Problem Set 6

This problem set is due Wednesday, October 26 at noon.

For this problem set it is important to know that the separation property is defined somewhat differently for contraction-bidimensional problems. Let (A, B, S) be a separation of G and $Z \subseteq V(G)$ be an optimal solution to a contraction bidimensional problem Π in G. Let G_A denote the graph obtained by *contracting* each connected component of $G[B]$ into its adjacent vertex of S with smallest index, and define G_B similarly. Let Z_A denote an optimal solution to Π in G_A and Z_B and optimal solution in G_B. We say Π has the separation property if

$$|Z_A| \leq |Z - B| + O(|S|) \text{ and } |Z_B| \leq |Z - A| + O(|S|) \ .$$

1. Show that the minimum connected dominating set problem admits a PTAS in apex-minor-free graphs.

   A dominating set in a graph G is a set $D \subseteq V(G)$ such that $D \cup N(D) = V(G)$, where $N(D)$ is the set of all vertices that are neighbors of some vertex of D. It is called a connected dominating set if $G[D]$ is connected.

2. Show that the connected vertex cover problem admits a PTAS in H-minor-free graphs.

   Recall that a vertex cover in a graph is a set of vertices Z such that every edge of the graph has at least one endpoint in Z; it is called a connected vertex cover if $G[Z]$ is connected.