6.885
L13.1
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Perimeter halving: every convex polygon P has an Aleksandrov gluing
- pick any point x on P's boundary
- let y be the point on P's boundary (diametrically opposite) such that the two boundary paths from x to y have the same length
- glue these two paths together:
  glue together the two points @ distance d from x, $\forall d$ (measured along P's boundary)

→ glue together 2 points, each with angle $\leq 180°$.
(& at x, y, glue together only one point)
⇒ Aleksandrov ⇒ forms unique convex polyhedron

EXPERIMENT: cut out a convex polygon
tape along a perimeter-halving gluing
see what you get

Mostly different: varying x, get uncountably many distinct polyhedra
- vary x along some edge (v_i, v_{i+1}), in open interval $(v_i, \frac{v_i + v_{i+1}}{2})$

⇒ both x & v_i are vertices
⇒ vertex-vertex shortest path takes on uncountable values
- if all these polyhedra that result are countable,
  each has a finite number of vertex-vertex shortest-path distances,
  so total number of these distances is countable,
  a contradiction ⨳

OPEN: can more than a finite number of polyhedra resulting from perimeter halving of P be equal?