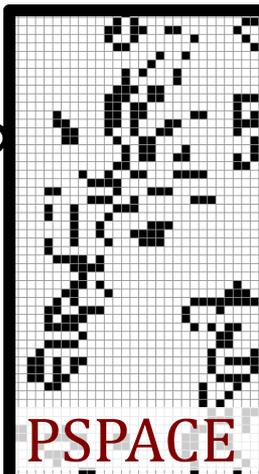


Complexity of Games & Puzzles

unbounded



PSPACE



PSPACE



EXPTIME



Rengo Kriegspiel?

Undecidable

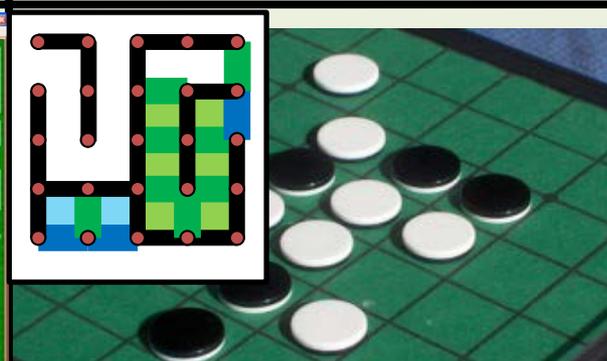
bounded



P



NP



PSPACE



bridge?

NEXPTIME

0 players
(simulation)

1 player
(puzzle)

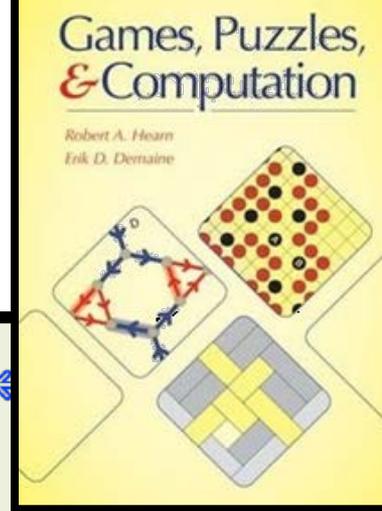
2 players
(game)

team,
imperfect info

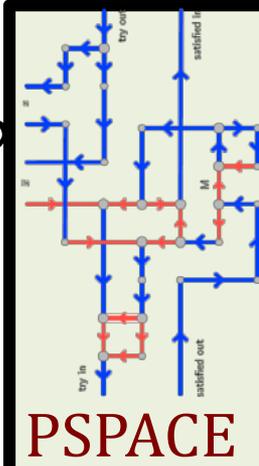


Constraint Logic

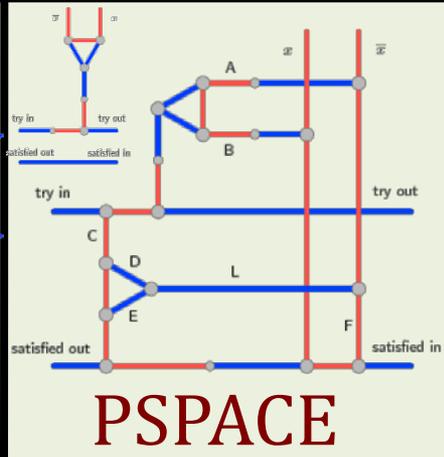
[Hearn & Demaine 2009]



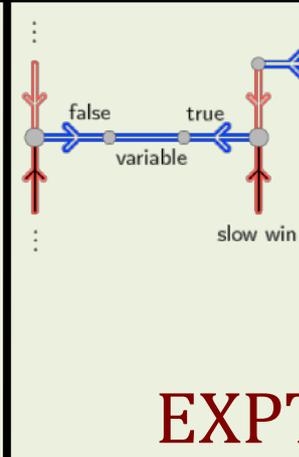
unbounded



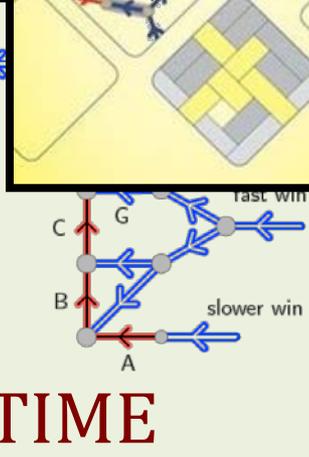
PSPACE



PSPACE

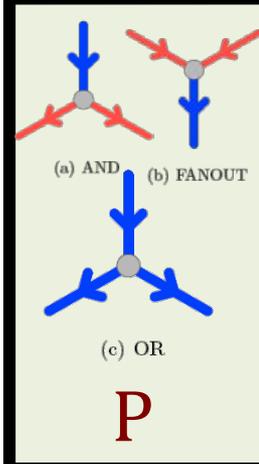


EXPTIME

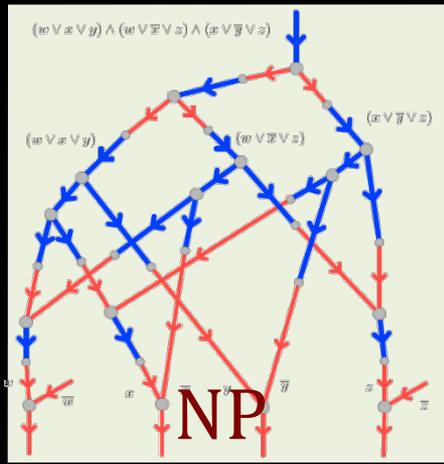


Undecidable

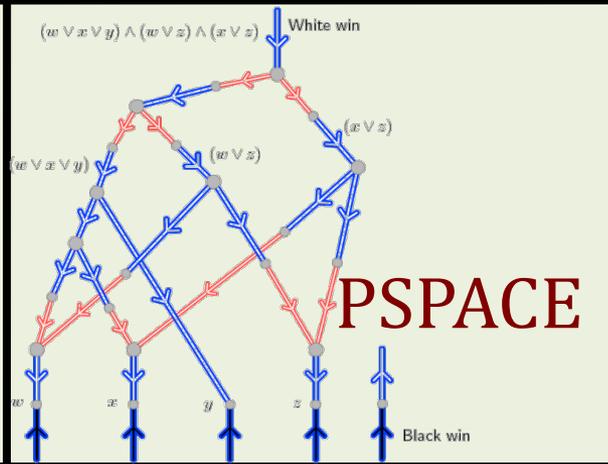
bounded



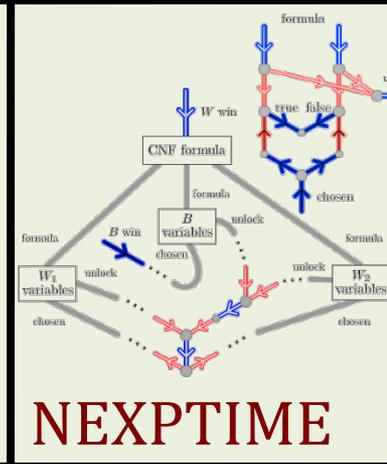
P



NP



PSPACE



NEXPTIME

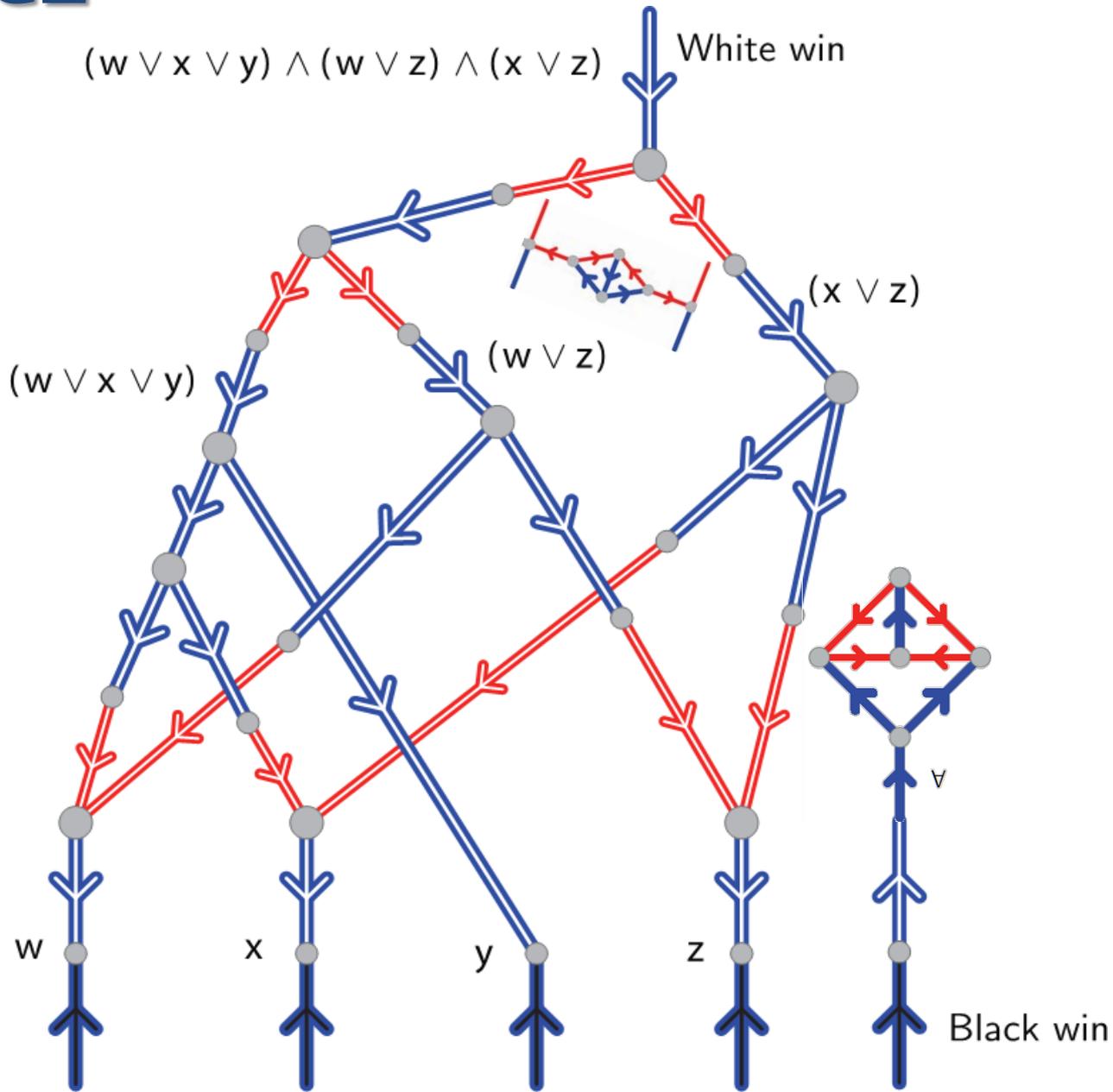
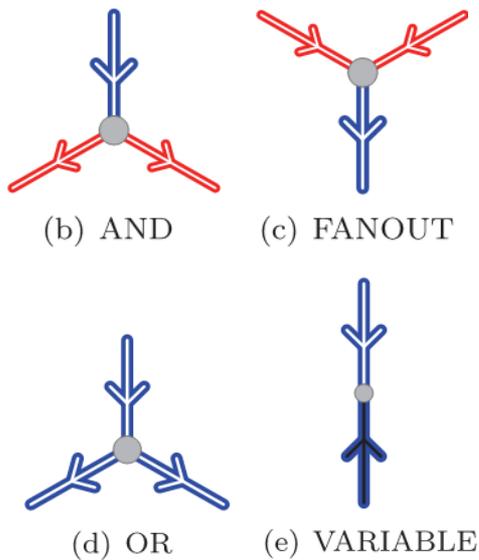
0 players
(simulation)

