6.885
L8.3
Oct. 4, 2004

Efficiency

- strip method impractical
- one measure: starting from square of paper, most of the area is wasted.
- what is the smallest square that can fold a given shape?

Cube wrapping: target = cube

[Catalano-Johnson & Loeb 2001]

- consider unit (1×1) square
- in $x \times x \times x$ cube, every point has an antipodal point of intrinsic distance $\geq 2x$ away

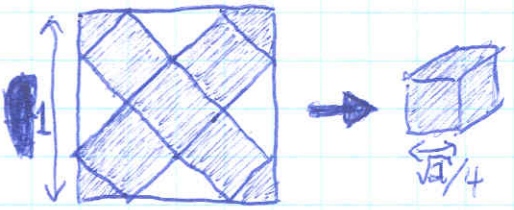
$\Rightarrow$ center of square must have distance $\geq 2x$ to someone (in particular, the corners)

[farthest "neighbor" from a point can only get closer by folding]

$\Rightarrow$ distance between opposite corners $\geq 4x$

$\Rightarrow$ side length $\geq 2\sqrt{2}\,x$

$\Rightarrow x \leq 1/2\sqrt{2} = \sqrt{2}/4$.



OPEN: regular tetrahedron, etc.?

OPEN: fold $x \times y$ rectangle into largest cube

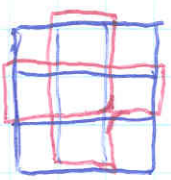
- as $x/y \to \infty$, strip method becomes efficient
- is the answer always $\Theta(1)$?

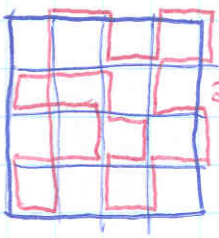
k×k Checkerboard folding [Montroll; Lang, ~1998]

- suppose color-reversal achieved only by paper boundary (not necessarily valid!)
- shortest cycle to cover all interior edges has $2k^2 - [2 \text{ if } k \text{ is odd}]$ edges
- unit square → c×c k×k checkerboard

$\Rightarrow 2ck - [2c/k \text{ if } k \text{ is odd}] \leq 4$

$\Rightarrow c \leq \dfrac{1}{k/2 - [2/k \text{ if } k \text{ is odd}]}$





	2×2	3×3	4×4	8×8
Predicted limit	1	5/6	1/2	1/4
Best known: Folding	2/3	3/5	1/2	1/4
Best known: Seamless folding	?	?	?	1/5

OPEN: any real lower bound? improve any foldings? 5×5? (particularly seamless)