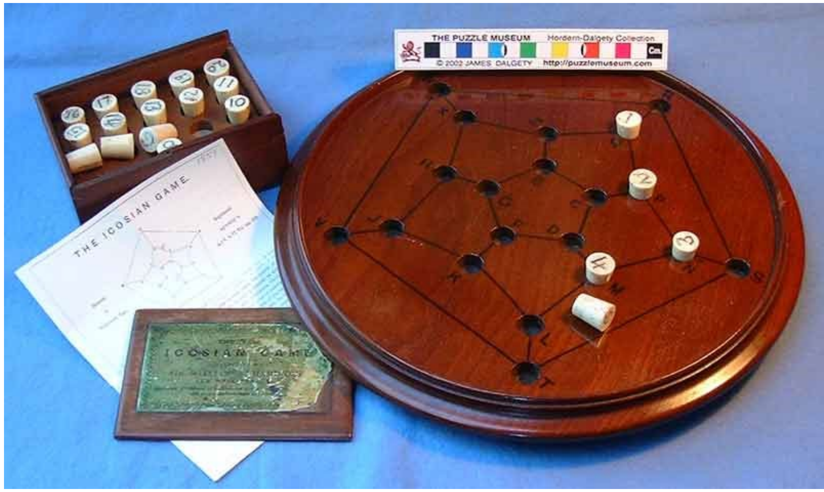






Approximating Hamiltonian Path

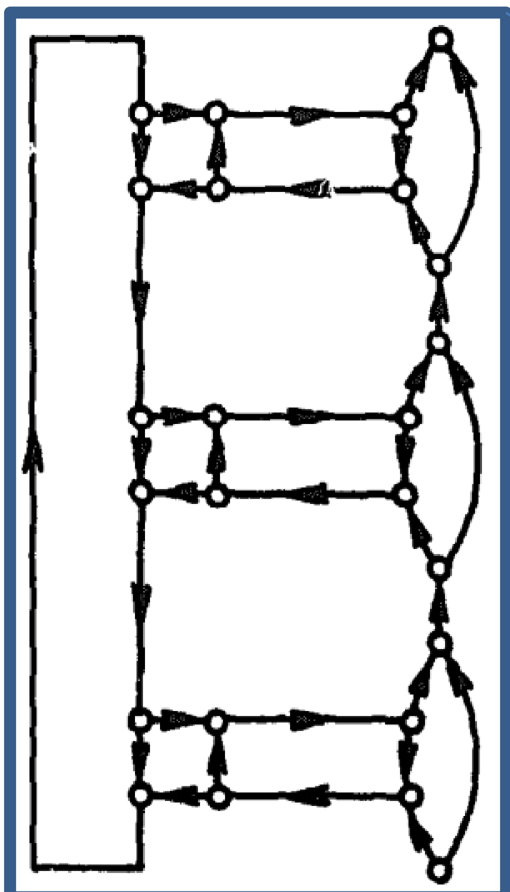
- **Maximum vertex-disjoint path cover:** select maximum number of edges that form vertex-disjoint paths
  - $\text{OPT} = |V| - 1 \Leftrightarrow$ Hamiltonian
  - $\frac{12}{17}$-approximation [Vishwanathan 1992]
  - NP-hard to $(1 - \varepsilon)$-approximate [Engebretsen 2003]
  - **Gap hardness:** NP-hard to distinguish between Hamiltonian and $\text{OPT} < 0.999999284\,|V|$ [here]



