6.885
L 10.2
Oct. 13, 2004

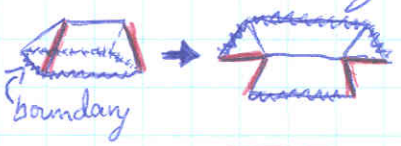
Curvature of a vertex = $360° - \sum$ incident face angles

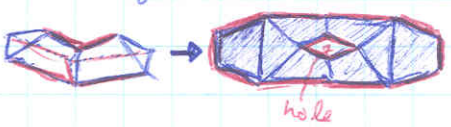
- locally:
  - positive ⇒ cone →
  - zero ⇒ flat →
  - } can be convex
  - negative ⇒ saddle →
  - } always non-convex

General properties of unfoldings & corresp. cuttings

- only zero-curvature vertices can be locally flattened
  ⇒ any cutting spans all nonzero-curvature vertices
  - in fact, curvature $< -k \cdot 360°$ ⇒ span with degree $> k$

(if polyhedron is homeomo. to sphere or disk)

- one-piece unfolding ⇒ no cycles in cutting
  ⇒ cutting is a spanning forest
- if polyhedron has no boundary & unfolding has no holes then cutting is a spanning tree
  (connected components of cutting = boundary components of unfolding)
- both conditions necessary: boundary ⇒ 

  holes ⇒ 

- convex polyhedron ⇒ cutting is spanning tree

Trivial bad example: [Bern, Demaine, Eppstein, Kuo 1999]
polyhedron with 1 vertex of negative curvature

- need ≥ 2 cuts from vertex
- can only go to boundary ⇒ disconnect (effective cycle)

⇒ no general unfolding