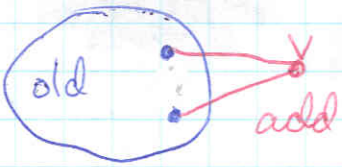
Henneberg characterization [Henneberg 1911]

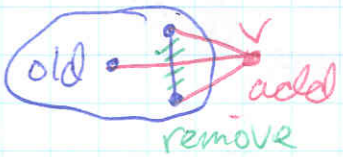
6.885
L3.4
Sept. 15, 2004

G is minimally gen. rigid in 2D if & only if G can be built from ⊶ using two ops:

Type I:



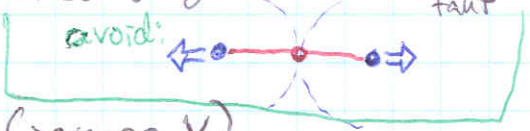
Type II:



true in higher dimensions as well

Lemma: G is generically rigid ⇔ G′ resulting from Type I mod. is generically rigid

Proof: - if G is flexible
then v can come along for the ride in that nontrivial motion (for a while) – because generic so v isn't taut



- if G′ is flexible
then restrict motion onto G (ignore v)
- nontrivial because original motion was & because v is rigidly attached □

Lemma: G is min. gen. rigid ⇔ G′ from Type I is

Proof: similar argument.

Lemma: G is [min.] gen. rigid ⇔ G′ from Type II is

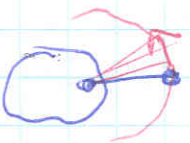
Proof: Harder. (Need infinitesimal rigidity I think.)

Proof of Henneberg:

(⇐) immediate from lemmas because ops. preserve min. gen. rigidity

only in 2D (or 1D)

(⇒) we showed $|E| = 2n-3$.
so average degree $= \frac{2|E|}{n} = 4 - 6/n < 4$.
⇒ there is a vertex of degree < 4.
no vertices of degree 1 (not rigid)



if degree 2: remove it (anti-Type I)
still min. gen. rigid by lemmas induct
if degree 3: ditto (with anti-Type II) □