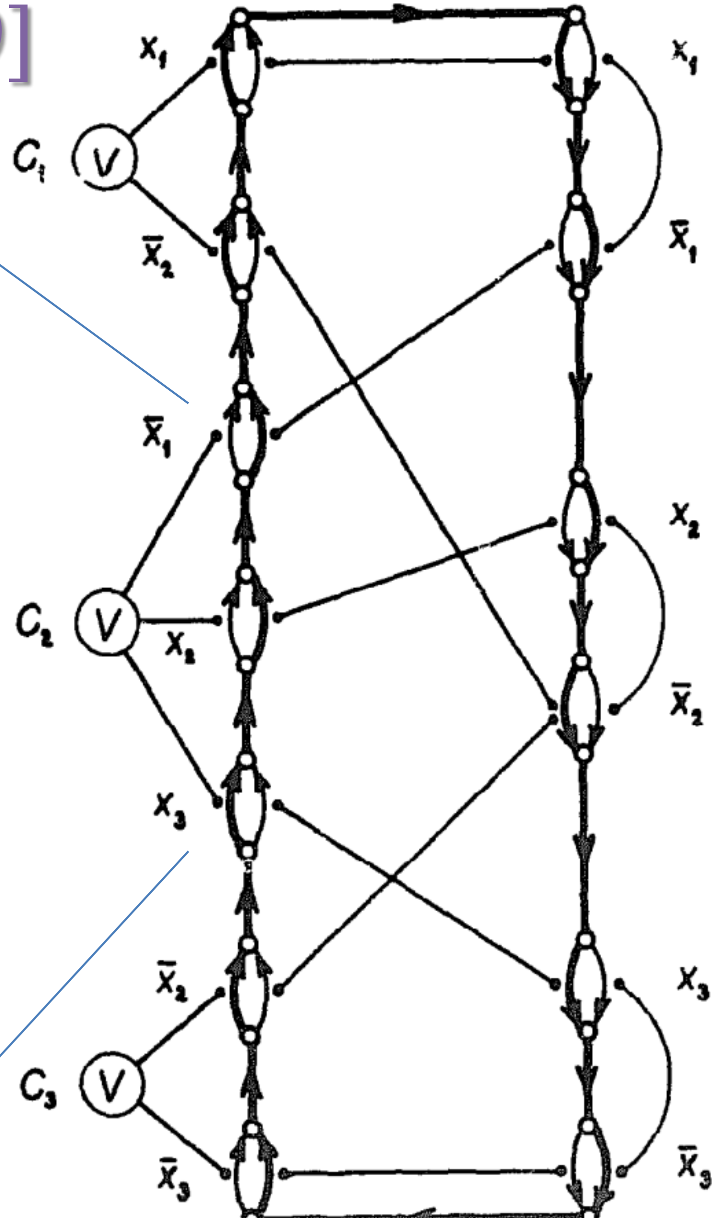
[Hamilton 1857]

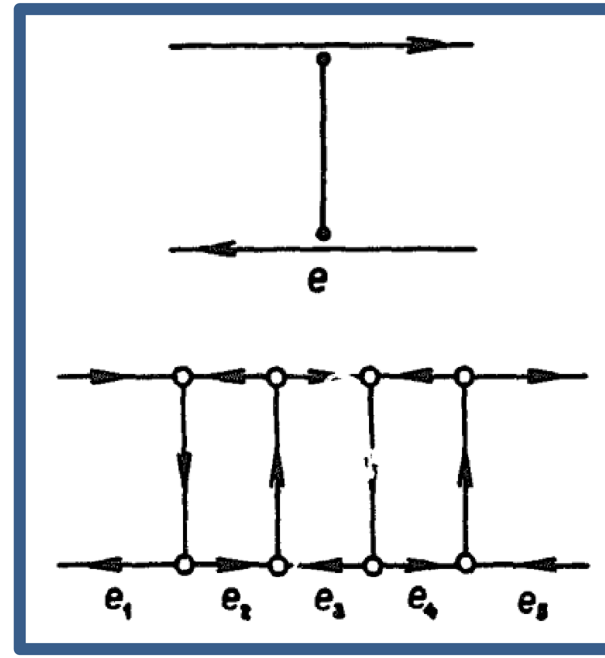
Planar Directed Max-Degree-3 Ham.

[Plesńik 1979]



