PS2: Practice problems

Answer Answers are shown in green. These answers have not been proofread by anyone but me, so there is substantial chance of error. Please let me know if you have any doubts. -lpk

1 Logic

1.1 Propositional sentences

For each of the sentences below, indicate whether it is valid, unsatisfiable, or neither. If neither, provide an interpretation that makes it true and one that makes it false.

1. $S \rightarrow F$

   Answer It is neither valid nor unsatisfiable. The interpretation $S = T, F = T$ makes it true; the interpretation $S = T, F = F$ makes it false.

2. $S \rightarrow S$

   Answer This is valid; that is, it is true in all interpretations.

3. $B \vee D \vee (B \rightarrow D)$

   Answer This is equivalent to $B \vee D \vee (\neg B \vee D)$, which is valid.

1.2 English to propositional logic

We'd like to formalize the sentence: *A person who is radical (R) is electable (E) if he/she is conservative (C), but is otherwise not electable.* For each of the logical sentences below, indicate whether it is a successful formalization of the English sentence. If it is not, explain why not.

1. $(R \wedge E) \leftrightarrow C$

   Answer This is not equivalent. It says that all (and only) conservatives are radical and electable.

2. $R \rightarrow (E \leftrightarrow C)$

   Answer This one is equivalent.

3. $R \rightarrow ((C \rightarrow E) \vee \neg E)$

   Answer This one is vacuous. It's equivalent to $\neg R \vee (\neg C \vee E \vee \neg E)$, which is true in all interpretations.

1.3 Quantifiers

Consider a finite domain $U = \{a, b, c\}$. For each of the sentences below, provide a sentence without any quantifiers that is equivalent to it in any interpretation with universe U.

1. $\forall x.P(x)$

   Answer $P(a) \wedge P(b) \wedge P(c)$

2. $\exists x.\forall y.Q(x, y)$

   Answer
   $(Q(a, a) \wedge Q(a, b) \wedge Q(a, c)) \vee (Q(b, a) \wedge Q(b, b) \wedge Q(b, c)) \vee (Q(c, a) \wedge Q(c, b) \wedge Q(c, c))$

1.4 English to FOL

Formalize each of these sentences in first-order logic.

1. All countries that border Ecuador are in South America.

   Answer $\forall c.\text{Borders}(c, \text{Ecuador}) \rightarrow \text{In}(c, \text{SouthAmerica})$

2. No two adjacent countries have the same map color.

   Answer $\neg\exists c_1.\exists c_2.\text{Adjacent}(c_1, c_2) \wedge \text{Color}(c_1) = \text{Color}(c_2)$

3. There is a barber who shaves all men, and only those men, who do not shave themselves.

   Answer $\exists b.\text{Barber}(b) \wedge \forall m.\neg\text{Shaves}(m, m) \leftrightarrow \text{Shaves}(b, m)$

For extra fun, figure out whether the last one is satisfiable, and provide a proof.

Answer This is unsatisfiable. We'll prove a contradiction. The barber part is irrelevant. So, the sentence is: $\exists b.\forall m.\neg\text{Shaves}(m, m) \leftrightarrow \text{Shaves}(b, m)$

We haven't really studied formal proof in this class, so I'll do it informally. Don't worry if you don't feel like you know how to do proofs in FOL.

1. First, rewrite the equivalence as two implications (always legal):

$$\exists b.\forall m.(\neg\text{Shaves}(m, m) \rightarrow \text{Shaves}(b, m)) \wedge (\text{Shaves}(b, m) \rightarrow \neg\text{Shaves}(m, m))$$

2. Let's just pick a particular constant name for the b without making any assumptions about him:

$$\forall m.(\neg\text{Shaves}(m, m) \rightarrow \text{Shaves}(\text{Figaro}, m)) \wedge (\text{Shaves}(\text{Figaro}, m) \rightarrow \neg\text{Shaves}(m, m))$$

3. Now, because this has to be true of all m, it has to be true of Figaro:

$$(\neg\text{Shaves}(\text{Figaro}, \text{Figaro}) \rightarrow \text{Shaves}(\text{Figaro}, \text{Figaro})) \wedge$$
$$(\text{Shaves}(\text{Figaro}, \text{Figaro}) \rightarrow \neg\text{Shaves}(\text{Figaro}, \text{Figaro}))$$

