General unfolding of convex polyhedra

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- consider shortest path from some point x to all other points y on polyhedron
  - each shortest path unfolds straight (geodesic), doesn't cross itself, and doesn't pass through pos.-curvature vertices
  - if two shortest paths touch elsewhere than x, then either one is a subpath of other (touch everywhere) or they touch only at termination (non-unique shortest path)
- cut locus / ridge tree = points with non-unique shortest paths from x
  - spanning tree of vertices of polyhedron (in fact, leaves = the polyhedron vertices)

Source unfolding [Sharir & Schorr 1986; Mount 1985; Mitchell, Mount, Papadimitriou 1987]

} discrete version

- cut along cut locus
- unfold shortest paths from x → straight
- ⇒ star-shaped polygon around x
- each ray from x is an unfolded shortest path

Star unfolding [Aleksandrov 1948; Aronov & O'Rourke 1992]

- cut along shortest paths from x to all vertices
  - ⇒ star of cuts
- much harder to prove nonoverlap

generic

OPEN: other general unfoldings?

Extensions:

- neither works for nonconvex polyhedra
- source unfolding works for higher dimensions [Miller & Pak 2003]
- source unfolding can be continuously "bloomed" w/o intersection [Langerman et al. 2004]

OPEN: other unfoldings, e.g., star? PSPACE-hard for nonconvex polyhedra?