- noncrossing: for any nonboundary touching points $p, q \in P$, $f(p) = f(q)$
  - slice local behavior of f around p & q in a d-sphere centered at $f(p)=f(q)$, radius ε
  - → get two $(d-1)$-manifolds on sphere, one for p & one for q
  - → each divides sphere into two halves, labeled + or − according to (nearby) normals

  - $\lambda(p,q) = \pm 1 \Rightarrow$ p's manifold contained in closure of $\pm$ side (half) of q's manifold
  - $\lambda(p,q)$ undefined $\Rightarrow$ ditto (for either + or −)

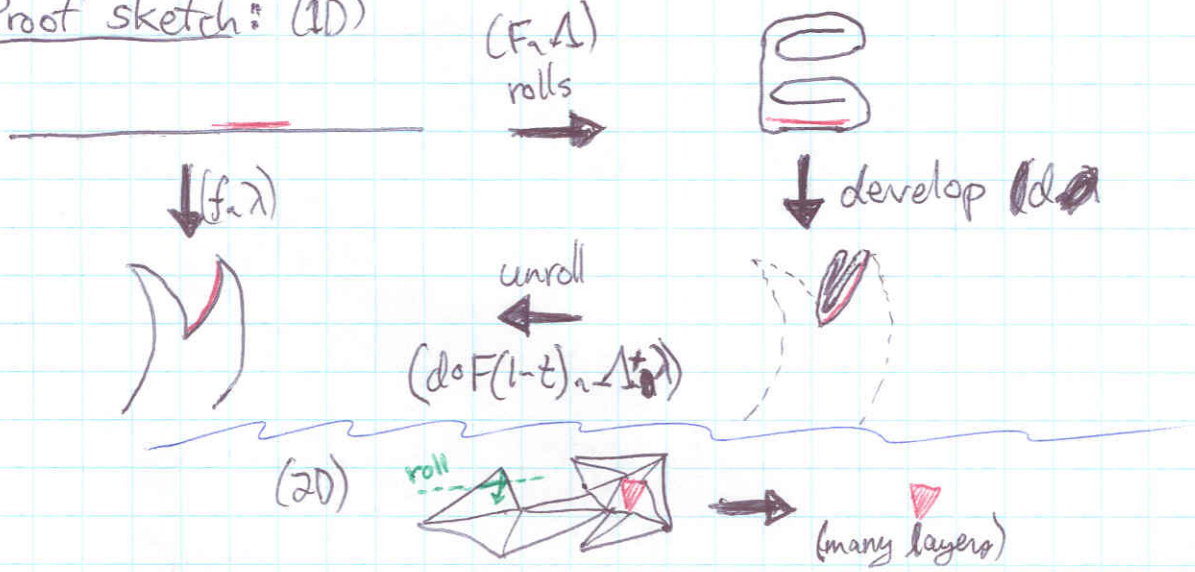
Folded state (f, λ) = geometry f + order λ.

OPEN: nice definition using (one level of?) infinitesimals

Folding motion (F, Λ) = $F: [0,1] \to \{\text{folded-state geometries}\}$ + $\Lambda: [0,1] \to \{\text{folded-state orders}\}$ such that $(F(t), \Lambda(t))$ varies continuously in t
- geometric continuity for F
- no "popping" for Λ. (skipping details)

Connectivity: [Demaine, Devadoss, Mitchell, O'Rourke 2004]
for every (piecewise-C^2) folded state (f, λ) of a polygonal piece of paper P (or line segment of paper P), there is a folding motion starting at $F(0) =$ identity (i.e. P) & ending at $(F(1), \Lambda(1)) = (f, \lambda)$.

Proof sketch: (1D)



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