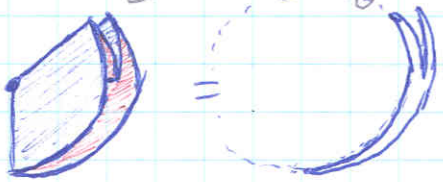
6.885
L7.2
Sept. 29, 2004

Single-vertex flat origami:

= disk of paper, creases emanate from center

Motivation: local foldability around a vertex of a general crease pattern

- crease pattern = sequence of angles $\theta_1, \theta_2, \ldots, \theta_n$
  normally, $\theta_1 + \theta_2 + \cdots + \theta_n = 360°$
  relax → $\leq 360°$ ⇒ allows cones too
- flat folding = folding of 1D circle (boundary of disk) on the circle



→ similar to 1D foldability — but significant differences!

Characterization of flat-foldable (single-vertex) crease patterns

[Kawasaki 1989; Justin 1989]

- angles θ_i measure travel along circle
- creases switch direction between clockwise & counterclockwise

⇒ total angular motion = $\pm(\theta_1 - \theta_2 + \theta_3 - \theta_4 \cdots + \theta_{n-1} - \theta_n)$

→ n must be even from alternation

$= 0$ because, can't reach around circle (Generally, 360° × winding #)

⇒ $\theta_1 + \theta_3 + \cdots + \theta_{n-1} = \theta_2 + \theta_4 + \cdots + \theta_n = 180°$ for $\Sigma\theta = 360°$.

- in fact, this condition is equivalent to flat foldability:
  - cut an "extreme" vertex in the closed walk
    → get an open walk
  - alternate mountain/valley in open walk
  - rejoin ends

[contrast: trivial for ~~of~~ 1D line segment of paper]

Other differences, for M-V patterns:

- is flat foldable but not "mingling" ← oops. No, not flat foldable.
- mingling ⇏ ∃ crimp: can have circular ···()()()···