1 player
(puzzle)

2 players
(game)

team,
imperfect info

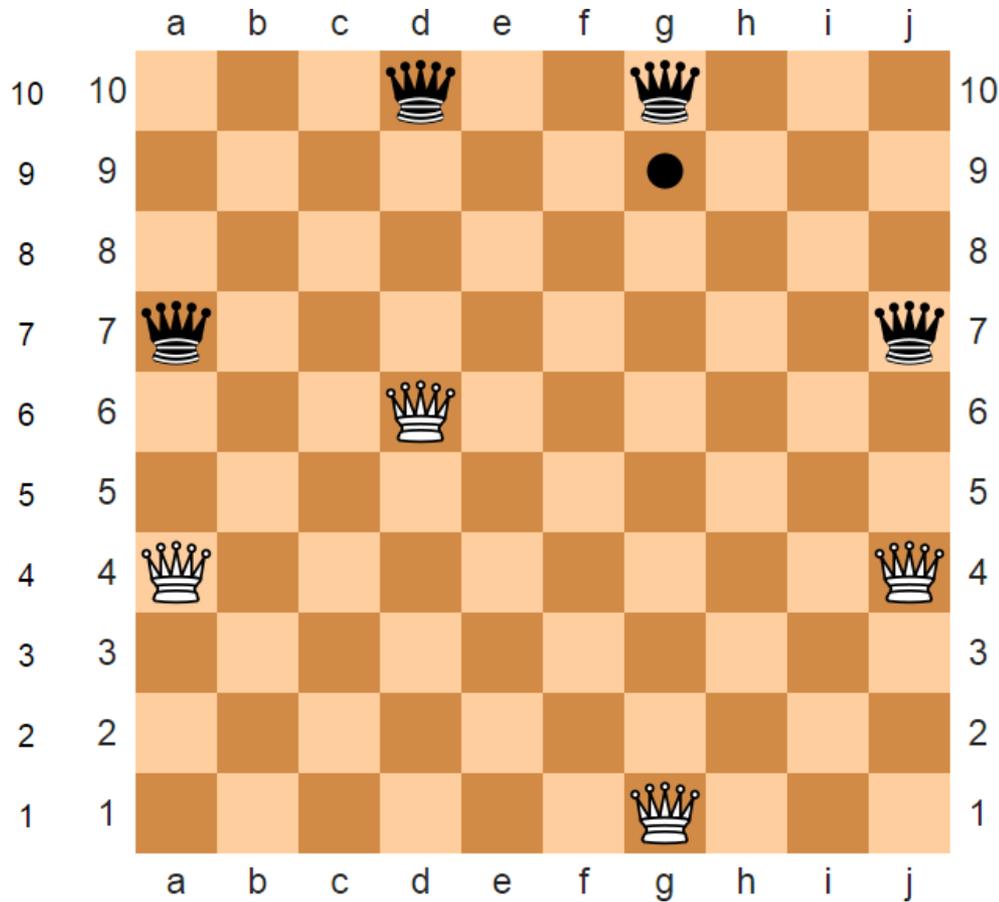
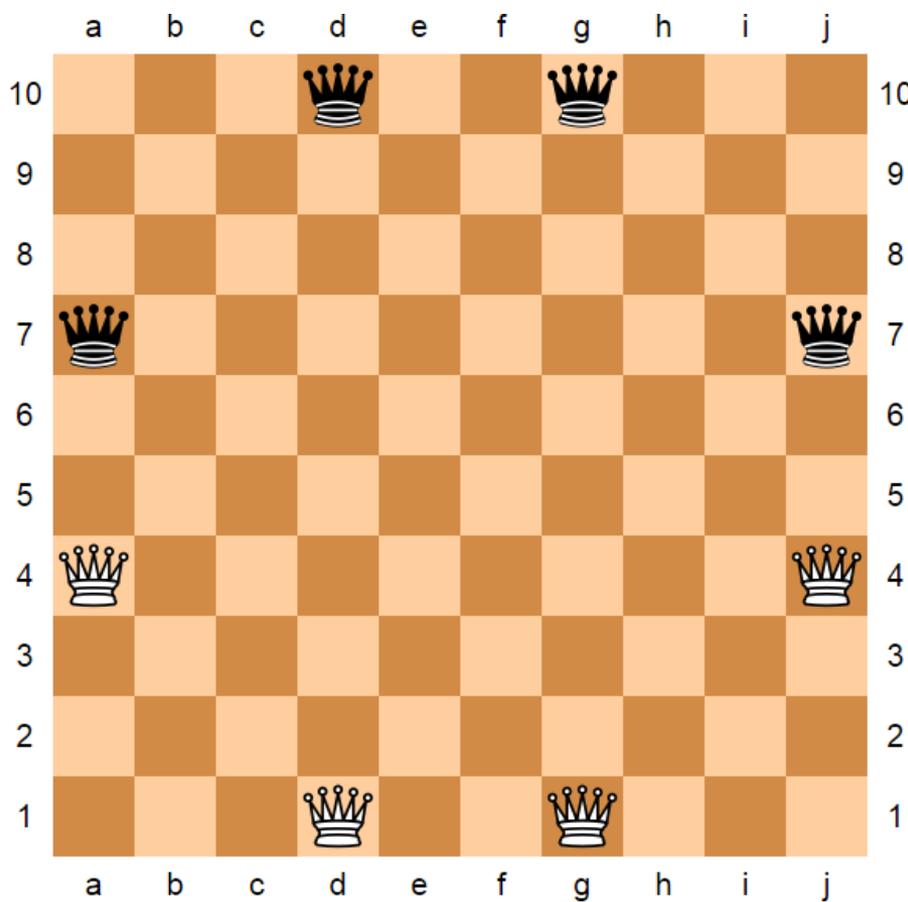
Bounded 2CL is PSPACE- complete





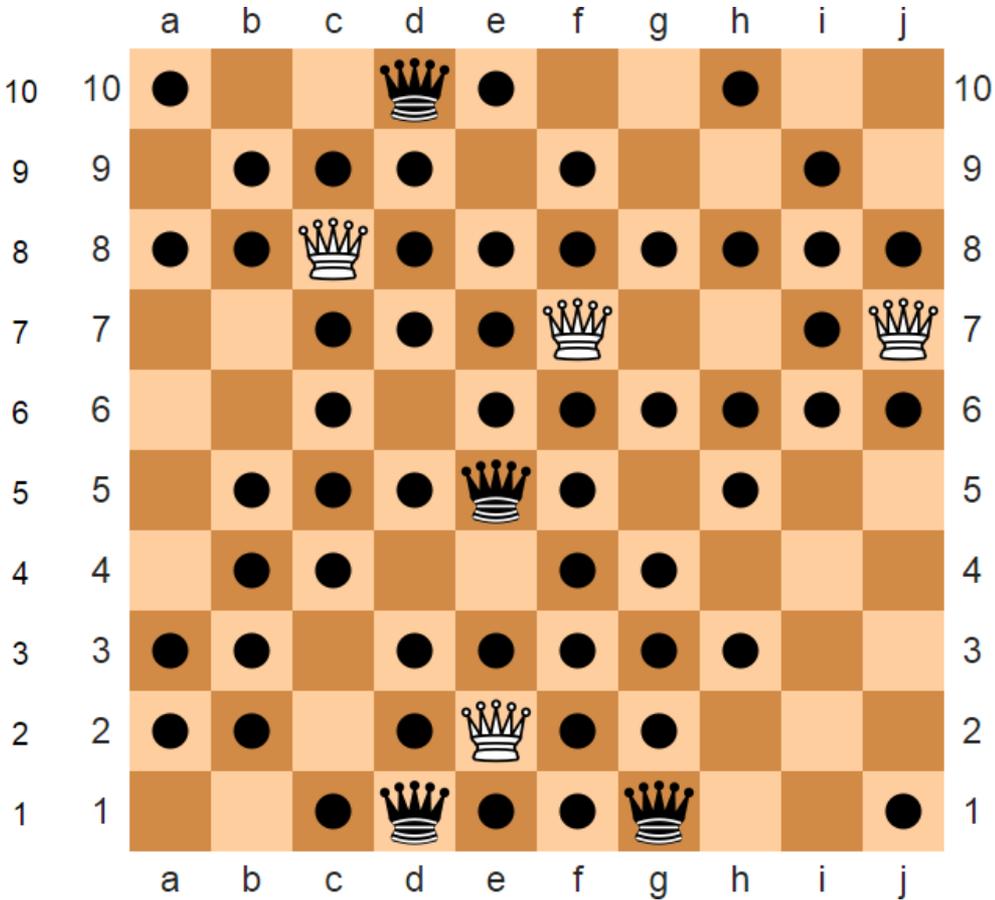
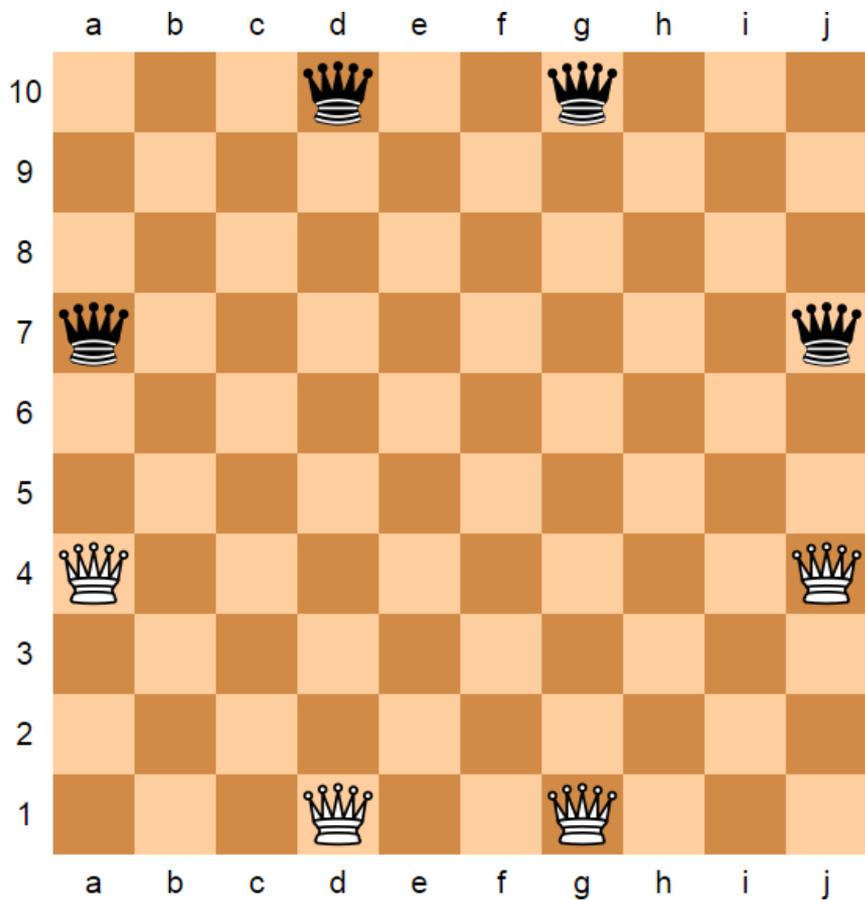
Amazons

