6.885
L5.1
Sept. 22, 2004

## Locked linkages: recall:

| | chains | trees |
|---|---|---|
| 2D | ✓never locked | →lockable |
| 3D | →lockable | lockable |
| 4D+ | never locked | never locked |

## Algorithms for unfolding 2D chains

① ordinary differential equation given by expansive infinitesimal motion [Connelly, Demaine, Rote 2003]
- each step expensive (convex program)
- need many steps for accuracy

② pseudotriangulations [Streinu 2000]
- each step expensive ~ close ties to rigidity
- $n^{O(1)}$ steps

PROJECT: implement algorithm

③ energy [Cantarella, Demaine, Iben, O'Brien 2004]
- each step $O(n^2)$
- pseudopoly. number of steps → $\text{poly}\left(n, \frac{\text{max edge length}}{\text{min vertex/edge dist.}}\right)$

Idea:
- define energy function on configurations:

$$E(C) = \sum_{\text{edge } \{v,w\}} \sum_{\text{vertex } u \notin \{v,w\}} \frac{1}{d(u, vw)}$$

- expansive motion decreases energy (indeed, each term)
⇒ energy-decreasing motions exist
⇒ downhill gradient of energy exists
- gradient motion avoids crossings
because crossings shoot E to +∞.

↳ easy to compute
↳ final bound on runtime: $O(n^{125} r^{41})$ ☺

OPEN: is minimum-energy configuration unique? what is it? } for closed chains
- known: for equilateral polygons, it's regular

OPEN: $O(n^{O(1)})$ algorithm?