Problem Set 1

This problem set is due Thursday, September 22 at noon.

1. Sparsity Lemma: Prove that for a planar a embedded graph in which every face has size at least three, $m \leq 3n - 6$, where m is the number of edges and n is the number of vertices.
2. Minimum Spanning Tree: Give a linear-time algorithm for minimum-weight spanning tree in a connected planar graph. Refer to the textbook for a discussion of representing embedded graphs in computations and efficiently performing basic operations such as contractions and deletions. You are encouraged to use the following results, which you need not prove.
   - Let G be a graph with edge-weights, and let v be a vertex. Let e be the minimum-weight edge incident to v. Then G has a minimum-weight spanning tree that includes e.
   - Let G be a graph with edge-weights, and let e be an edge contained by some minimum-weight spanning tree of G. Let T be a minimu- weight spanning tree of the graph obtained from G by contracting e. Then $T \cup \{e\}$ is a minimum spanning tree of G.
3. Rootward Computation: Give a linear-time algorithm for the following problem and argue its correctness.
   input: a planar embedded graph G with face weights and a spanning tree T of G.
   output: a table giving, for each nontree edge uv, the weight enclosed by the fundamental cycle of uv with respect to T.