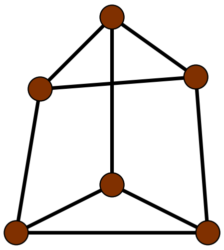
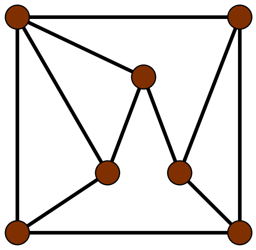
6.885: Folding and Unfolding in Computational Geometry
Fall 2004 — Prof. Erik Demaine

Problem Set 1

Due: Monday, November 8, 2004

Problem 1. Give a Henneberg construction for each of the following minimally generically rigid graphs.

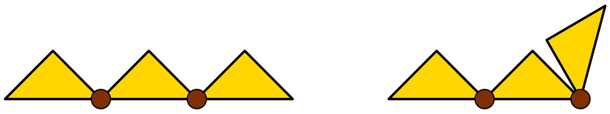

(a)


(b)

Problem 2. Prove that every pointed pseudotriangulation is minimally generically rigid. A *pseudotriangulation* of a set of points in the plane is a collection of edges between these points, including the convex hull of the point set, such that every face except the outside face has exactly three convex vertices. A pseudotriangulation is *pointed* if every vertex has an incident reflex angle (angle of more than 180°). Problem 1(b) shows an example of a pointed pseudotriangulation.

Problem 3. Prove that no four-bar 3D open chain is locked.

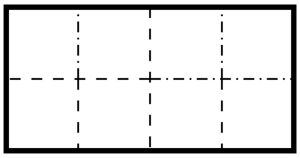
Problem 4. Find a locked planar hinged open chain of convex polygons. An *open chain of polygons* is a collection of polygons $P_1, P_2, \ldots, P_n$ where each polygon P_i is joined along one vertex to a vertex of P_{i-1} (if $i > 1$) and joined along a different vertex to a vertex of P_{i+1} (if $i < n$). A *configuration* of such a chain is a rigid placement of the polygons such that joined pairs of vertices are geometrically coincident, and such that no two polygons intersect in their interiors. A *motion* is a continuum of configurations. A chain is *locked* if there are two configurations that cannot reach each other by a motion. Two configurations of the same open chain of polygons are shown below:

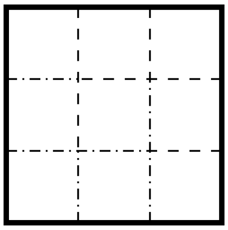


Problem 5. Derive and prove a characterization of flat-foldable single-vertex crease patterns where $\theta_1 + \theta_2 + \cdots + \theta_n$ might be larger than 360°. You must show that your condition is both necessary and sufficient and that it can be checked in linear time. Give an example of a flat-foldable single-vertex crease pattern that does not satisfy $\theta_1 + \theta_3 + \cdots + \theta_{n-1} = \theta_2 + \theta_4 + \cdots + \theta_n$.

Problem 6. Find a two-vertex crease pattern that is locally flat foldable (at each vertex) but is not (globally) flat foldable.

Problem 7. Fold the following maps flat using the specified mountain-valley assignment. (Dashed edges are valleys; dot-dashed edges are mountains.)





Problem 8. Fold a pleated hyperbolic paraboloid from a square of paper according to the following directions.