Problem Set 9

This problem set is due Wednesday, November 16 at noon.

1. Prove that the Steiner tree spanner as constructed in the lecture actually contains a Steiner tree of the given terminals of weight at most $(1 + c\epsilon)\text{OPT}$ (where c is an absolute constant).

   Hint: Recall that the structure theorem guarantees that we can transform any forest inside a brick into another forest of slightly larger weight that includes E and W and that preserves connectivity among the boundary and has at most α joining vertices, where $\alpha = 2c'\kappa\epsilon^{-2.5} = O(\epsilon^{-5.5})$. However, we do not know the joining vertices; hence we greedily selected $\theta = 2\alpha\epsilon^{-1} \cdot \ell(\text{MG})$ portals around each brick.

2. Let G be a plane graph, P be a $(1+\epsilon)$-short path in G and T be binary tree in G with no vertices of degree 2, rooted at r, and having all leaves on P.

   (a) How many joining vertices does T have with P if its depth is at most ϵ^{-1}?

   (b) Show that if the depth of T is at least ϵ^{-1} then there is a level k such that the total weight of all edges of T at level k is small and give an upper bound on that weight.

   (c) Show how to transform T into another tree T' of weight at most $(1+2\epsilon)\ell(T)$ that spans r and $V(T) \cap V(P)$ and has $f(\epsilon)$ joining vertices with P. Give an upper bound on $f(\epsilon)$.