clause gadget



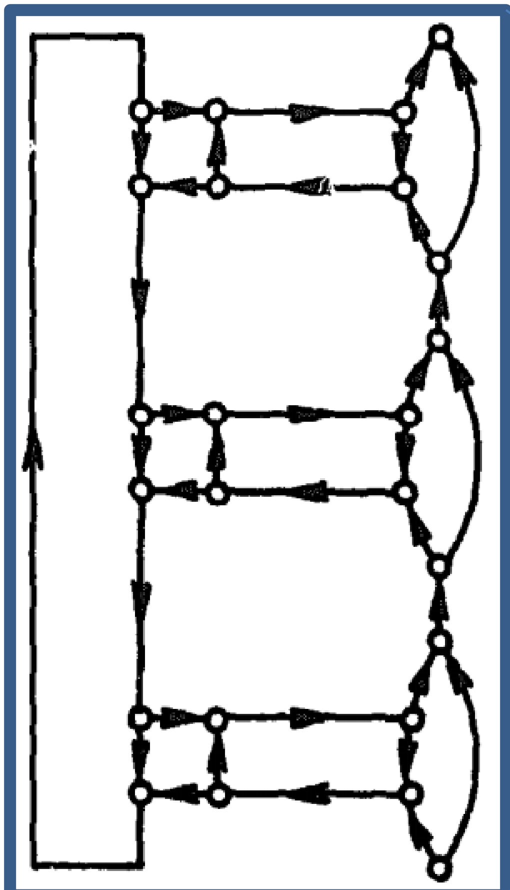


XOR gadget

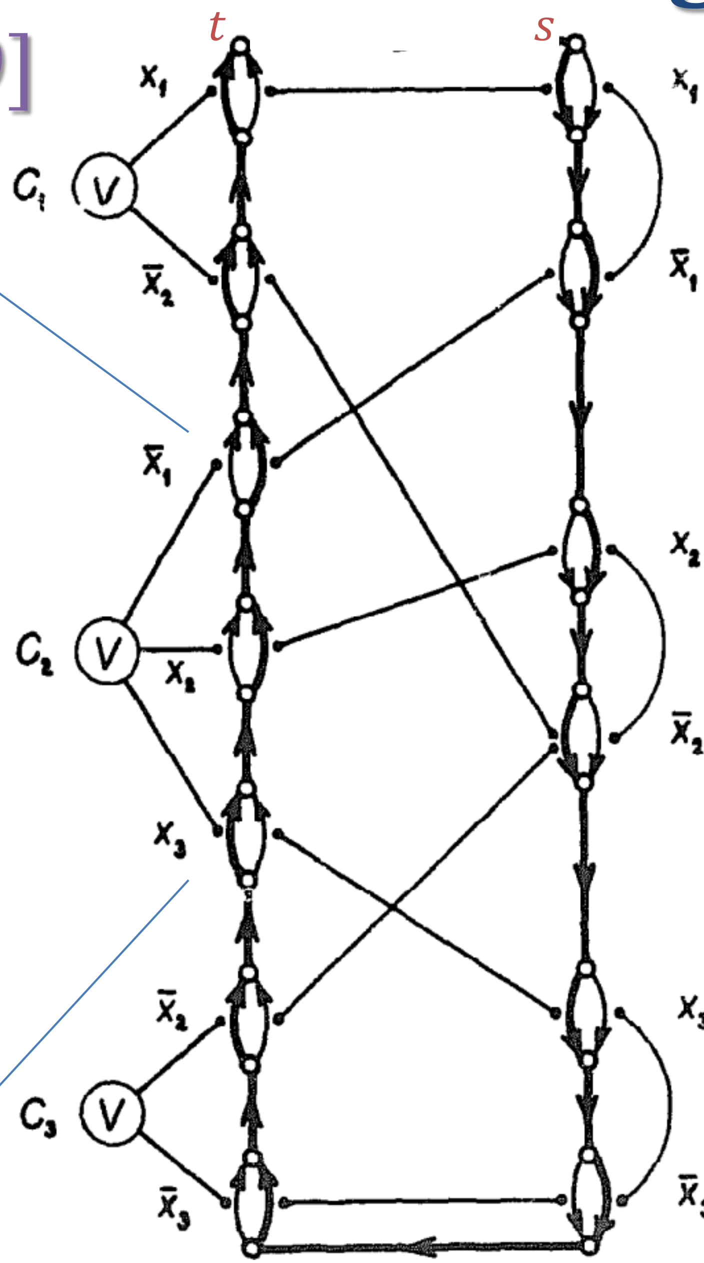
$$(x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3})$$

