Interlocked 3D chains [Demaine, Langerman, O'Rourke, Snoeyink 2002 & 2003]

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- know smallest 3D chain has 5 bars } [Cantarella & Johnston 1996]
  - 6 bars if chain is closed
- can we use smaller chains if we have more than one?
- what is the smallest "interlocked" pair of chains?
  - two-dimensional optimization

Interlocked set of chains = cannot be separated arb. far by motion avoiding self-intersection

Motivation: Lubiw's problem [2000]

OPEN: what is the minimum number of vertices that need to be "cut" to make a worst-case chain with n bars not locked?

- certainly $\geq \lfloor \frac{n-1}{4} \rfloor$ by repeated knitting needles:

- $\lfloor \frac{n-1}{4} \rfloor$ cuts guarantees that each piece is not locked but two or more pieces might be interlocked...
- best upper bound: $\lfloor \frac{n-3}{2} \rfloor$

Main results:

		OPEN CHAIN			CLOSED CHAIN		
		2	3	4	3	4	5
OPEN chain	2	sep.	sep.	sep.	sep.	sep.	sep.
	3	sep.	sep.	interlock	sep.	interlock	interlock
	4	sep.	interlock	interlock	interlock	interlock	interlock

OPEN: smallest k for which 2-chain interlocks with k-chain?

- $k \leq 16$ [Glass, Langerman, O'Rourke, Snoeyink, Zhong 2004]

OPEN: triples or larger groups of chains.

- three open 3-chains can interlock