Rigidity theory

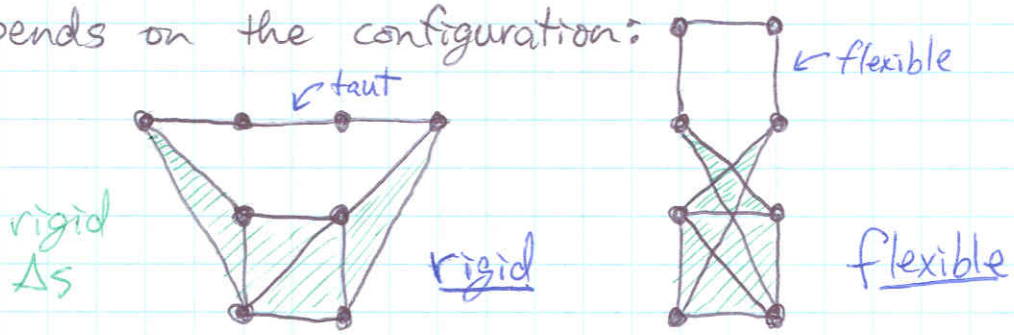
6.885
L3.1
Sept. 15, 2004

Review (simplified — no pinning)

Linkage (G, ℓ) where $G=(V,E)$ is a graph
and $\ell: E \to \mathbb{R}^{\geq 0}$ assigns edge lengths

Configuration $C: V \to \mathbb{R}^d$ such that $\|C(v)-C(w)\| = \ell(v,w) \quad \forall \{v,w\} \in E$

- a configuration of a linkage is **flexible** if there is a nontrivial motion [redacted] of the linkage starting at that configuration
  - ↳ positive duration
  - ↳ not a rigid motion (translation + rotation)
- otherwise **rigid** — isolated point in configuration space
- depends on the configuration:



(in particular, # degrees of freedom is a local property)

OPEN: complexity of deciding rigidity

- topological universality ⇒ checking for isolated point in a general algebraic set
- deciding whether (max.) dimension (intrinsic) of an algebraic set is $\geq d$ is $NP_{\mathbb{R}}$-[redacted] complete [Koiran 1999]
  (& NP-complete if inputs are integers)

$\Rightarrow$? $NP_{\mathbb{R}}$-complete

($NP_{\mathbb{R}}$ defined in Blum, Shub, Smale 1989)