[Walter Zamkaskas 1988]



Amazons

[Walter Zamkaskas 1988]





Amazons

[Hearn 2005]

⋮



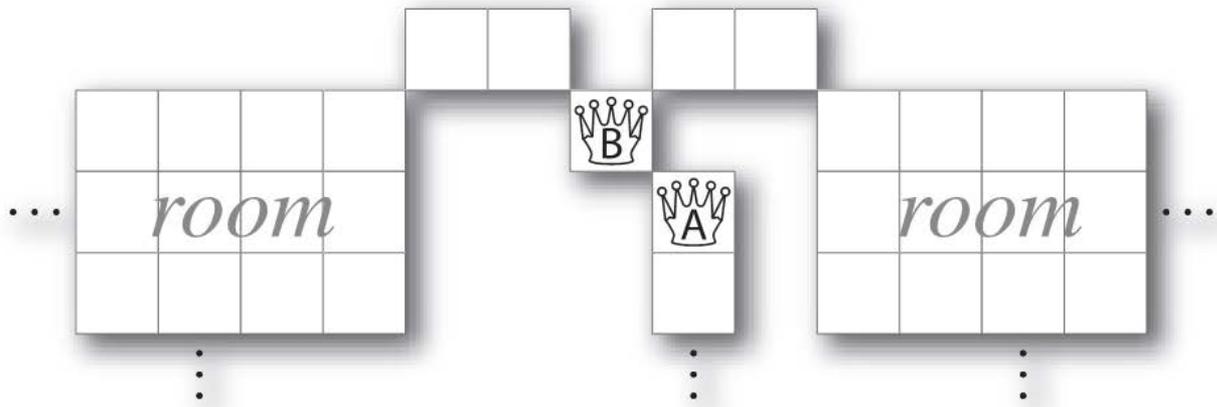
wire



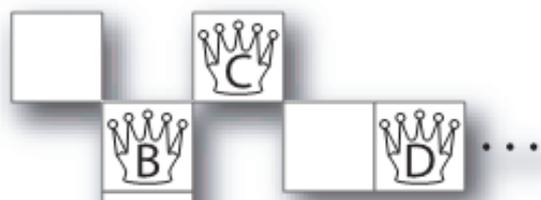
shift



⋮



victory



turn &
one way

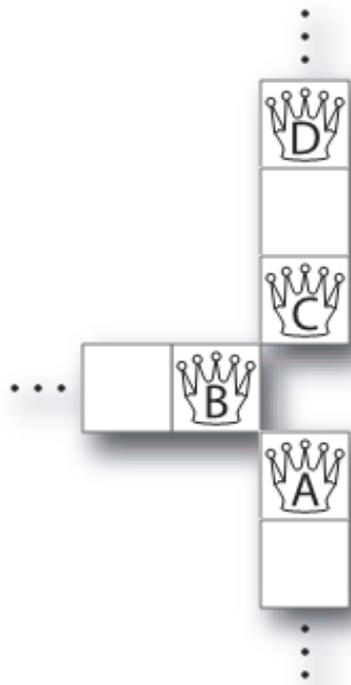
⋮



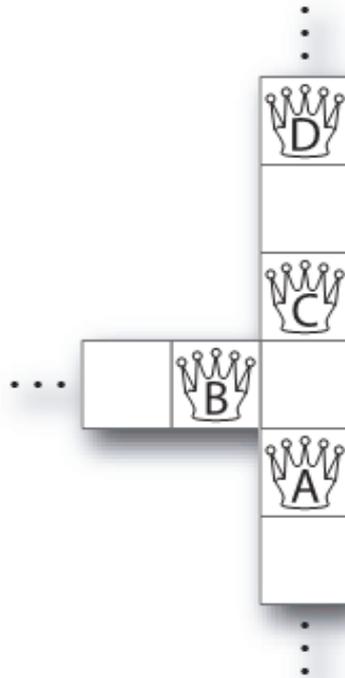
variable

Amazons is PSPACE-complete

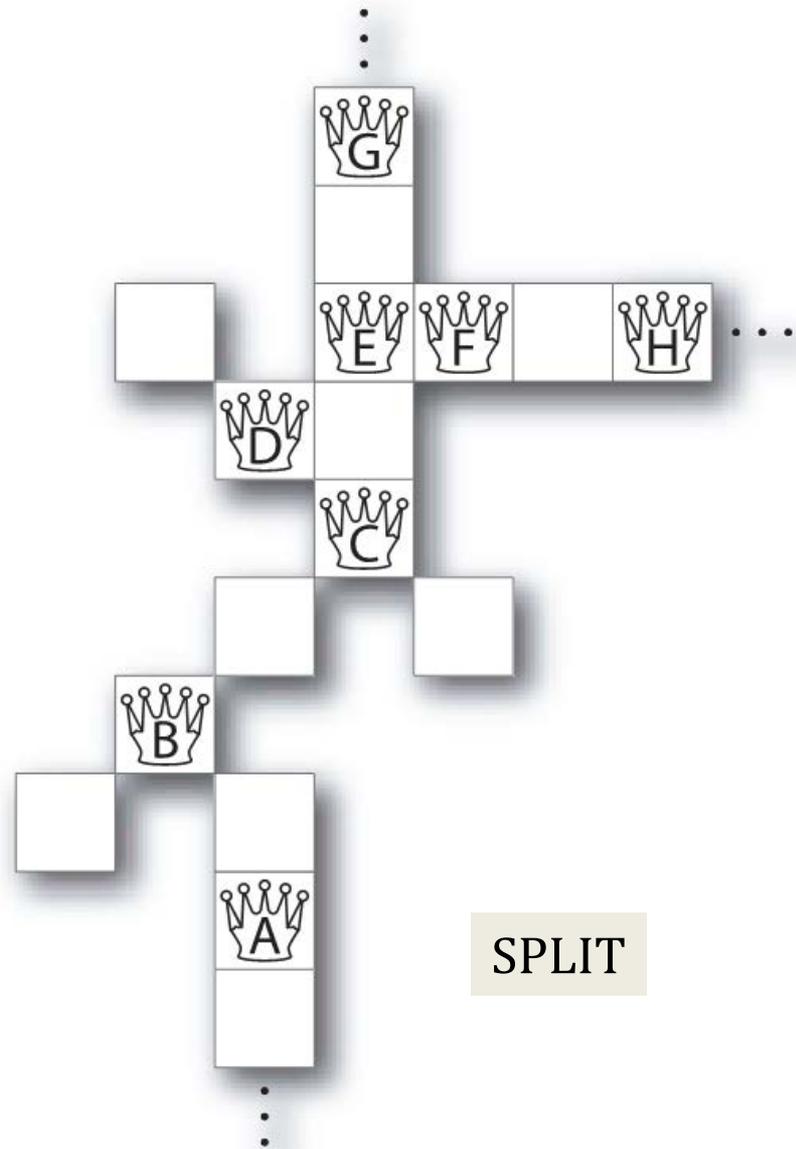
[Hearn 2005]



AND



OR &
CHOICE



SPLIT



Konane [Hawaii <1778]

photo by Thane Plambeck, 2005

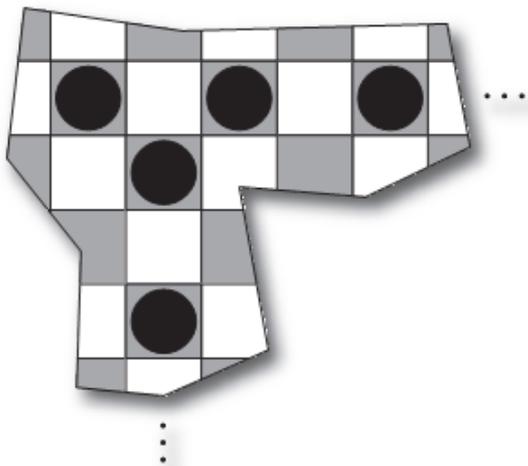




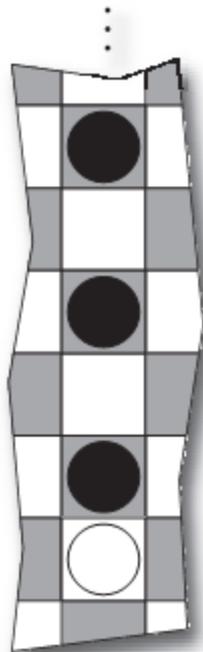
Konane is PSPACE-complete

[Hearn 2005]

