6.885
L22.2
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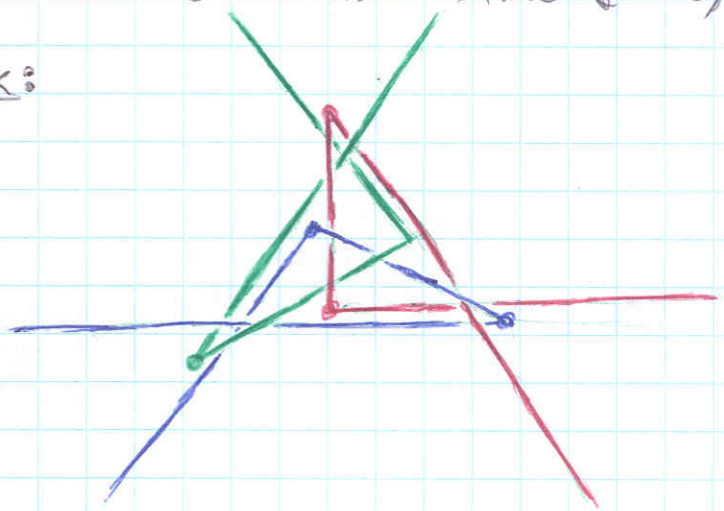
No finite set of 2-chains can interlock

... even if 2-chains are held rigid!

- explosion motion: [Dawson 1984, de Bruijn 1954]
  - scale every point $p \in \mathbb{R}^3$ to $(1+t)p$, time $t \geq 0$.
  - resulting config. is non-self-intersecting (affine transform) but lengths are wrong — all too long
  - shorten bar lengths to original lengths, at all times t
  - subset of original $\Rightarrow$ non-self-intersecting
  - separate as $t \to \infty$
- basic property we're using: 2-chains are starshaped sets

Two 3-chains can't interlock

... even with finitely many 2-chains added in

- perturb chains so that no nonincident bars are coplanar & no three vertices are collinear
- find plane parallel to two middle bars such that middle bars are on opposite sides
- re-orient to make it the xy plane
- explode via $(x, y, z) \mapsto (x, y, (1+t) \cdot z)$ $\quad t \geq 0$.
- middle bar lengths preserved
- other lengths increase $\rightsquigarrow$ trim appropriately
- avoid self-intersection
- 3-chains separate vertically
- all chains separate if initial z coords. distinct (perturb)

Three 3-chains can interlock: