Locked Linkages

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- linkage is locked if its config. space is disconnected
  i.e. there are two configs. unreachable from each other
- summary:

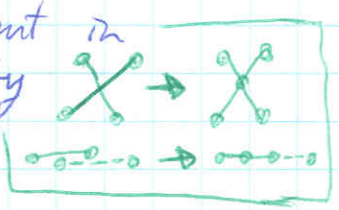
	chains	trees
2D	never locked	lockable
3D	lockable	lockable
4D+	never locked	never locked

Carpenter's rule theorem [Connelly, Demaine, Rote 2000/03]

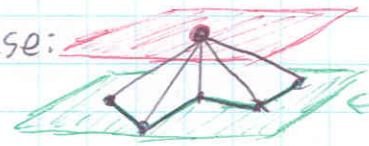
given any configuration of a linkage of max. degree 2, there is a motion that
- straightens/convexifies all outermost open/closed chains
- expansive: distance between any two vertices never decreases
  ($\Rightarrow$ noncrossing by Δ inequality)

Proof sketch:

- consider tensegrity from adding all $\binom{n}{2}$ struts (except where there are already bars)
- tensegrity is infinitesimally flexible
  $\Leftarrow$ every equilibrium stress is zero (on struts)
  $\Leftrightarrow$ every polyhedral lifting is flat (on struts)
  - detail: need to show stresses are equivalent in tensegrity vs. "planarized" tensegrity



- slice hypothetical polyhedral lifting @ maximum z:
  - peak case:



  - convex vertices ≡ mountains ⇒ bars
  - reflex vertices ≡ valleys
  - polygon has ≥3 convex vertices ⇒ bars degree 3 ✗
  - in general:

  ⇒ flat except possibly interior to convex polygons
- piece together infinitesimal motions using (ordinary) differential equation.