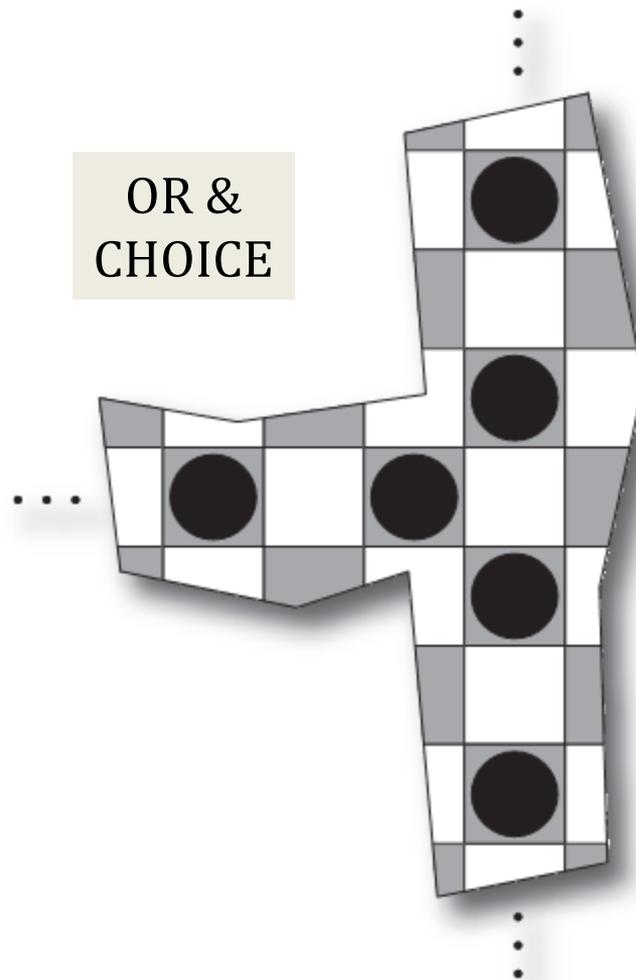
turn



variable

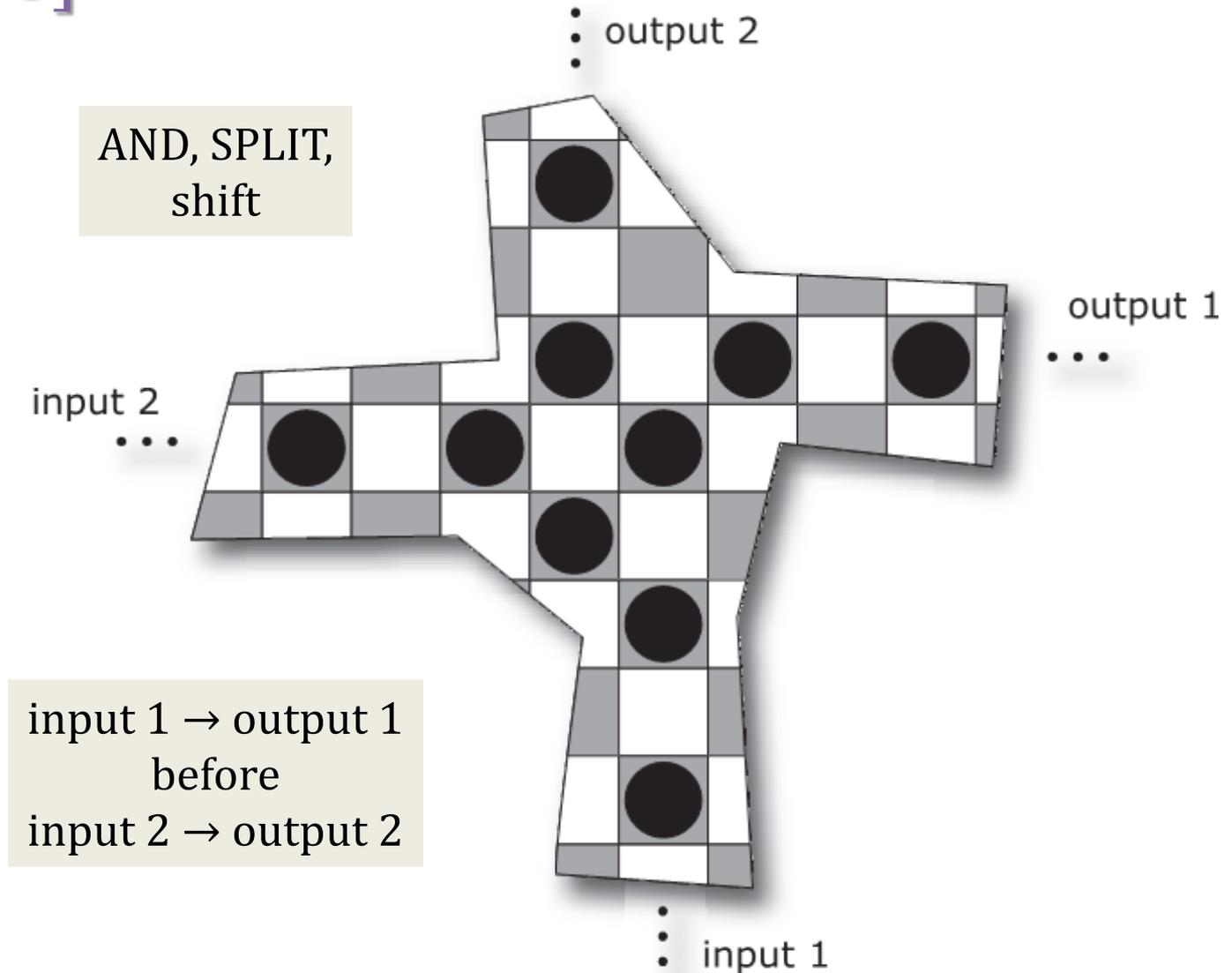


OR &
CHOICE



Konane is PSPACE-complete

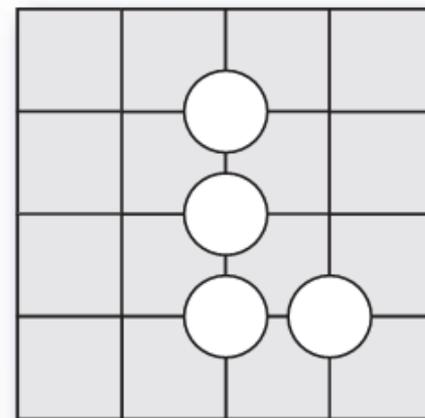
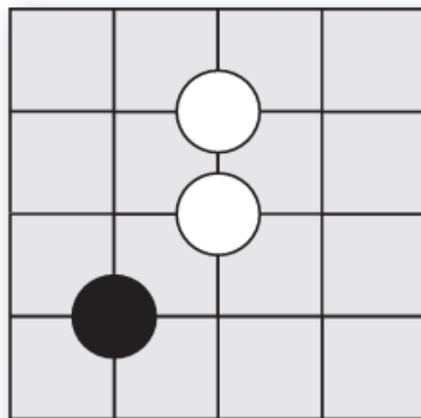
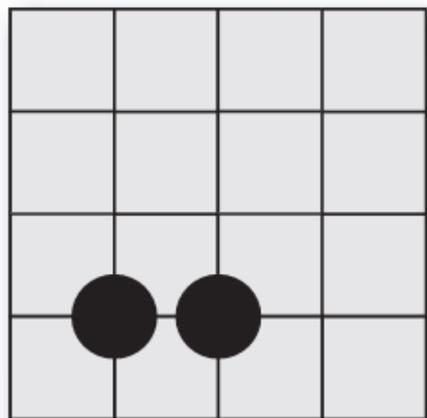
[Hearn 2005]





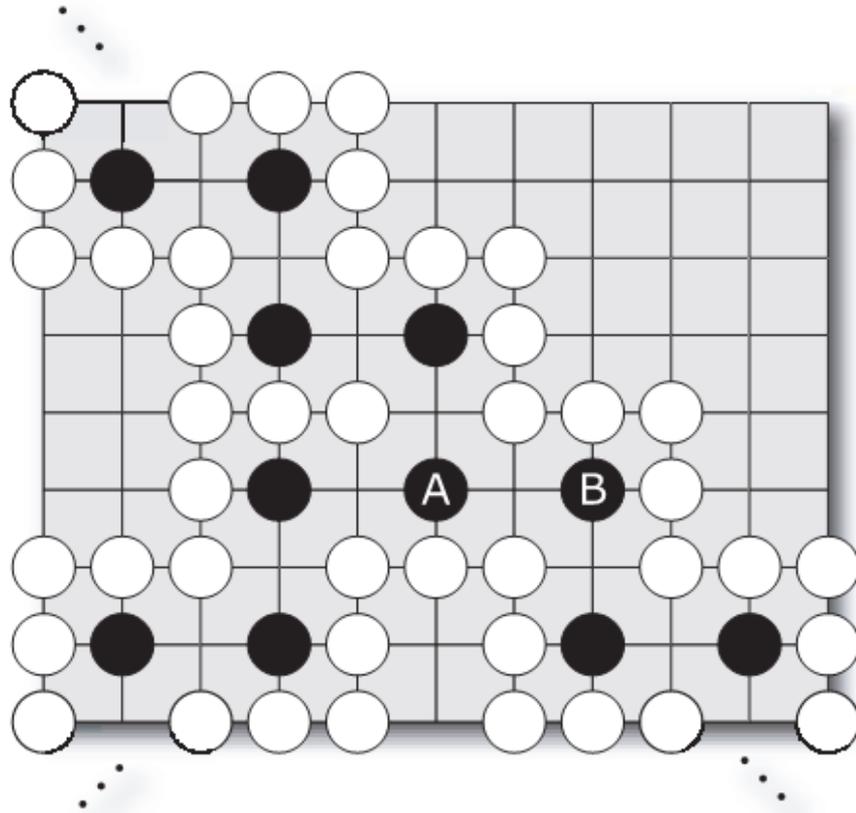
Cross Purposes

[Michael Albert 2004]

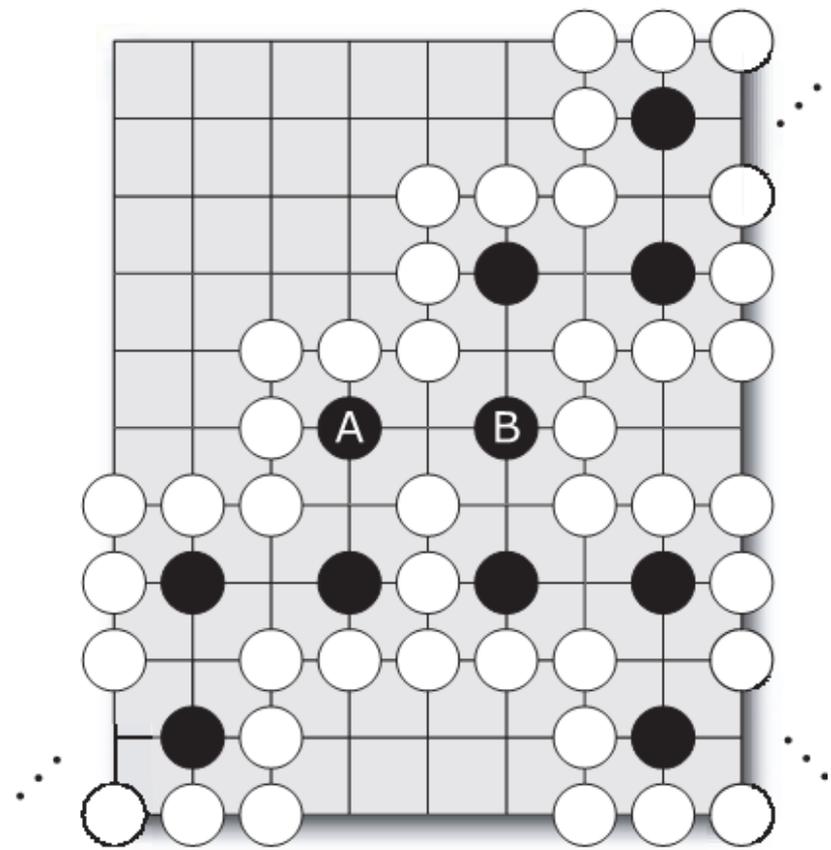


Cross Purposes is PSPACE-complete

[Hearn 2005]



