6.885
L6.4
Sept. 27, 2004

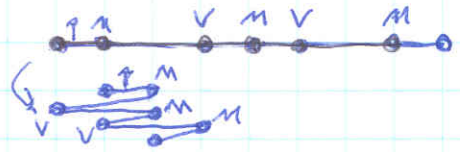
Crease pattern = set of creases i.e. $(d-1)$-D manifolds on P
Flat folding = folded state (f, λ) with $f(P)$ (image) in a d-flat (essentially $\mathbb{R}^d$)

Flat foldable: crease pattern = precisely the creases of a flat folding

Mountain crease = normals point apart
Valley crease = normals point together

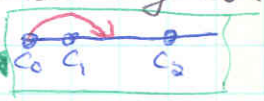
Mountain-valley assignment = map from creases in pattern to $\{M, V\}$
Mountain-valley pattern = crease pattern + M-V assignment

1D flat folding [Arkin et al. 2004]

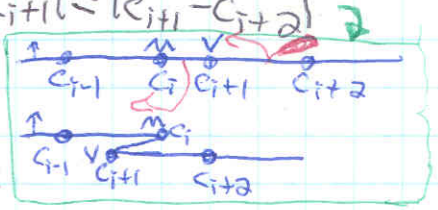


- all crease patterns are flat foldable: accordion / zig-zag fold
- not all mountain-valley patterns:

  - denote creases by $c_1, c_2, \ldots, c_n$; ends by c_0, c_{n+1}
  - end c_0 is **foldable** if $|c_0 - c_1| \leq |c_1 - c_2|$

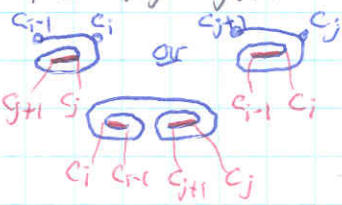


  - creases c_i, c_{i+1} are **crimpable** if one's mountain, other's valley, & $|c_{i-1} - c_i| \geq |c_i - c_{i+1}| \leq |c_{i+1} - c_{i+2}|$



  - both ops. produce smaller M-V patterns
  - M-V pattern is flat foldable
    $\Leftrightarrow$ there's a sequence of crimps & end folds
    $\Leftrightarrow$ **mingling**: for any maximal sequence $c_i, c_{i+1}, \ldots, c_j$ all the same M/V, either $|c_{i-1} - c_i| \leq |c_i - c_{i+1}|$ or $|c_i - c_{i+1}| \geq |c_j - c_{j+1}|$

Proof:
- flat foldable $\Rightarrow$ mingling
  - $c_i, \ldots, c_j$ & incident segments form spiral
  - one end must be short
- mingling $\Rightarrow$ end fold or crimp
  - for each max. $c_i, \ldots, c_j$, write
    - (if "left mingling" i.e. $|c_{i-1} - c_i| \leq |c_i - c_{i+1}|$
    - [else; then) or] for "right mingling"
  - all sequences $\Rightarrow$ $[)[) \cdots [)(] \cdots (](]$ or $(](] \cdots$
    (crimp; end fold)



- crimp/end fold preserves flat foldability
  - convert flat folding to one startable with crimp (or end fold)
- induct $\Rightarrow$ all equivalent $\square$

- linear-time algorithm by maintaining avail. crimps/end folds