Infinitesimal rigidity

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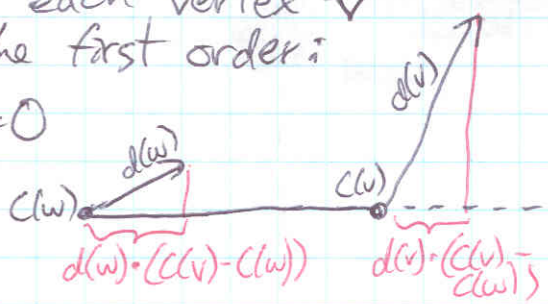
Sept 20, 2004

- **infinitesimal motion** = first derivative (w.r.t. time) of a motion at time 0.

i.e. assign velocity vector $d(v)$ to each vertex v preserving bar lengths to the first order:

$$(C(v)-C(w)) \cdot (d(v)-d(w)) = 0$$



- constraints form linear system:

$$|E| \left\{ \begin{pmatrix} 0\cdots0\ c_x(v)-c_x(w)\ 0\cdots0\ c_y(v)-c_y(w)\ 0\cdots0 \end{pmatrix} \right. \begin{pmatrix} d_x(v_1) \\ d_y(v_2) \\ \vdots \\ d_x(v_n) \\ d_y(v_n) \end{pmatrix} = 0$$

$d|V|$

↳ **rigidity matrix** R

- infinitesimal motions = kernel R = nullspace R
- rank-nullity theorem: rank R + nullity R = #cols. $d|V|$.
  (nullity R = $\dim(\ker)$)
- infinitesimally rigid $\Leftrightarrow$ nullity $R = \binom{d+1}{2}$ ~ rigid motions
  $\Leftrightarrow$ rank $R = dn - \binom{d+1}{2}$

(so all about rank)

Generic means all nonzero minors of R (complete graph K_n)
(induced square submatrices on subsets of rows & cols.)
have nonzero determinant
($\Rightarrow$ rank doesn't decrease artificially)

- at generic configurations, rigidity $\Rightarrow$ inf. rigidity
- if any realization is inf. rigid, then G is generically rigid
- flexibility $\Rightarrow$ inf. flexibility i.e. inf. rigidity $\Rightarrow$ rigidity
  but not vice versa: