Angle limitations in β-producible chains

- when v_{i+1} is inside cone, (times between i & $i+1$)
  v_i is outside cone & (v_i, v_{i+1}) touches apex
  $\Rightarrow$ v_i is in complementary cone B_β
  - ↳ if $\beta \le 90°$, B_β = reflection of C_β thru xy
  - ↳ if $\beta \ge 90°$, B_β = complement/exterior of C_β & v_{i+1} is in xy-reflection of B_β

$\Rightarrow$ v_i, v_{i+1} on opposite sides of double-cone of half-angle $\min\{\beta, 180°-\beta\}$

- at time $i+1$ when v_{i+2} is created inside C_β, both (v_i, v_{i+1}) and (v_{i+1}, v_{i+2}) are in the double-cone, because v_{i+1} is about to leave C_β.

$\Rightarrow$ turn angle at $v_{i+1} \le 2 \min\{\beta, 180°-\beta\}$.

$\Rightarrow$ max turn angle α in β-producible chain $\le 2 \min\{\beta, 180°-\beta\}$.
i.e. $\alpha/2 \le \beta \le 180° - \alpha/2$

($\le\alpha$)-chain = chain with max. turn angle $\le \alpha$