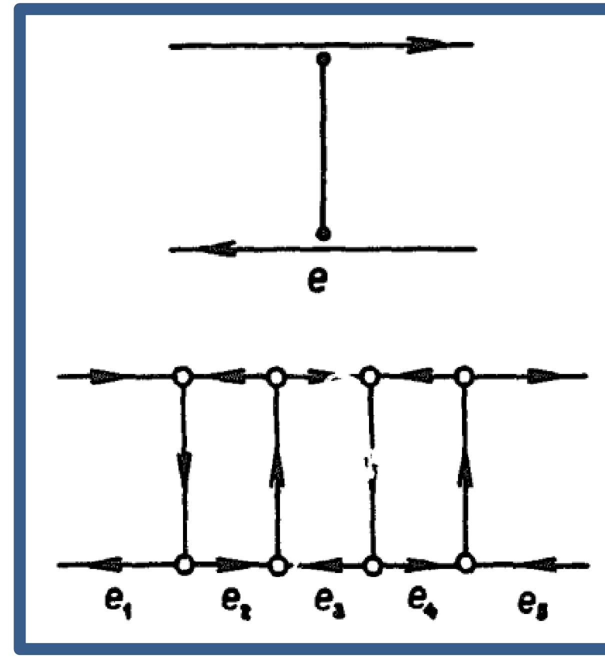
Planar Directed Max-Degree-3 Ham.

[Plesńik 1979]



