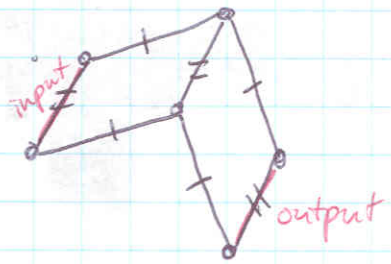
Kempe's proof (cont'd)

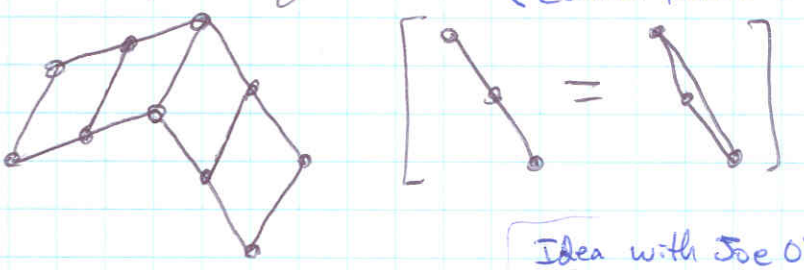
6.885
L2.4
Sept. 13, 2004

Translator: two parallelograms
- opposite edges remain parallel
- make adjacent edges long for reach



mostly works... but:

Bug: [Kapovich & Millson]
- parallelograms can flip into contraparallelograms via degenerate configuration (get more curves than desired)
- fix for translator: bracing



- **OPEN:** easy fix for contraparallelograms?

Idea with Joe O'R: cable from rigid Δ prevents crossing...

- complicated fixes by Kapovich & Millson 2002; King 1999 (because they prove a more general result)

OPEN: number of bars required in Kempe construction?
- Gao & Zhu & Chou & Ge claim $O(n^4)$ — but this may be just for bounded-degree polynomials
- obvious upper bound is exponential

OPEN: can you do better?
- are the other proofs better?

OPEN: without crossings? [posed by Don Shimamoto]

→ also generalize to surfaces in 3D (but beware Kempe bug!)

OPEN: curves in 3D, etc.

Topological generalization: [Kapovich & Millson 2002; King 1999; Thurston?]
for any compact real algebraic set, (solution to set of polynomial equations in many unknowns (very general))
there is a linkage whose config. space is a union of finitely many pieces each analytically isomorphic to given set

↳ mapping & inverse have power-series expansions (local)