All-layers simple foldability

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- view as string folding problem:
  $S = l_0\ d_1\ l_1\ d_2 \cdots d_{n-1}\ l_{n-1}\ d_n\ l_n$
  (indices: l_0 = 1, d_1 = 2, l_1 = 3, d_2 = 4, d_{n-1} = $2n-2$, l_{n-1} = $2n-1$, d_n = $2n$, l_n = $2n+1$ ← index)
  - each $d_i \in \{M, V\}$
  - each $l_i \in \mathbb{R}^+$ ~ in grid cases, $\le \max\{\text{width}, \text{height}\}$
  - N = # symbols = $2n+1$.
- allowable fold at index i (crease $c_{i/2}$)
  = don't "cover up" any creases with uncreased length & creases "match up" in direction when folded
  $\equiv S[i-x] = \overline{S[i+x]}$ for all $1 \le x \le \min\{i-1, N-i\}$.
  ↳ $\overline{M} = V$, $\overline{V} = M$, $\overline{l} = l$
  except that $S[1]$ & $S[N]$ (end lengths) can be shorter than their complements. [$x = \min\{i-1, N-i\}$]

NECESSARY
- any all-layers simple folding consists only of allowable folds (otherwise you make a crease unusable for future)

SUFFICIENT
- any allowable fold preserves all-layers simple foldability
  i.e. S is simply foldable
  $\Leftrightarrow$ result of allowable fold at i is simply foldable

($\Rightarrow$) ↳ this is a substring of original ⇒ easier to fold (any simple folding of S induces folding of substring)

($\Leftarrow$) trivial – concatenate fold at i with remaining folds)

⇒ simple $O(n^3)$ algorithm
- $O(n)$ to test whether i is allowable
- $O(n^2)$ to find an allowable fold
- $O(n^3)$ to find a sequence or get stuck (simple folding)