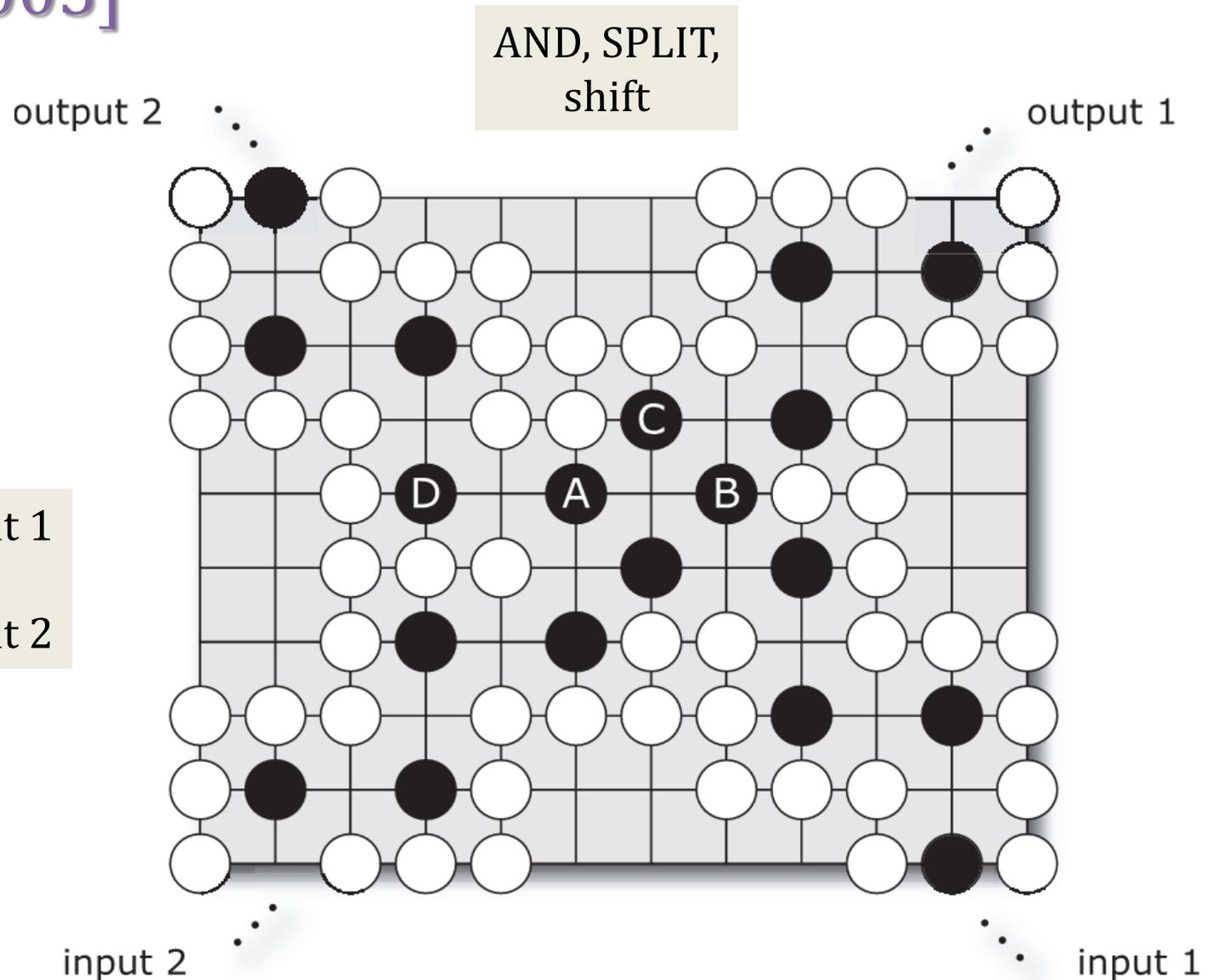
protected OR



CHOICE

Cross Purposes is PSPACE-complete

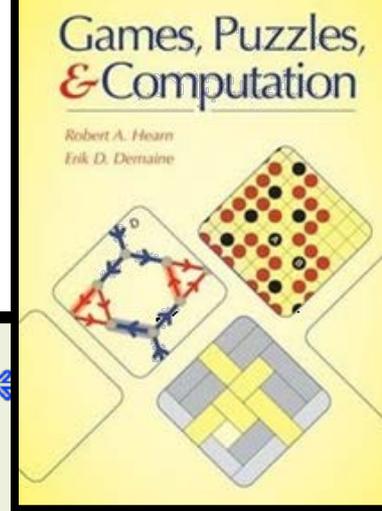
[Hearn 2005]



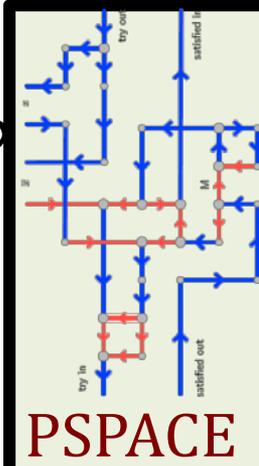


Constraint Logic

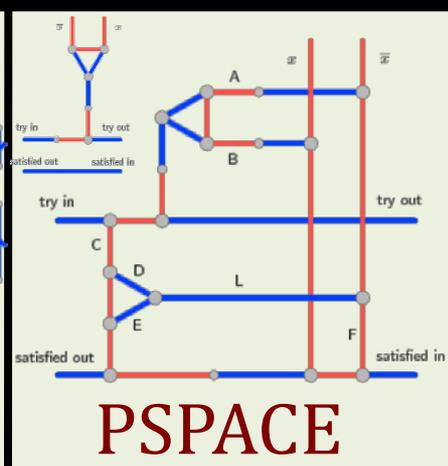
[Hearn & Demaine 2009]



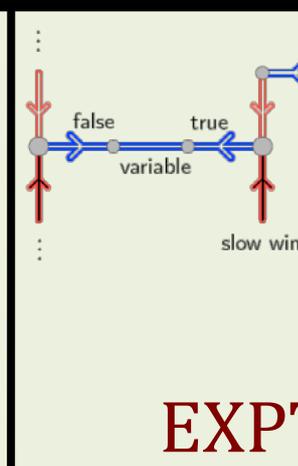
unbounded



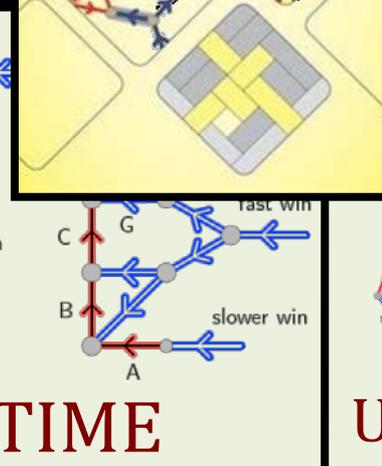
PSPACE



PSPACE

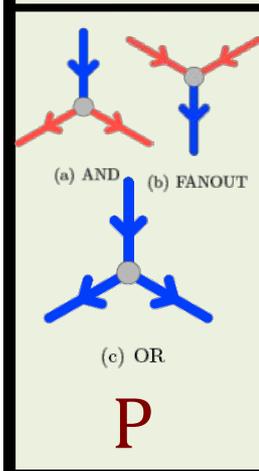


EXPTIME

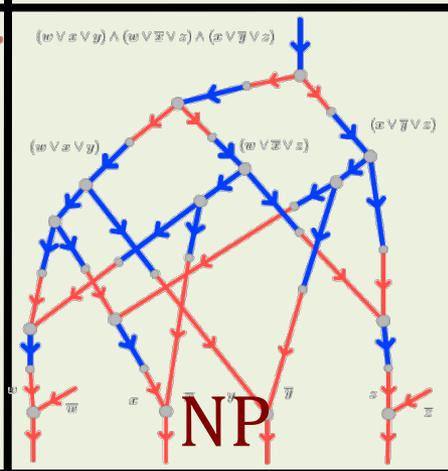


Undecidable

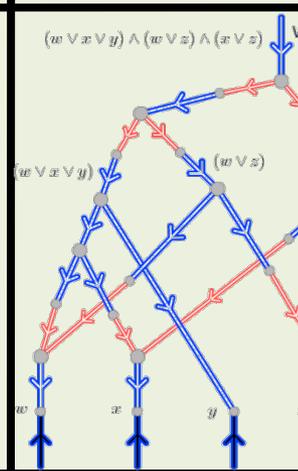
bounded



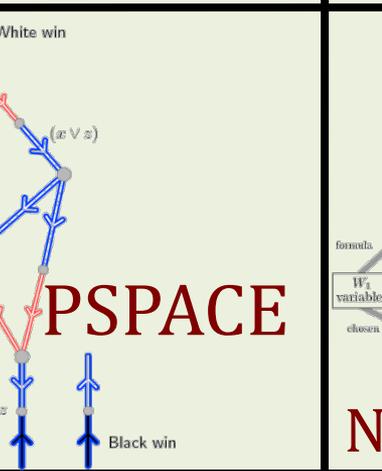
P



NP



PSPACE



NEXPTIME

0 players
(simulation)

1 player
(puzzle)

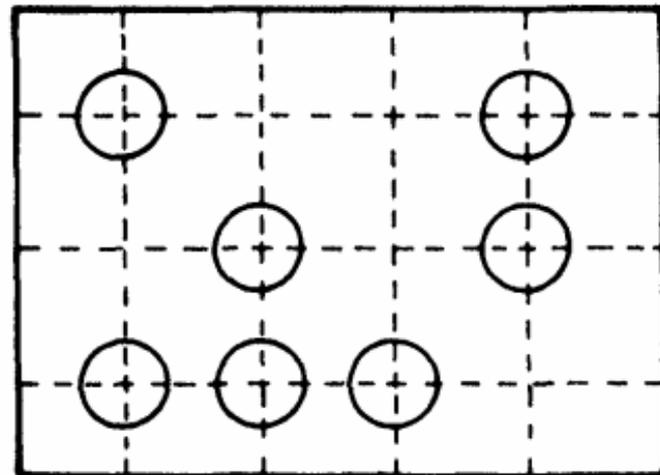
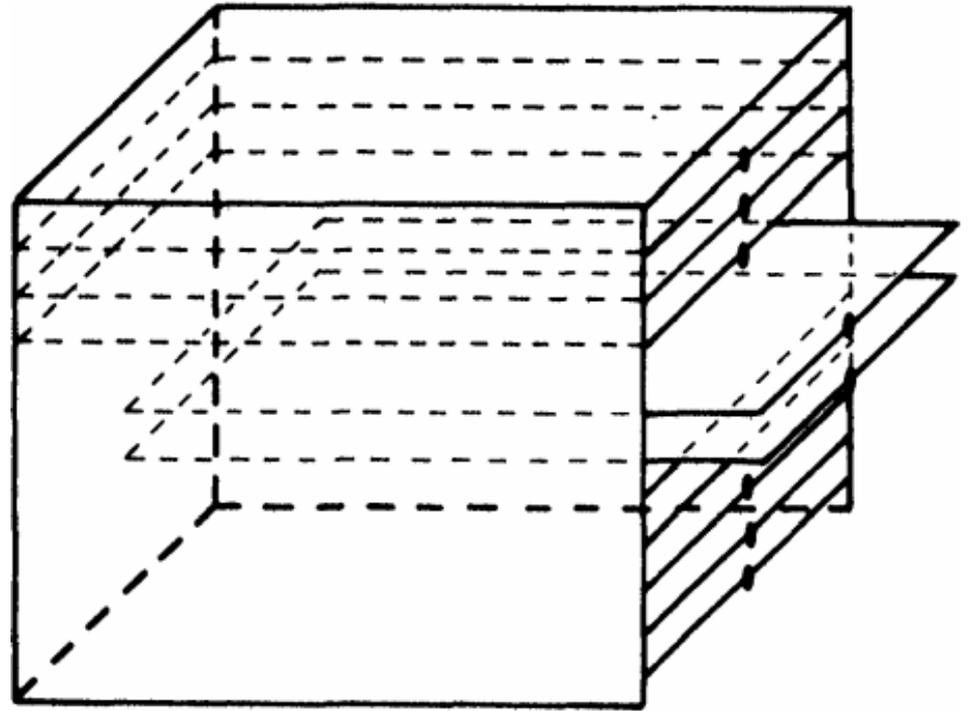
2 players
(game)

