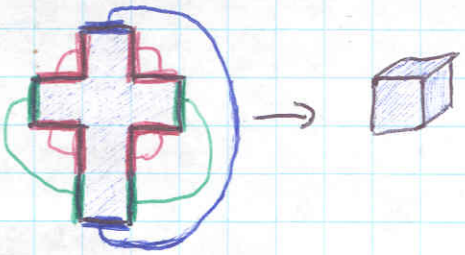
# Folding polyhedra



6.885
L12.12
Oct. 25, 2004

**Decision problem:**

Given a polygon (or any (connected, metrizable) polygonal 2-manifold)
can its boundary be glued
(in a perfect pairing of open intervals of boundary
whose closures cover the whole boundary)
such that the resulting (boundaryless) manifold can be folded
into exactly a convex polyhedron?
↳ no double covering / multiple layers like origami

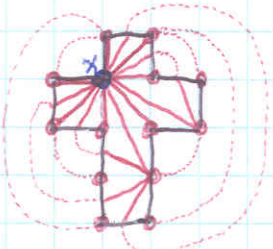
**Enumeration problem:** if yes, list them all

**Combinatorial problem:** how many can there be?

**OPEN:** if the goal is any nonconvex polyhedron,
perhaps homeomorphic to a sphere,
is the answer YES for all polygons? [O'Rourke 2004]

**It's all about the gluing...**

- polygon + gluing induces a metric by shortest-path lengths between all pairs of points
  - metric is polyhedral:
    all but finitely many points
    have zero curvature
    (locally look like disks)
  - metric is convex if
    the finite exceptional points
    have positive curvature
    (locally look like convex cones)




shortest paths from x
to all vertices
given just polygon
+ gluing