4. Rewriting the implications as disjunctions:

$$(\text{Shaves}(\text{Figaro}, \text{Figaro}) \vee \text{Shaves}(\text{Figaro}, \text{Figaro})) \wedge$$
$$(\neg\text{Shaves}(\text{Figaro}, \text{Figaro}) \vee \neg\text{Shaves}(\text{Figaro}, \text{Figaro}))$$

5. Simplifying:

$$\text{Shaves}(\text{Figaro}, \text{Figaro}) \wedge \neg\text{Shaves}(\text{Figaro}, \text{Figaro})$$

6. This is clearly a contradiction. So, it cannot be that there is a barber who shaves all and only the men who do not shave themselves. (Unless she is a woman!)

2 Planning

2.1 Monkey and bananas

A monkey wants bananas, but is too short to grab them. Help him make a plan!

There are three locations. The monkey is at L1, a box is at L2 and the bananas are hanging at L3. The monkey can **move** to any location if he is not elevated. The monkey can **climb up** on top of the box if he is at the same location as the box; if he climbs on the box, he will be elevated. The monkey can **climb down** from the box if he is on it, and then he will no longer be elevated. The monkey can **push** the box to any location if he and the box are at the same location to start with and he is not elevated. The monkey can **grab** the bananas if he is at the same location as the bananas and is elevated, at which point, he will have the bananas.

Write STRIPS or PDDL-like action schemas (syntax isn't important) for the monkey's operations above.

AnswerMove(start, target):

- **Pre:** At(Monkey, start), not Elevated(Monkey)
- **Result:** not At(Monkey, start), At(Monkey, target)

ClimbUp(loc):

- **Pre:** At(Monkey, loc), At(Box, loc), not Elevated(Monkey)
- **Result:** Elevated(Monkey)

ClimbDown():

- **Pre:** Elevated(Monkey)
- **Result:** not Elevated(Monkey)

PushBox(start, target):

- **Pre:** At(Monkey, start), At(Box, start), not Elevated(Monkey)
- **Result:** not At(Monkey, start), not At(Box, start), At(Monkey, target), At(Box, target)

GrabBananas():

- **Pre:** At(Monkey, L3), At(Bananas, L3), Elevated(Monkey)
- **Result:** Holding(Monkey, Bananas), not At(Bananas, L3)

2.2 Regression

The monkey has the goal: $\text{Holding}(\text{Monkey}, \text{Bananas}) \wedge \text{At}(\text{Monkey}, \text{L1})$.

- Which actions are *relevant* to this goal?

  Answer An action is relevant if it can make a fluent in the goal true. So the relevant actions are **GrabBananas()**, **Move(L2, L1)**, and **Move(L3, L1)**. (STRIPS usually implicitly assumes that each of the arguments of an operator has to be different, so we don't have **Move(L1, L1)** as well.)

- Which actions are both *relevant* and *applicable*?

  AnswerGrabBananas() is not applicable, because it has as a precondition **At(Monkey, L3)** which is in contradiction with a fluent in the goal. (Actually detecting that contradiction requires either reasoning outside of STRIPS or putting **not At(Monkey, L1)** and **not At(Monkey, L2)** in the goal. Another strategy is to have a fluent **Loc(Monkey)** that takes on locations as values; this also takes us outside of the realm of basic STRIPS.)

  Only the move actions are both relevant and applicable.

- For each of the actions that are both relevant and applicable, provide the regression of the goal under that action.

  Answer For **Move(L2, L1)**, the regression of the goal is
  Holding(Monkey, Bananas), At(Monkey, L2), not Elevated(Monkey).

  It is similar for the other move operation.

2.3 Graphplan

If you have a goal $G_1 \wedge G_2 \wedge \ldots \wedge G_n$, then one heuristic function to use might be $\sum_i \text{lc}(G_i)$, where $\text{lc}(G_i)$ is the level cost of G_i in the plan graph. Is this heuristic admissible? Why or why not?

Answer It is not admissible, because it might be that the same actions that cause G_1 to be true can cause all of the goal fluents to be true as well.