6.885
L17.3
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Canonical configuration for ($\leq\alpha$)-chains

- direction $d(v_i, v_{i+1}) = \frac{v_{i+1} - v_i}{\|v_{i+1} - v_i\|}$
  of an edge (v_i, v_{i+1}) is a point on unit sphere
- choice of directions + v_0 determines configuration
- canonical config. puts $d(v_i, v_{i+1})$ on cone $C_{\alpha/2}$
  i.e. circle of radius $\alpha/2$ ($< 180°$)

- place $d(v_0, v_1)$ at point with max. x coord.
- place $d(v_i, v_{i+1})$ relative to $d(v_{i-1}, v_i)$:
  intersection of circle of radius θ_i = turn angle at v_i
  centered at $d(v_{i-1}, v_i)$ on sphere
  with $C_{\alpha/2}$ circle on sphere
  ↳ take ccw first intersection
- intersection exists because $\theta_i \leq \alpha$
  radius of first circle ↗ ↖ diameter of second circle
- a kind of spiral ~ similar to helix.

- contained in cone $C_{\alpha/2}$: by induction

- in fact, strictly inside cone $C_{\alpha/2}$ except for first link
  because $d(v_0, v_1) \neq d(v_1, v_2)$
  (indeed, $d(v_i, v_{i+1}) \neq d(v_{i+1}, v_{i+2})$)