Exam 1
Assigned: 03/16/2004
Due: 03/18/2004 (Midnight)

Problem 1 *Normalized correlation and SSD*

(a) Given a square image patch in one image, if we search for a corresponding patch in a second image using correlation or least squares differences integrated over the path will we, in general, get the same answer? Either provide an informal proof or counterexample.

Hint: Here correlation is defined by a simple dot product between window vectors and SSD is defined by the squared L2 norm between vectors.

(b) What if all patches are normalized to zero mean and unit norm? Provide an informal proof or counterexample.

(c) Normalization is often useful but can have undesired behavior, especially when there is structured noise in the image. Describe an example where normalized correlation will match image patches that a human observer would never perceive as matching.

Problem 2 *Pinhole camera*

When does a smaller pinhole lead to a sharper image?

Problem 3 *Affine spaces and barycentric combinations*

Solve exercise 12.2 in Forsythe and Ponce.

Problem 4 *Color space conversion*

Color conversion is often performed using an equation of the form $y = Mx$ where M is a non-singular matrix. Given a tuple of the form $x = (\mathrm{R,G,B})^T$ is it possible to do so when converting to (a) CIE XYZ? (b) CIE xy? (c) CMY? (d) HSV?

Problem 5 *Rotation matrices*

Show that rotation matrices are characterized by the following properties: (a) the inverse of a rotation matrix is its transpose and (b) its determinant is 1.

Problem 6 *Orthographic, weak, and paraperspective projection*

With the camera center at the origin, the image plane at ($z = -1$) and a scene reference point at $(0, 1, 10)$ describe where scene points at $(0, 2, 10)$, $(3, 3, 10)$, and $(0, 1, 11)$ will project under (a) orthographic, (b) weak perspective, and (c) paraperspective projection.

Problem 7 *Structure from motion*

The rank constraint for multiple affine motions described in the text was rank=$4k$, where k is the number of independently moving objects. But the rank constraint for Tomasi and Kanade's factorization algorithm is rank=3. Explain this inconsistency.

Problem 8 *Image pyramids*

Which of these pyramid representations are overcomplete: Laplacian, QMF, Steerable filters? Why might a complete representation cause difficulty for early vision tasks such as orientation representation? Please explain in a few sentences.

Problem 9 *Small object motions*

Solve exercise 10.3 in Forsythe and Ponce.

Problem 10 *SIFT*

Are SIFT descriptors strictly invariant to: translation? rotation? affine transformation? perspective transformation? Please explain your answers.

Problem 11 *Aliasing*

Aliasing takes high spatial or temporal frequencies to low frequencies. Explain why the following effects occur: (a) white shirts with thin dark pinstripes often generate a shimmering array of colors on television, and (b) in old cowboy films that show wagons moving, the wheel often seems to be stationary or rotating in the wrong direction (i.e., the wagon moves from left to right and the wheel seems to be turning counterclockwise). Please provide a few sentence explanation for each question.

Problem 12 *Perspective projection of points and lines*

(a) In perspective projection, points project to points. What do planes project to?

(b) Describe a physical configuration of a camera and world object that leads to degenerate perspective projection of a line. Describe another configuration that leads to degenerate perspective projection of a plane.

Problem 13 *Cross ratio property of projective spaces*

Solve exercise 13.6 in Forsythe and Ponce.