Fold & cut problem

6.885
L9.1
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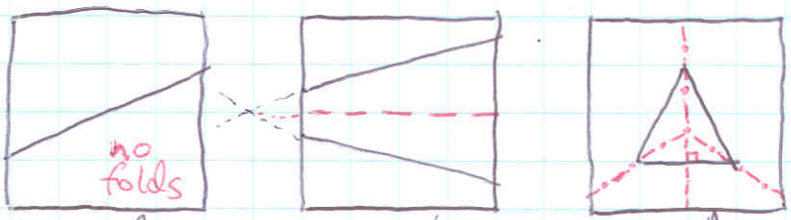
① Fold flat
② Make one complete straight cut
③ Unfold
} what shapes/patterns of cuts can result?

History:
- Kan Chu Sen 1721 ~ Japanese puzzle book
- Betsy Ross (1873 story) ~ ☆ in American flag
- Harry Houdini 1922 ~ ☆ in *Paper Magic* (ghost?)
- Gerald Loe 1955 ~ *Paper Capers*
- Martin Gardner 1960 ~ *Scientific American*

Universality: any set of line segments can be cut
- two methods: [Demaine, Demaine, Lubiw 1998] ← almost... but "nicer"
  & [Bern, Demaine, Eppstein, Hayes 1998]

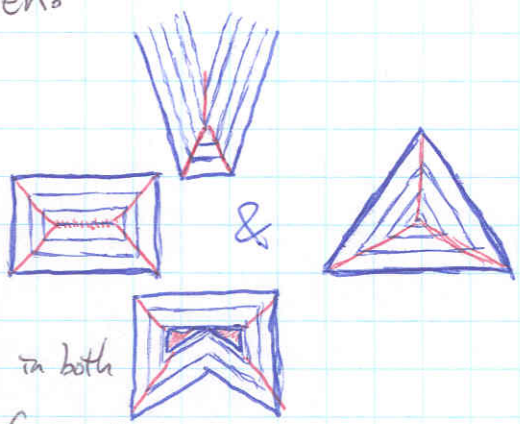
Warm-ups:



trivial — bisector — 3 angular bisectors (meet) + 1 perpendicular

Method 1: "straight skeleton" + "perpendiculars" [DDL 1998]

Straight skeleton [Aichholzer et al. 1995 & 1996]
- simultaneously shrink each face in desired cut pattern so that edges remain parallel to the originals & perpendicular distance to originals is uniform.
- 3 things (events) can happen:
  ① edge shrinks to ∅ ⇒ drop it
  ② entire face collapses to zero area ⇒ add it all
  ③ face "splits" ⇒ recurse in both



- straight skeleton = trace of vertices