Infinitesimally locked linkages [Connelly, Demaine, Rote 2002]

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L 5.2
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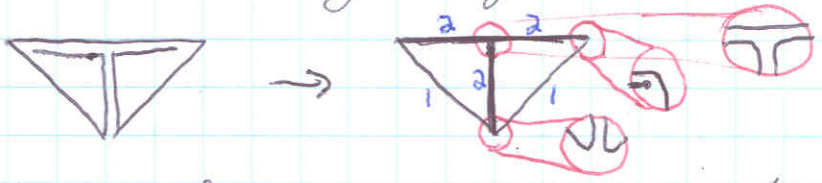
OPEN: characterize locked linkages
e.g. locked trees in 2D
- might be polynomial
- PSPACE-hard to decide reachability between two particular configurations [Alt, Knauer, Rote, Whitesides 2004]

Locked within ε: configuration C whose connected component in config. space (disallowing self-intersection) is contained in a ball of radius ε centered at C.

Idea: look at self-touching linkages (configurations) with infinitesimal (combinatorial) separations



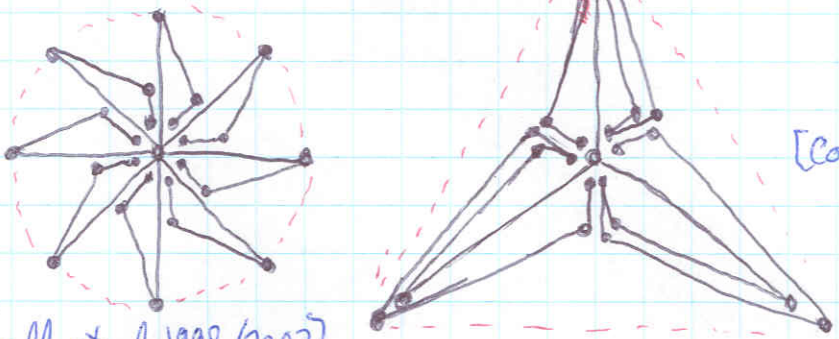
- now the configuration might be rigid (locked within $\emptyset$)
- infinitesimal rigidity:

  - linear constraint: u should remain right of vw
  - not always linear (or convex):
    - ignore such constraints

- can (conservatively) test infinitesimal rigidity using linear programming
- often easy to do by hand with equilibrium stresses
- infinitesimal rigidity ⇒ rigidity
- rigidity ⇒ strongly locked:
  - for any $\varepsilon > 0$ there is a $\delta > 0$ such that δ-perturbations are locked within ε
    - move vertices within δ-disks, preserving combinatorial sidedness

PROJECT: implement locked linkage designer/tester

Eg:



[Biedl et al. 1998/2002]

[Connelly, Demaine, Rote 2002]

OPEN: infinitesimal carpenter's rule theorem? even in 1D!