clause gadget



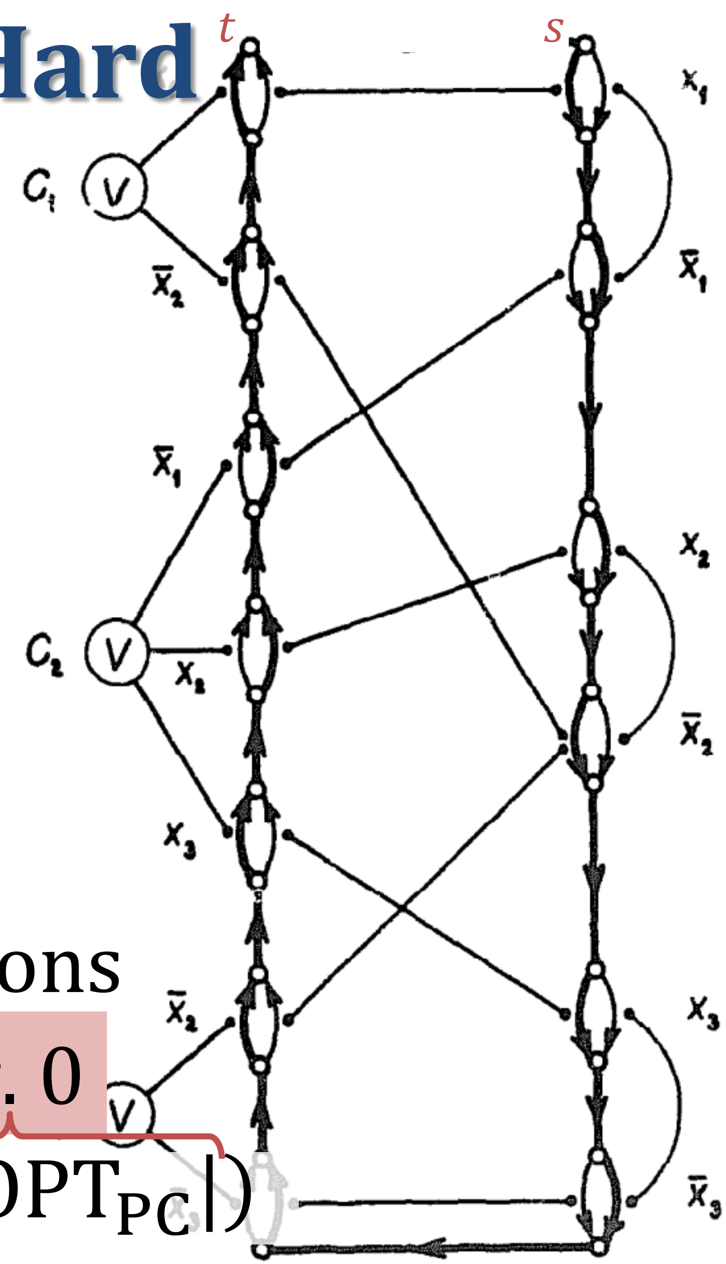


XOR gadget

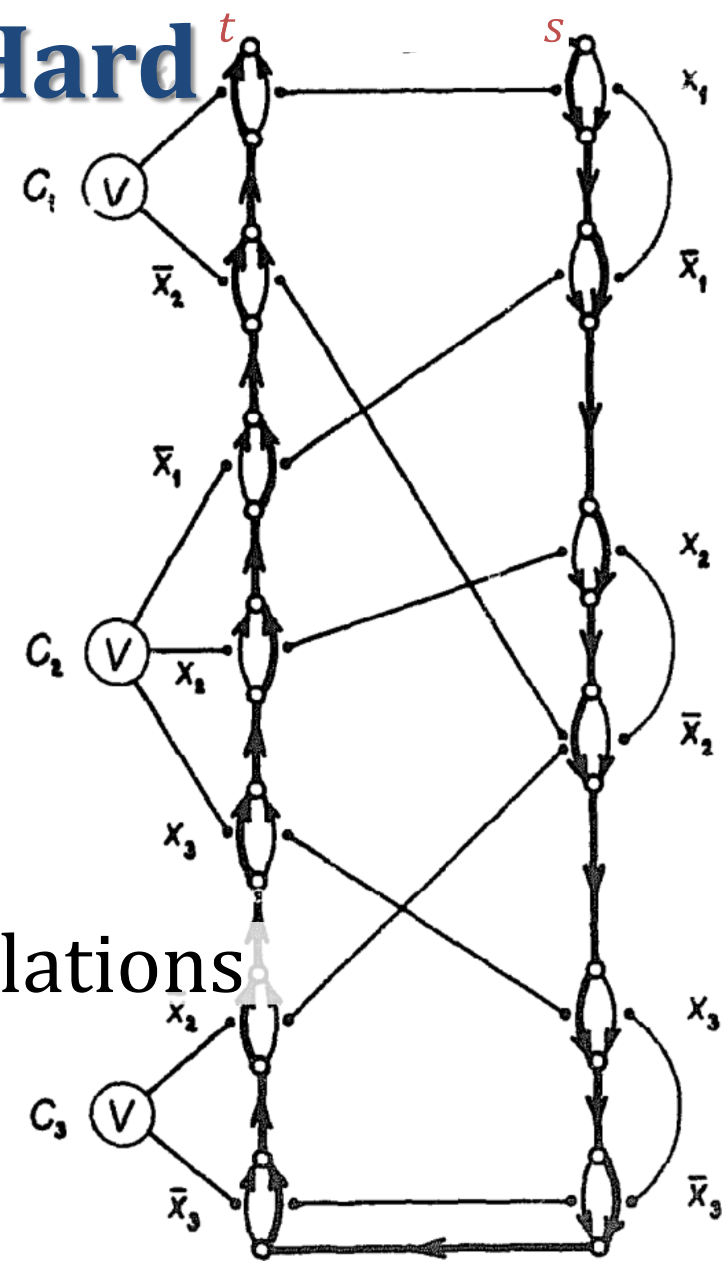
$(x_1 \lor \overline{x_2})$
$\land (\overline{x_1} \lor x_2 \lor x_3)$
$\land (\overline{x_2} \lor \overline{x_3})$

Directed Max-Degree-3 Max Vertex-Disjoint Path Cover is Hard

- Reduce from Max 3SAT-29
  - Each variable in $\leq$ 29 clauses
- Look at which gadgets have path endpoints (2 per path)
  - Charge to corresponding clause or variable, then $\leq$ 29 corresponding clauses
- k paths $\Rightarrow O(k)$ 3SAT violations
- L-reduction condition:
  $|O(k) - \text{OPT}_{\text{SAT}}| \not\leq O(\underbrace{|k - \text{OPT}_{\text{PC}}|}_{\text{e.g. } 0})$



Directed Max-Degree-3 Max Vertex-Disjoint Path Cover is Hard



- Reduce from Max 3SAT-29
  - Each variable in ≤ 29 clauses
- Look at which gadgets have path endpoints (2 per path)
  - Charge to corresponding clause or variable, then ≤ 29 corresponding clauses
- $\varepsilon\, n$ paths $\Rightarrow O(\varepsilon\, n)$ 3SAT violations
- 3SAT $< 1 - \varepsilon$ satisfiable $\Rightarrow \Omega(\varepsilon\, |V|)$ endpoints

Power of Gap Reductions

- These **gap preservation** arguments are easy
- Key: We always compare to **ideal solution** (Hamiltonian, fully satisfiable, no blanks, etc.)
- Contrast with L reductions, which compare $|\mathrm{APX} - \mathrm{OPT}|$, or PTAS reductions $\left(\frac{\mathrm{APX}}{\mathrm{OPT}}\right)$
  - Difficult/impossible in these examples
- Proves stronger gap hardness result (comparing to ideal) which implies inapproximability (comparing to OPT)
- Only downside is not proving APX-hardness