Silhouette folding & gift wrapping

6.885
L8.1
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- what 2D polygons & 3D polyhedra can be folded.
  [Bern & Hayes 1996] [Akiyama, Gardner]
  say from a square of paper?

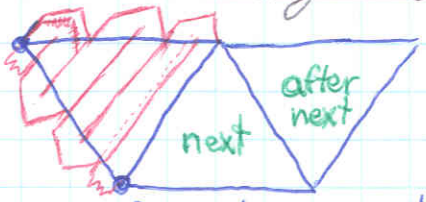
holes, too

- with bicolor (duo) paper, extra challenge of getting a desired visible 2-color pattern

Universality: [Demaine, Demaine, Mitchell 2000]

Every connected union of polygons in 3D,
each with a specified visible color (on each side),
can be folded from a sufficiently large square of paper
(or a sufficiently large scaling of any 2D shape of paper)

Proof:
- fold paper down to long, thin strip (!)
- triangulate the polygons
- choose a "path" of the triangles, possibly with repeats
- to cover a triangle: zig-zag parallel to next edge (starting at opposite corner)



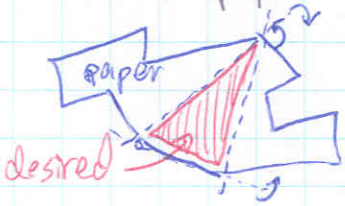
choose parity of zig-zag to end at right vertex

- for zigs/zags and turns to next △, use turn gadget:

fold bottom layer

fold away excess (potentially many folds)

- hide excess paper underneath each triangle (∵ convex)



fold repeatedly underneath along lines extending desired edges

- color-reversal gadget:

fold bottom layer

ditto

□