Flattening polyhedra

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- piece of paper = polyhedral surface (e.g., cube)
- can it be folded flat? (without tearing)

Connection to fold & cut:

- 3D fold & cut: given 3D solid of paper marked with a set of 2D polygons.
  fold (through 4D) into 3D to align precisely given polygons
- flattening = boundary of this problem

OPEN: d-D fold & cut for $d \geq 3$? at least convex polyhedra?

OPEN: folding that aligns all k-D faces, for all $0 \leq k \leq d$?
even for $d = 2$?

Continuity: seems difficult — no canonical tiny state

OPEN: is configuration space of polyhedra (as piece of paper) connected?

- note: in more typical models of folding polyhedra (finite/no additional creases),
  convex polyhedra are rigid
  & all polyhedra conserve volume (No-Bellows theorem)

Flat folded state exists for all polyhedra homeomorphic to sphere (or disk) [Demaine, Demaine, Hayes]

- same method as disk-packing fold & cut

OPEN: higher genus?

OPEN: arbitrary polyhedral complexes?

OPEN: flattening based on the 3D straight skeleton?

- possible for "thin convex prismatoids" (thin slices of convex polyhedra) [Demaine, Demaine, Lubiw]