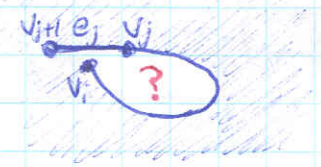
General gluing dynamic program [Lubiw 2000] (also [Hirata 2000])

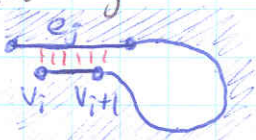


- now two types of subproblems:
  - chain $v_i \cdots v_j$, with v_i glued to v_j
  - chain $v_i \cdots e_j$, with v_i glued to a point along the edge $e_j = (v_j, v_{j+1})$

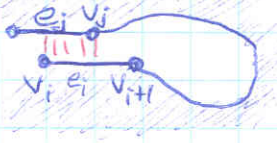
  [- symmetrically: chain $e_i \cdots v_j$]
- answer to subproblem = list of options, each specifying:
  - total angle of material glued at v_i
  - in $v_i \cdots e_j$ case, interval $\subseteq (0, |e_j|)$ of points along e_j to which v_i can glue in this option
  - combinatorial type of gluing ($v_5 \hookrightarrow e_7 \to v_6$, etc.)
- to solve subproblem $v_i \cdots e_j$ (say), two cases:

  ① zip: locally glue interval after v_i to interval before point on e_j to which v_i glues, with nothing else gluing in
  - stop at a vertex ~ 2 choices:

    Ⓐ try zipping all of (v_i, v_{i+1}) & recurse on $v_{i+1} \cdots e_j$
    ↳ v_i's interval along e_j = v_{i+1}'s interval, offset by $+|e_i|$, capped at $|e_j|$

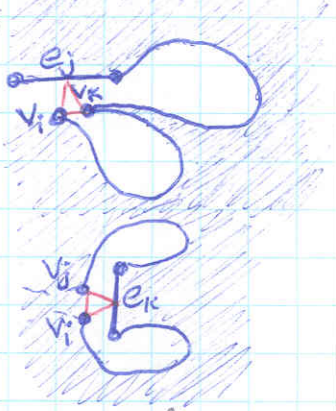
    

    Ⓑ try zipping e_j all the way to v_j & recurse on $e_i \cdots v_j$
    ↳ v_i's interval along e_j = v_j's interval along e_i, capped at $|e_j|$

  - total material = $\theta_i + 180°$

  ② tug: glue another vertex v_K against v_i (together with point along e_j)
  - consider smallest k ($> i$) where this happens
  - ⇒ recursively try to zip $v_i \cdots v_K$
  - also recurse (zip or tug) on $v_K \cdots e_j$
  - v_i's interval along e_j = v_K's interval
  - add θ_i to total material



- for $v_i \cdots v_j$ subproblem, tug can also glue in a point along an edge e_K against v_i
- time = $O(n^3) \times$ # gluings at any stage = $\begin{cases} 2^{O(n)} \\ n^{O(1)} \end{cases}$ in general for bounded sharpness