Dimensionality / degree-of-freedom analysis

6.885
L3.3
Sept. 15, 2004

minimally generically rigid graph in d-D has exactly $dn - \binom{d+1}{2}$ edges

($2n-3$ for 2D)

Proof: Without bars, config. space has dn dimensions.

Each bar-length constraint either
- changes nothing (algebraically dependent)
- decrease dimension by 1 (algeb. independent)
  - ↳ only in degenerate cases can dimension decrease by 2 or more (in some places)
    - surfaces touching in lower-dim. regions

(bar-length constraint: polynomial surface, $dn-1$ dims.)

Rigid once config. space reaches $\binom{d+1}{2}$ dimensions

(space of rigid motions)