6.885
L10.5
Oct. 13, 2004

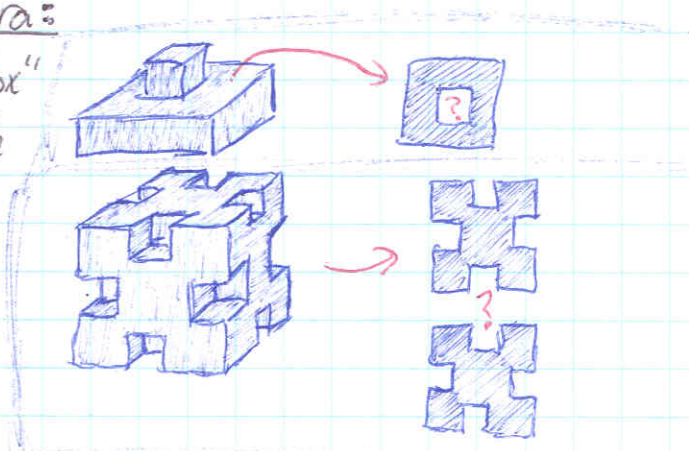
Fewest nets: edge-unfold convex polyhedron with a "small" number of pieces

- conjecture (?): 1
- certainly $\le F$ = # faces (cut out each)
- $\le 2/3 \cdot F$ by pairing together 2/3 of the faces [Spriggs 2003]
- $\le 1/2 \cdot F$ by fancier argument [Dujmović, Morin, Wood 2004]

OPEN: $o(F)$?

Edge unfolding nonconvex polyhedra:

- trivial ununfoldable: "double box" insufficient area in "donut hole"



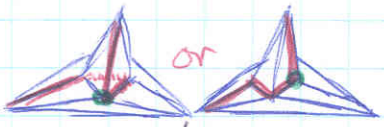
- all faces ~ disks: "cube with bites" can't connect two X's

[Biedl et al. 1999]

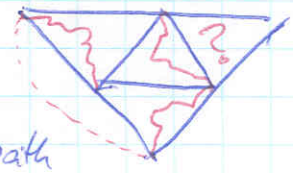
- all faces triangles ($\Rightarrow$ "topologically convex")

[Bern, Demaine, Eppstein, Kuo, Mantler, Snoeyink 2003]

- suppose base vertices of spike have negative curvature without 1 spike Δ
  (floor angle $= 300° - \varepsilon$; spike angle $= 90° - \varepsilon$)
- claim: can't edge unfold a single hat (1-piece)
  - ≥ 2 leaves of spanning, not at neg. curvature
    - $\Rightarrow$ one at peak, one on base
    - 2 possibilities
    - each leaves all but 1 spike Δ at some vertex



$\Rightarrow$ must be a path of cuts from one base vertex to another, interior to each hat

- these force eventual disconnect (4 vertices, 4 Δs)

$\Rightarrow$ if path from spike misses any neg.-curvature vertices, they will need to be spanned, forcing desired path