6.885
L14.2
Nov. 1, 2004

Bounded sharpness = every polygon angle $\leq 360° - \varepsilon$ for some $\varepsilon > 0$ indep. of n

i.e. reflex angles aren't too sharp

- prevents sharp star example with $360° - O(\frac{1}{n})$

- now every leaf in gluing tree introduces $\geq \varepsilon$ curvature
  (fold points introduce $180°$ — assume $\varepsilon \leq 180°$)
- $720°$ total curvature $\Rightarrow O(1/\varepsilon)$ leaves

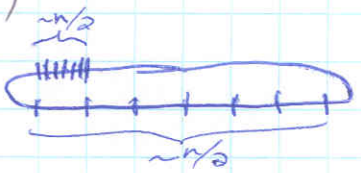
Bound on # gluings in terms of L = # leaves in gluing tree

- $< L$ internal junctions of degree ≥ 3
- $2^{O(L)}$ unlabeled trees on these junctions & the leaves
- $n^{O(L)}$ assignments of vertices & edges to these junctures
- between 2 vertex junctures, measure out length to find all vertices in between

- trouble: fold points (edges & leaves)
  $\Rightarrow$ cycles where geometry is not "nailed down"
  (really happens e.g. with rolling belts)
  - $O(n^2)$ possible combinatorial types for each cycle
    ($O(n^2)$ events during "rolling")
  - $\Rightarrow n^{O(1)}$ possibilities for all (≤ 4) such cycles

$\Rightarrow n^{O(L)}$. (probably actually $\binom{O(n)}{O(L)}$, which is $2^{O(n)}$)

- in fact, $O(n^{2L-2})$ & $O(n^4)$ for $L = 4$ & $O(n^2)$ for $L \leq 3$

OPEN: better bounds? (for $L \geq 4$)

- $\Theta(n^2)$ tight for $L = 2$ (rolling belt)