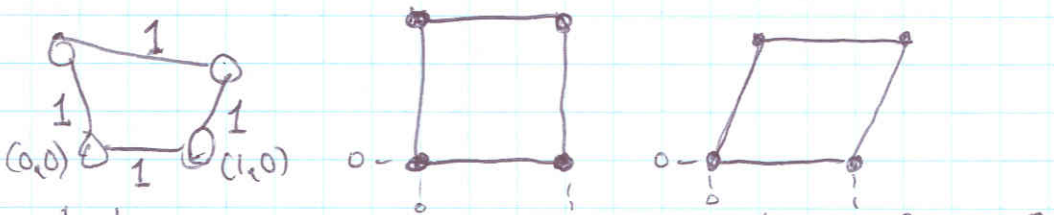
6.885
L2.1
Sept. 13, 2004

Linkage (G, ℓ, f)
= graph $G=(V,E)$
+ edge length assignment $\ell: E \to \mathbb{R}^{\geq 0}$
+ fixing of pinned vertices f: (subset of V) $\to \mathbb{R}^d$

Configuration C of linkage (G, ℓ, f) in $\mathbb{R}^d$
= vertex coordinate assignment $C: V \to \mathbb{R}^d$
consistent with f on pinned vertices (dom f)
& satisfying edge lengths:
$\|C(v) - C(w)\| = \ell(v,w)$ for all $\{v,w\} \in E$

Example:



linkage — a config. in $\mathbb{R}^2$ — another config. in $\mathbb{R}^2$

Configuration can be viewed as a (special) point in $\mathbb{R}^{dn}$, $n=|V|$
– number the vertices $v_1, v_2, \ldots, v_n$
– $C \equiv (\underbrace{\cdot\ \cdot}_{\text{coords. for } v_1}, \underbrace{\cdot\ \cdot}_{\text{coords. for } v_2}, \ldots, \underbrace{\cdot\ \cdot}_{\text{coords. for } v_n})$

Configuration space of linkage (G, ℓ, f)
= subspace of $\mathbb{R}^{dn}$ of points corresponding to configurations of (G, ℓ, f)

Motion of linkage = path/curve in configuration space

Example: $n=4$, $d=2$ $\Rightarrow$ config. space lives in 8D
really only 4 of those dimensions matter
(other 4 are fixed by pinning)
essentially just a circle of motions

LOCAL PROPERTY

Degrees of freedom = intrinsic dimension of config. space
typically: $d \cdot$ (# nonpinned vertices) – # bars
(but need to ignore "extraneous bars" — more later)

Trace of vertex v_i = projection of config. space down to coordinates for v_i
i.e. everywhere v_i can get in configurations