$O(nm)$ algorithm

($n = |V|$, $m = |E|$)
[Jacobs & Hendrickson 1997]

6.885
L3.6
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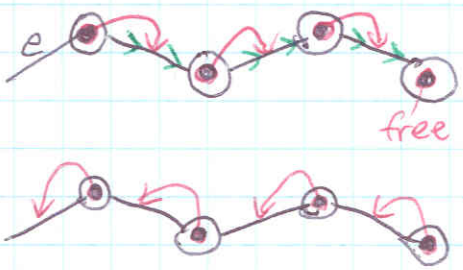
Equivalent formulation of Laman:

- G is min. gen. rigid $\Leftrightarrow |E| = 2n-3$ & edges in E are "indep."
- edges in E are independent
  $\Leftrightarrow \forall$ edge $\{v,w\}$ $\forall$ induced subgraph S of G
  $|E(S')| \leq 2|V(S')|$ where $S' = S + \{v,w\}$ (quadruple)

Algorithm:

- repeatedly try to add an edge to indep. set of edges ($O(m)$ outer loop over E)
- give each vertex 2 pebbles
- vertex can assign each pebble to any incident edge
- to see whether e can be added:
  - add 4 copies of e ~ unpebbled
  - try to place a pebble on all edges by searching for directed path from endpt. of e to a free pebble

$O(n)$ time



directed graph associated with pebbling

  - succeed if and only if e is independent from edges already

(by equiv. characterization — only need to check $\{v,w\} = e$)

↳ YES: add to set of indep. edges
↳ NO: discard as redundant

- there is a similar $O(n^2)$ algorithm based on bipartite matching [Hendrickson 1992]

OPEN: is there a faster algorithm? $O(m \text{ polylog})$? $O(m\sqrt{n})$? ← faster for sparse graphs

Ideas: dynamic graph connectivity; better matching algorithms