team,
imperfect info



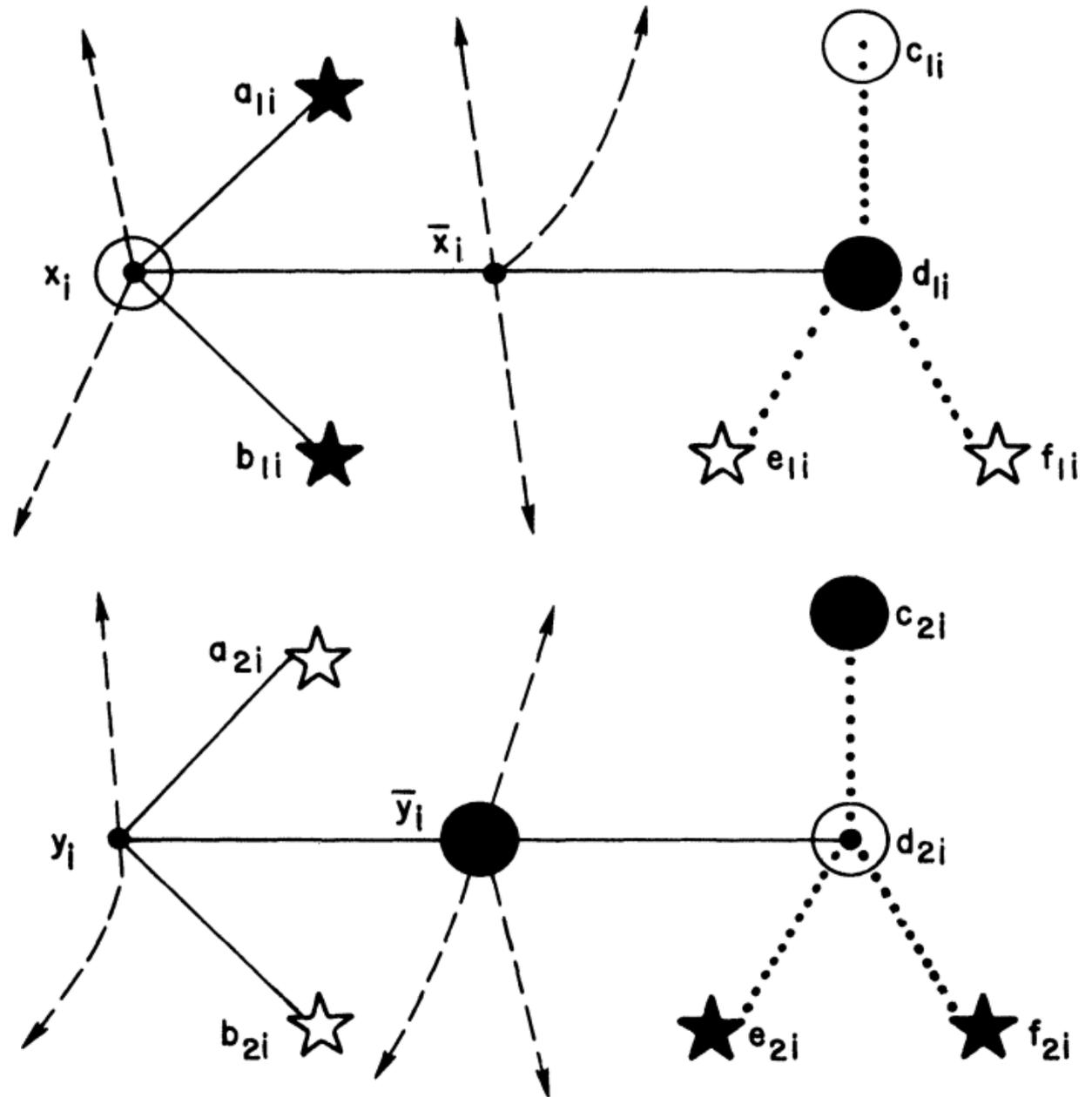
Peek

[Stockmeyer &
Chandra 1979]



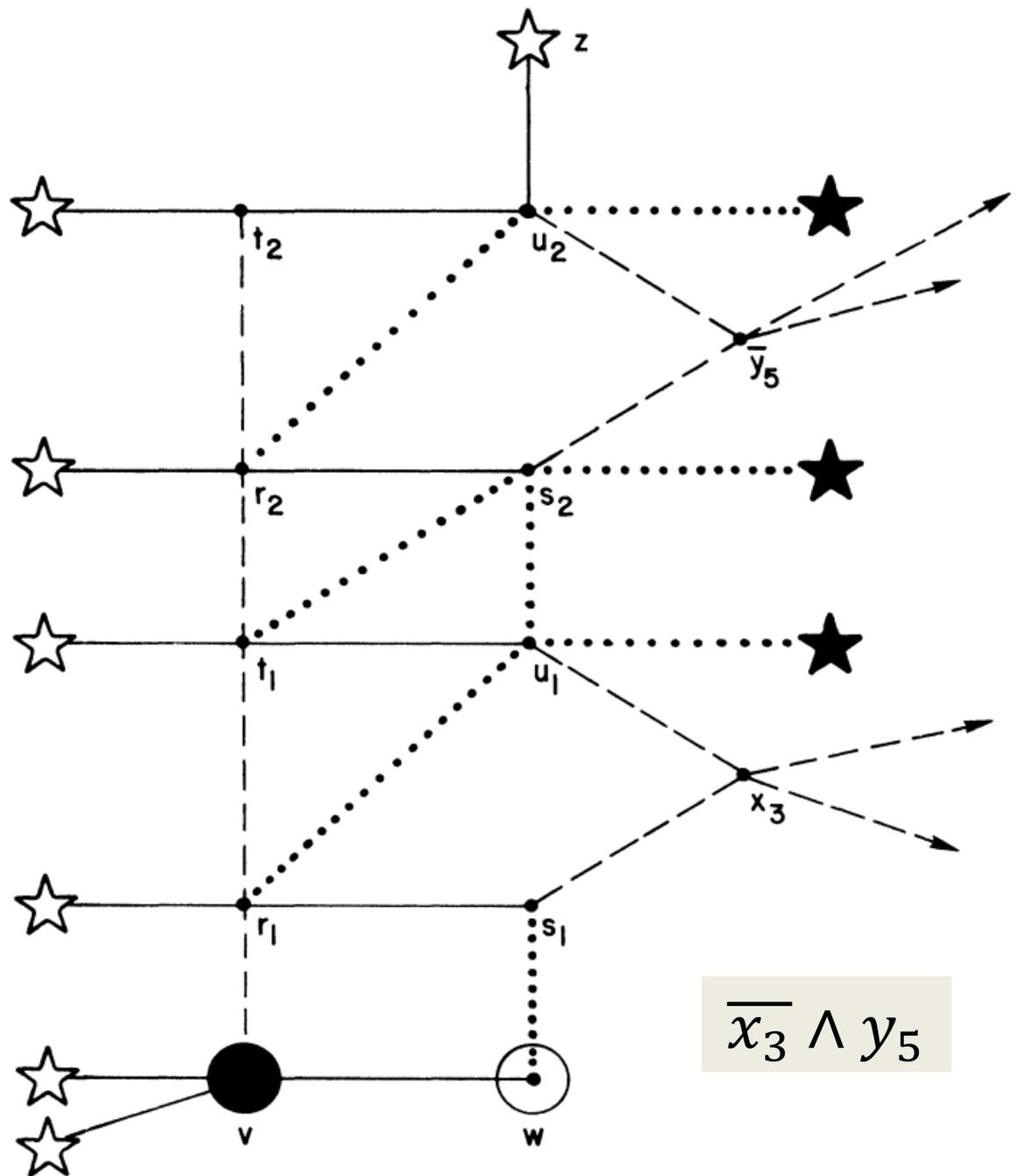
BLOCK is EXPTIME-complete

[Stockmeyer & Chandra 1979]



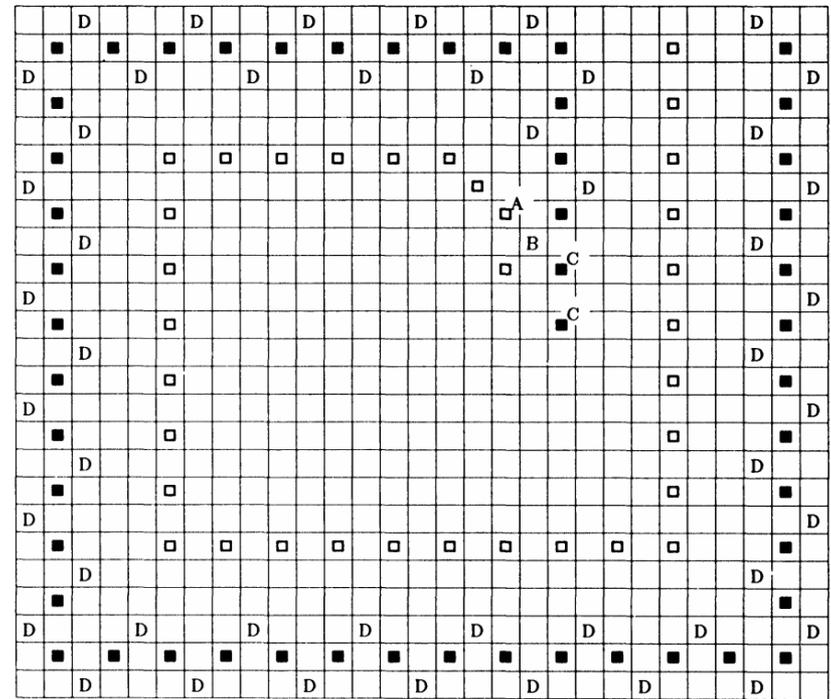
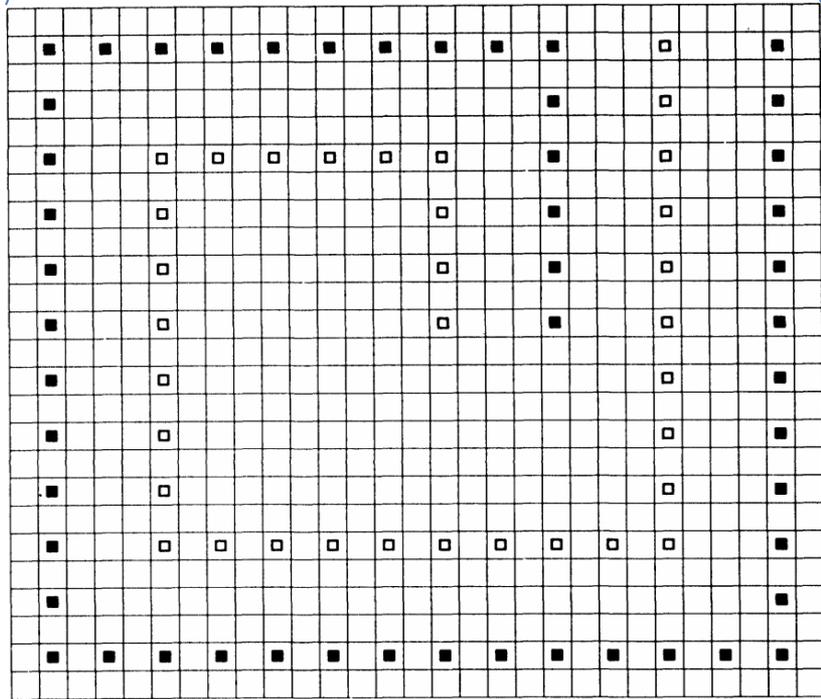
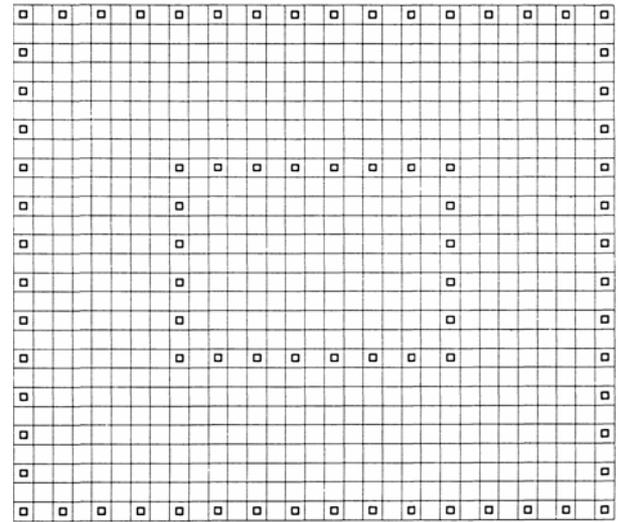
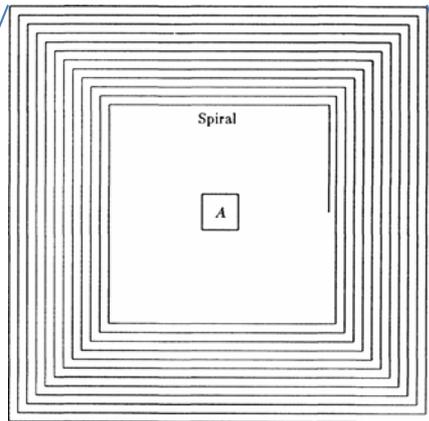
BLOCK is EXPTIME-complete

[Stockmeyer & Chandra 1979]



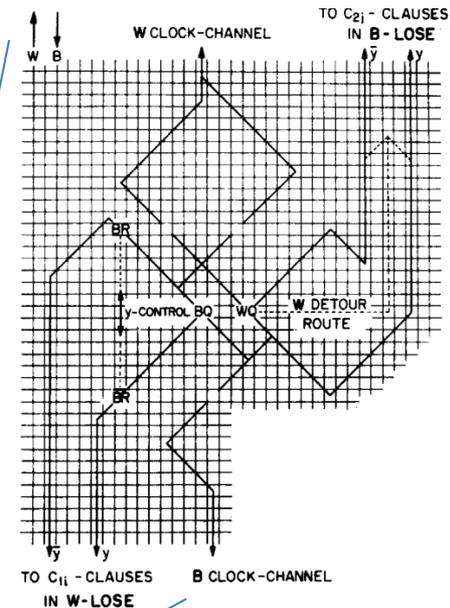
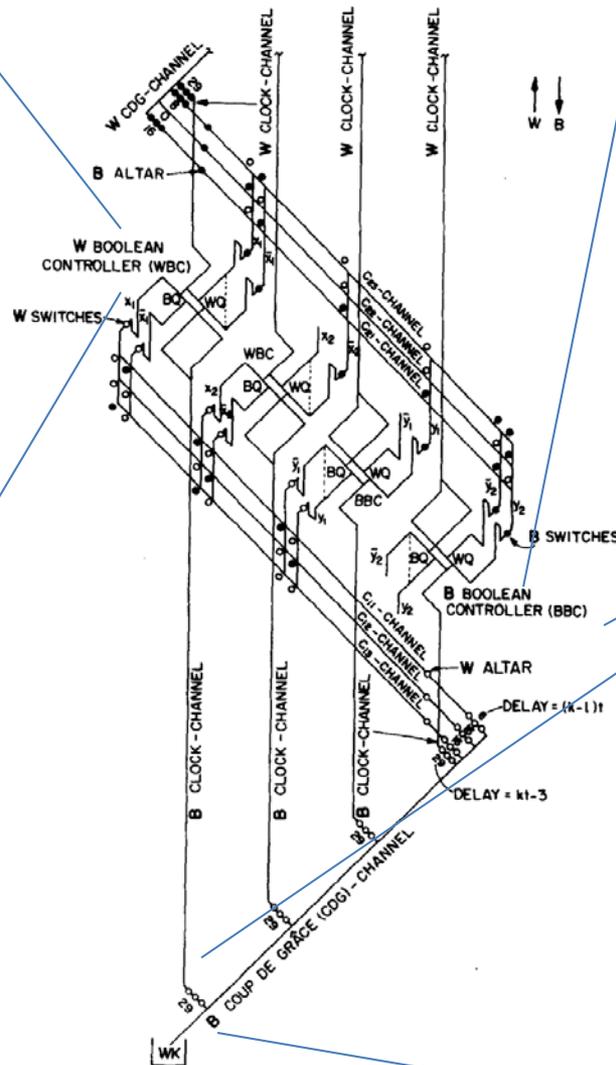
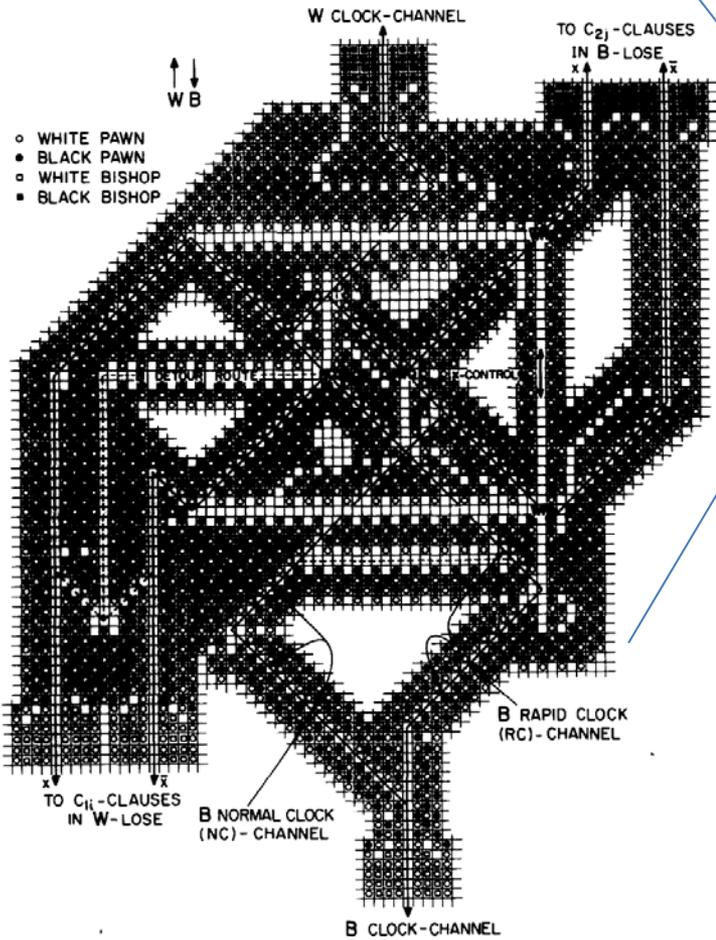
Checkers is EXPTIME-complete

[Robson 1981]

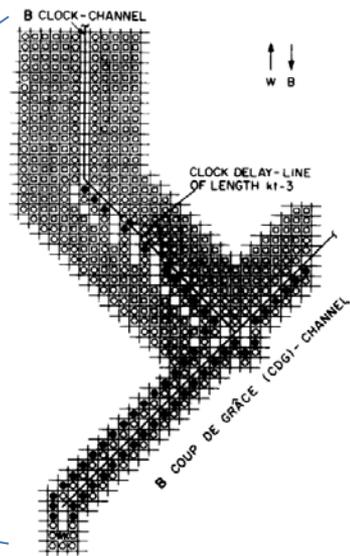


Chess is EXPTIME-complete

[Fraenkel & Lichtenstein 1981]



TO C_{1i} -CLAUSES IN W-LOSE



reduction from G_3

Go is EXPTIME-hard [Robson 1982]

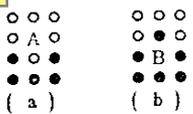


Figure 1.

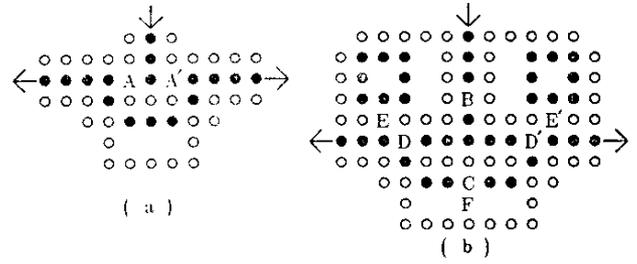


Figure 4.

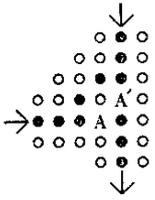


Figure 5.

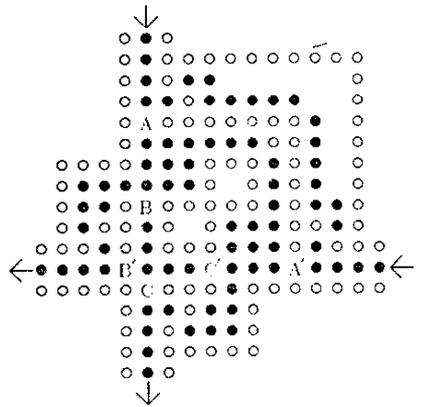


Figure 6.

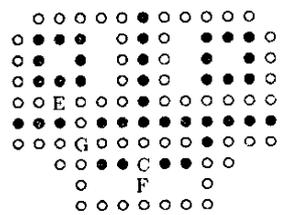


Figure 8.

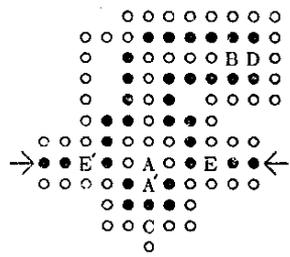


Figure 7.

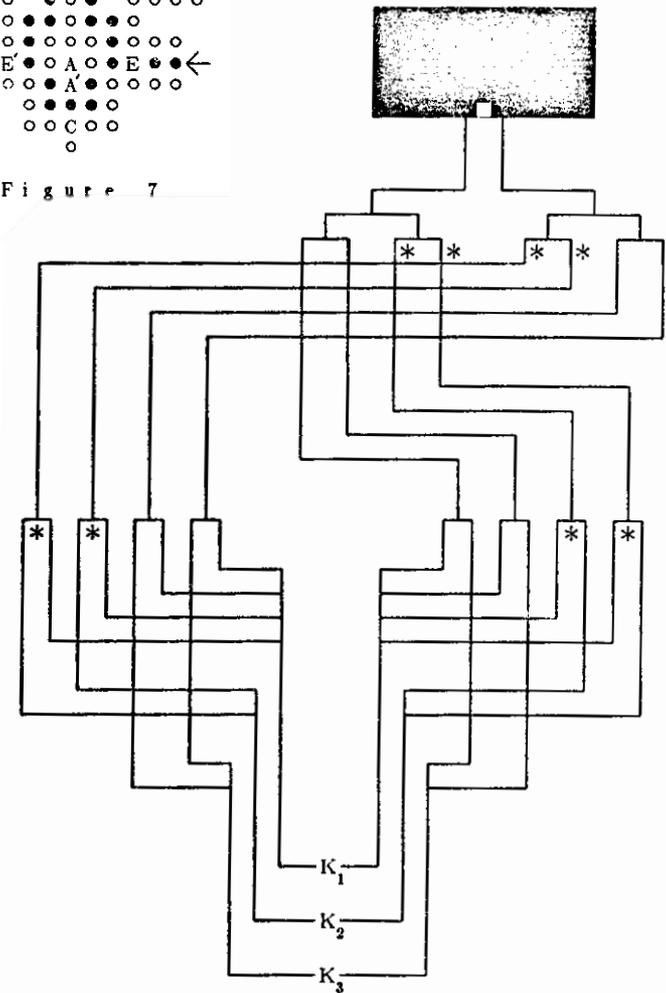


Figure 2.

almost surrounded
Black group with one eye

Connection paths start at
only two gaps in White's
surrounding wall

Black choice forks

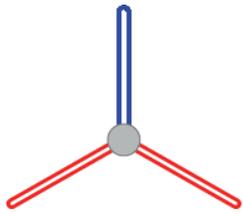
crossovers

White choice forks

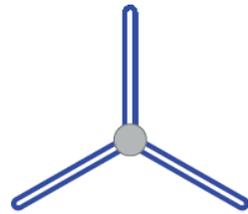
crossovers
and joins

KOs

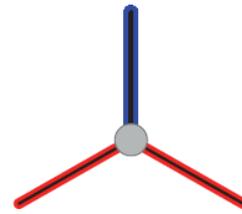
2CL is EXPTIME-complete



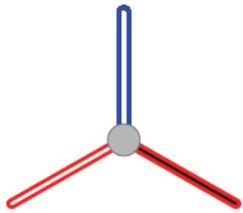
(a) White AND



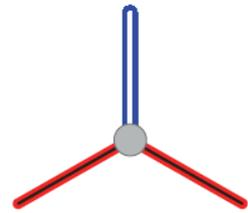
(b) White OR



(c) Black AND



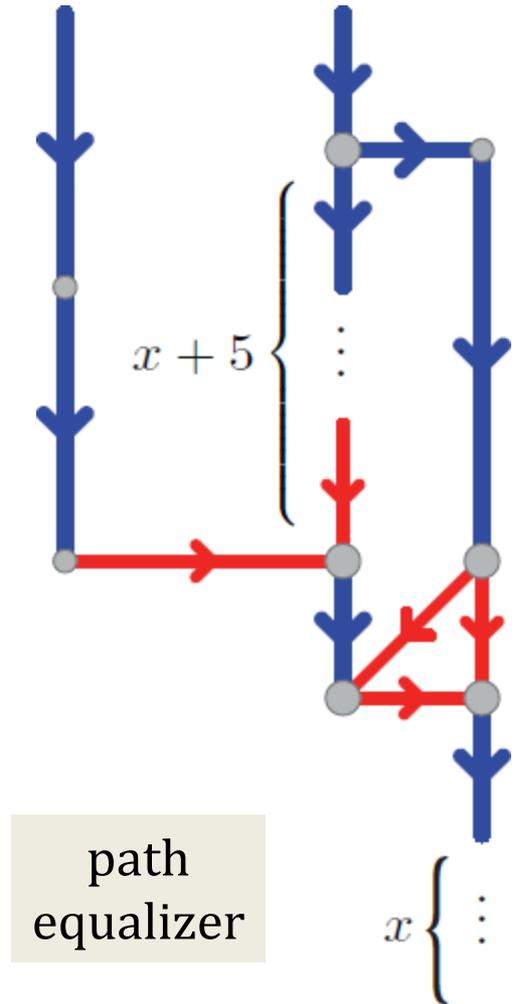
(d) Multiplayer AND 1



(e) Multiplayer AND 2

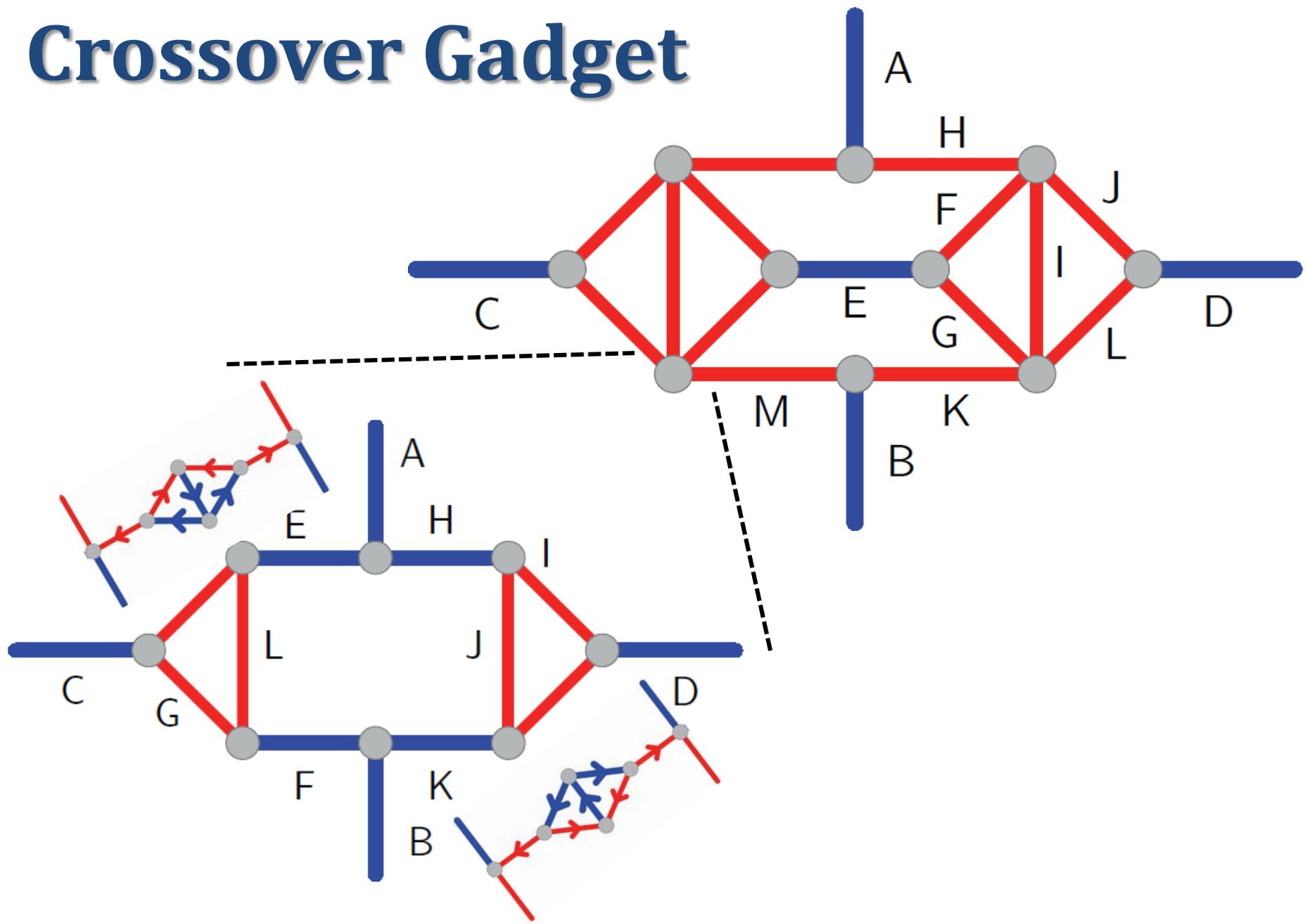


(f) Black-White





Crossover Gadget



No-Repeat Games

No-repeat rule:

[Robson - MFCS 1984]

lose if ever repeat a past game configuration
⇒ G_1, G_2, G_3 become EXPSPACE-complete
as do Chess & Checkers

- OPEN: is Go with superko (no-repeat) }
EXPSPACE-complete? (as in USA & China)

Conditional no-repeat rule:

[Robson - MFCS 1984]

- two special variables x & y
- lose if ever repeat a past game configuration
& at most 1 of x & y have changed since
⇒ G_1 becomes 2EXPTIME-complete

Private-Information Games

Private-information games: [Reif-JCSS 1984]

you can see some but not all of opponent's state
⇒ G_1 5DNF, G_2 DNF become 2EXPTIME-complete
↳ version of Peek with half of winning holes visible to each player

Blind games: [Reif-JCSS 1984]

player 1's entire state is hidden from player 2
⇒ G_2 DNF becomes EXPSPACE-complete
↳ version of Peek above



Private & Blind Peek

[Reif 1984]

