6.892	Class 6	Mar. 13, 2019

Class	Complete SAT Problem	
$\Sigma_1 = NP$	$\exists X_1 : \varphi(X_1)$ ← set of variables	SAT
$\Pi_1 = coNP$	$\forall X_1 : \varphi(X_1)$	UNSAT
Σ_2	$\exists X_1 : \forall X_2 : \varphi(X_1, X_2)$	
$\Pi_2 = co\Sigma_2$	$\forall X_1 : \exists X_2 : \varphi(X_1, X_2)$	PH
Σ_k	$\exists X_1 : \forall X_2 : \cdots X_k : \varphi(X_1, \ldots, X_k)$	
$\Pi_k = co\Sigma_k$	$\forall X_1 : \exists X_2 : \cdots X_k : \varphi(X_1, \ldots, X_k)$	
PSPACE	$\exists X_1 : \forall X_2 : \cdots X_n : \varphi(X_1, \ldots, X_n)$	QSAT
	or $\forall X_1 : \exists X_2 : \cdots X_n : \varphi(X_1, \ldots, X_n)$	

Complements: $\exists \equiv \neg\forall\neg$ & $\forall = \neg\exists\neg$

$\Rightarrow \neg\exists X_1 : \varphi(X_1)$ UNSAT

$\equiv \forall X_1 : \neg\varphi(X_1)$

Σ_2-complete: [Schaefer & Umans – SIGACT News 2002 + www]

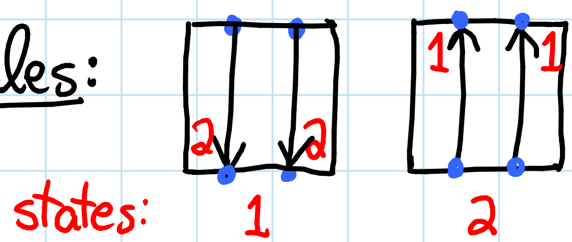
- L1, L2 — $\exists : \forall :$ 3SAT — $\exists : \forall :$ NAE 3SAT
- SP5 — $\exists S_1 \subseteq M_1 : \forall S_2 \subseteq M_2 : S_1 \cup S_2$ is a 3DM
- L13 — is circuit A = circuit B with some inputs fixed?
- GT19 — $\exists$ k-coloring: no maximal clique is monochromatic
- L21 — $\exists : \exists! :$ planar 3SAT or planar 1-in-3SAT
- — $\exists$ k "clues" to force unique answer to:
  - planar 3SAT — planar 1-in-3SAT
  - Akari — Shakashaka — Sudoku

[Demaine, Ma, Schvartzman, Waingarten, Aaronson – TCS 2018]

Gadget general model: (for robot motion planning)
- locations (entrances/exits)
- states
- transitions $(\ell_1, s_1) \to (\ell_2, s_2)$ — transition graph; s_1: $\ell_1 \xrightarrow{s_2} \ell_2$ — visual notation

[Demaine, Grosof, Lynch, Rudoy – FUN 2018]
[Demaine, Hendrickson, Lynch – arXiv 2018]

Examples:



2-toggle
[Demaine, Grosof, Lynch – CIAC 2017]

Gadget types:
- k-tunnel = all transitions are along edges of perfect matching on locations (states can control traversability & directions)
- deterministic = transition determines state change
- reversible = every move can be then undone

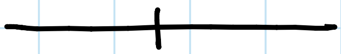
Characterization: motion planning with deterministic reversible k-tunnel gadgets is PSPACE-complete iff:
- some gadget has interacting tunnels: traversal of some tunnel affects traversability of another tunnel
- even with planar connections of gadgets

[Demaine, Hendrickson, Lynch – arXiv 2018]

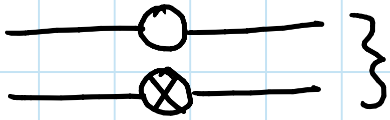
2-state characterization:

[Demaine, Grosof, Lynch, Rudoy – FUN 2018]

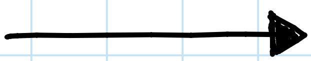
Tunnel types:



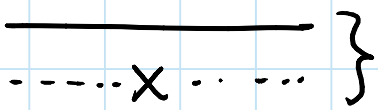
tripwire: always bitraversable & traversal flips state



lock: bitraversable in one state not in other



toggle: traversal in one direction & toggles state + direction



trivial: always/never bitraversable

Hard 2-tunnel gadgets: